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A TREATISE ON SURVEYING

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A TREATISE ON SURVEYING, VOLUME I (Instruments
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A

TREATISE ON SURVEYING

Sixth Edition

VOLUME TWO

More Advanced Techniques and Developments

Rewritten under the editorship of

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NOTE

The chapters have been rewritten as follows:

C. D. Reed	1, 6
D. H. Fryer	2, 3, 4, 5, 11, 13
W. V. Shearer	7, 15, Appendix
D. H. Macmillan	8
P. G. Mott	9, 10
S. C. O'Grady	12
R. E. Fryer	14

PLATES

(Between pages 292 and 293)

- I. Zeiss TAL photo-theodolite
- II. Wild E2 rectifying enlarger
- III. Cutting a slotted template
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- V. Mirror stereoscope with parallax bar
- VI. Multiplex plotter
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PREFACE TO THE SIXTH EDITION

DEVELOPMENT in new materials, improvement in optical devices and the rapid increase in the number of electronic instruments available to the scientist have all had important effects on the craft of surveying. This new edition of an old and respected work has been compiled by a number of authors—all expert in their chosen field—who, while trying to maintain the carefully detailed advice to the reader which has always been a feature of this work, have widened the scope of their presentation to include the most recent developments.

The result is not intended merely as a textbook for the beginner or the student engineer, but is also meant to assist the experienced surveyor who has not found time to follow the rapid changes in technique so noticeable in the surveying field in the war and post-war periods. Much of the new work covered cannot be found easily except by a search through the published literature, and several authors have, from their wide experience, provided original solutions to specific surveying problems.

The first volume of *A Treatise on Surveying* deals with the elementary aspects of surveying as applied in practice in the United Kingdom or for small surveys overseas, and covers the first-year syllabus of the universities and engineering colleges, where survey extends over two years. Volume II is designed for the use of civil engineers and land surveyors in Great Britain and overseas and for the preparation of civil-engineering students for final degree papers. It covers also, with Volume I, the Intermediate Examinations of the Royal Institute of Chartered Surveyors, except for one or two special aspects of survey work such as land registration which require specialized study.

W. FISHER CASSIE
General Editor

NEWCASTLE UPON TYNE
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CHAPTER I

ERRORS AND ADJUSTMENTS OF SURVEY OBSERVATIONS

IN ORDER to understand the correction of observed quantities in surveying it is necessary to consider the causes of inaccuracy. Careless observation, booking or computation leads to discrepancies which are classified as *mistakes*. Common mistakes include misreading of verniers, incorrect counting of the total number of chain lengths, mistaking the foot on the levelling staff, booking numbers incorrectly, and numerous other slips. Such mistakes cannot possibly be rectified except by a complete repetition of the work involving the mistake, and the only safeguard is the careful double checking of every reading and booking and a cross check on all calculations.

ERRORS IN OBSERVATIONS

If mistakes are entirely eliminated, discrepancies would still exist between observed and true values. These discrepancies or residuals are due to unavoidable inaccuracies of instrument or observation and are classified as *errors*. Errors can be subdivided into two distinct categories, (i) systematic errors and (ii) accidental errors.

SYSTEMATIC ERRORS. Systematic errors are non-compensating and cumulative. They always tend to distort the true values in the same direction, so that the errors are all positive or all negative. Systematic errors include such conditions as the steel band of incorrect length and the observer who always tends to read the vernier slightly too high. This type of error can easily be dealt with as its correction is a simple multiple concerning the number of observations taken. For example, suppose a 100 ft. steel band is $\frac{1}{8}$ in. short, then in a distance of 800 ft. the cumulative error would be $8 \times \frac{1}{8}$ in. or 1 in.

ACCIDENTAL ERRORS. Accidental errors tend to compensate one another, and it is equally likely that they are positive or negative. Accidental errors arise from many causes. One source which affects all types of measurement is defective vision, which is liable to give either positive or negative errors through a combination of imperfect estimation of readings and imperfect setting of instruments entirely beyond the control of the observer. Other sources of accidental error include the limited graduation of instruments, small momentary changes of temperature and parallax.

Accidental errors cannot be eliminated by cross-checking the calculations, double-checking the readings or the simple adjustment of the observations as in the case of systematic errors. It is obvious that for precise surveying some adjustment must be made to such accidental errors, and the observations are therefore adjusted to give the result which is most probably correct. In order to carry out this adjustment it is necessary to consider the basic principles of the law of error of observed quantities.

Suppose that $X_1, X_2, X_3 \dots X_n$ are n observations of a quantity x . If these observed values vary, then the observations are not perfectly correct, and the errors $E_1, E_2, E_3 \dots E_n$ of the various observations are related to the value of x by the following n equations:

$$\begin{aligned} x - X_1 &= E_1 \\ x - X_2 &= E_2 \\ x - X_3 &= E_3 \\ &\dots\dots\dots \\ x - X_n &= E_n \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This gives $n + 1$ unknown ($E_1, E_2, E_3 \dots E_n$ and x) and only n equations, and it is therefore necessary to involve another condition in order to complete the solution.

METHOD OF LEAST SQUARES. It is known that in a large series of observations of a quantity the accidental errors involved conform to certain rules:

1. They are not systematic, i.e. positive and negative errors have equal frequency.
2. Small errors occur more frequently than large errors, and very large errors do not occur at all.

In other words, the true value of the quantity must lie somewhere within the range of the observed values, and it will be found that the majority of the observations group near the midpoint of the

range. Therefore, having regard to the fact that the errors are likely to be equally positive and negative, the most probable value of x is the one which is symmetrical to all the observations. The arithmetic mean is symmetrical to the observed values. Let the arithmetic mean be x_o , then $x_o = \frac{\Sigma(X)}{n}$ (2)

Add the n equations (1) to give

$$\begin{aligned} nx - \Sigma(X) &= \Sigma(E) \\ \therefore x &= \frac{\Sigma(X)}{n} + \frac{\Sigma(E)}{n} \\ &= x_o + \frac{\Sigma(E)}{n} (3) \end{aligned}$$

Since the most probable value of x is centrally situated in the range of observation $\Sigma(E)$ tends to be very small, and if n is large $\frac{\Sigma(E)}{n}$ tends to zero and therefore $x = x_o$. That is, the arithmetic mean is the most probable value of the number of observations provided the observations are taken with equal precision.

The difference between the most probable value of the quantity and the observed value is called the residual error or residual. Denote the residuals by $r_1, r_2, r_3 \dots r_n$, then

$$\begin{aligned} x_o - X_1 &= r_1 \\ x_o - X_2 &= r_2 \\ x_o - X_3 &= r_3 \\ &\dots\dots\dots \\ x_o - X_n &= r_n (4) \end{aligned}$$

Add the n equations (4) to give

$$nx_o - \Sigma(X) = \Sigma(r)$$

But from (2) the left-hand side of this equation is zero and therefore

$$\Sigma(r) = 0$$

Consider any value of the observed quantity other than the arithmetic mean. Let this value be X_o and let the residuals to this value be given by the equations

$$\begin{aligned} X_o - X_1 &= R_1 \\ X_o - X_2 &= R_2 \\ X_o - X_3 &= R_3 \\ &\dots\dots\dots \\ X_o - X_n &= R_n (5) \end{aligned}$$

Square equations (4) and add together

$$nx_o^2 - 2x_o\Sigma(X) + \Sigma(X^2) = \Sigma(r^2) \quad . \quad . \quad . \quad (6)$$

Square equations (5) and add together

$$nX_o^2 - 2X_o\Sigma(X) + \Sigma(X^2) = \Sigma(R^2) \quad . \quad . \quad . \quad (7)$$

Hence

$$\begin{aligned} \Sigma(R^2) &= nX_o^2 - 2X_o\Sigma(X) + \Sigma(r^2) + 2x_o\Sigma(X) - nx_o^2 \\ &= \Sigma(r^2) + nX_o^2 - 2X_o\Sigma(X) + \frac{2\Sigma(X)}{n} \cdot \Sigma(X) - n \frac{\Sigma(X)}{n} \cdot \frac{\Sigma(X)}{n} \\ &= \Sigma(r^2) + n \left[\frac{\Sigma(X) \cdot \Sigma(X)}{n^2} - \frac{2X_o\Sigma(X)}{n} + X_o^2 \right] \\ &= \Sigma(r^2) + n \left[\frac{\Sigma(X)}{n} - X_o \right]^2 \end{aligned}$$

Now $\left[\frac{\Sigma(X)}{n} - X_o \right]^2$ is always positive

Hence $\Sigma(R^2) > \Sigma(r^2)$

But R is the residual found by taking any value X_o of the quantity, so that the sum of the squares of the residuals r , taking the arithmetic mean, is less than the sum of the squares of any other residuals R taking a value other than the arithmetic mean. This principle is used in the solution of problems concerning the adjustment of observations and is called the *method of least squares*.

THE CURVE OF PROBABILITY. In order to make a comparison of the relative reliability of a number of observations or the observations of a number of observers, it is necessary to consider the probability of the occurrence of errors of varying magnitude. With carefully made observations, small errors are more likely to occur than large ones, and there will be a limit to the possible size of the error. If the value a is taken as the maximum possible error, then all the errors of observation will lie between $+a$ and $-a$ with a preponderance in the neighbourhood of zero. The probability of the occurrence of an error can be assumed to be a function of the value of the error.

Let the probability that an error lies between zero and E be a function of E equal to $f(E)$

Then the probability of the error being between zero and $E + \delta E$ is equal to $f(E + \delta E)$

So that the probability of the error lying between E and $E + \delta E$ is

$$\begin{aligned} f(E + \delta E) - f(E) \\ = f'(E)\delta E \end{aligned}$$

which may be taken as the probability of occurrence of the error E since δE is small. The function $f'(E)$ is called the law of distribution of error. Hence it follows that the probability that an error does not exceed the possible error a is

$$\int_{-a}^{+a} f'(E) dE.$$

$f'(E)$ can be represented by a curve whose form can be determined from the principles of probability. The probability of the simultaneous occurrence of a complete system of errors, E_1, E_2, E_3 , etc., is the product of the respective probabilities and equals

$$f'(E_1) \cdot f'(E_2) \cdot f'(E_3) \dots \delta E_1 \cdot \delta E_2 \cdot \delta E_3 \dots \quad (8)$$

If this value is made a maximum, the most probable value of the unknown x is derived. Now when this value is a maximum its logarithm is also a maximum. So that if the logarithm of equation (8) is taken, it can be differentiated with respect to x and the result equated to zero, giving

$$\frac{f''(E_1)}{f'(E_1)} \frac{dE_1}{dx} + \frac{f''(E_2)}{f'(E_2)} \frac{dE_2}{dx} + \dots = 0$$

which can be written in the form

$$\frac{f''(E_1)}{f'(E_1)} + \frac{f''(E_2)}{f'(E_2)} + \dots = 0$$

since $x - X_1 = E_1$ etc., giving

$$\frac{dE_1}{dx} = \frac{dE_2}{dx} = \dots = 1$$

So that
$$\frac{f''(E_1)}{E_1 f'(E_1)} \cdot E_1 + \frac{f''(E_2)}{E_2 f'(E_2)} \cdot E_2 + \dots = 0 \quad (9)$$

and from the principle of arithmetic mean if the number of observations is large then

$$E_1 + E_2 + E_3 + \dots = 0 \quad (10)$$

(9) and (10) constitute equations which must be satisfied simultaneously. So the coefficients can be equated to give

$$\frac{f''(E_1)}{E_1 f'(E_1)} = \frac{f''(E_2)}{E_2 f'(E_2)} = \dots = \text{a constant}$$

It can be shown that this constant must be negative since the product of the probabilities was a maximum value, and it is convenient to equate

$$\frac{f''(E_1)}{E_1 f'(E_1)} = \frac{f''(E_2)}{E_2 f'(E_2)} = \dots = -\frac{1}{\sigma^2}$$

So if any value of error E is taken by integration, the probability of occurrence of the error E is given by the equation

$$f'(E) = c e^{-E^2/2\sigma^2}$$

where c is a constant and e is the base of Napierian logarithms (2.71828). It is convenient to assign the value unity to the combined probabilities giving

$$c \int_{-a}^{+a} e^{-E^2/2\sigma^2} dE = 1$$

but, since the value a varies with the particular observation, it is better to take the extreme limits of error $+\infty$ and $-\infty$ giving

$$c \int_{-\infty}^{+\infty} e^{-E^2/2\sigma^2} dE = 1$$

hence

$$c = \frac{1}{\sigma \cdot \sqrt{2\pi}}$$

and

$$f'(E) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-E^2/2\sigma^2} \quad . \quad . \quad . \quad (11)$$

and σ is a quantity expressed in the same units as x .

Equation (11) represents a curve of probability which can be plotted with O as the origin, distances from O along the horizontal axis in either direction representing the errors E positive or negative and the ordinates representing the probability of such an error. The form of the curve is shown in Fig. 1. It will be seen that the curve is symmetrical about the vertical axis through O , indicating that errors of opposite sign have an equal probability. The curve also indicates the maximum probability of having zero or very small error as compared with the small probability of having large errors. The horizontal axis is an asymptote to the curve indicating that infinitely large errors have a probability, while it is known that errors above a certain value are not possible. The curve, however, has such a form that the relative probabilities fall away so rapidly above a certain value of error that for all practical

purposes the small values of probability for large errors may be taken as zero. This slight inconsistency is due to taking the integral limits between $+\infty$ and $-\infty$ instead of $+a$ and $-a$ when calculating the value of the constant c , and gives a more general case.

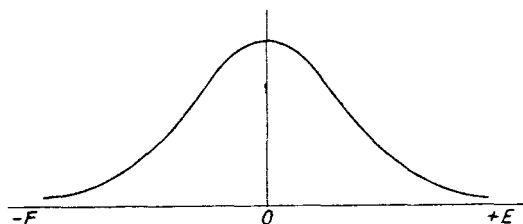


FIG. 1

It is obvious that the quantity σ plays an important part in the shape of the probability curve. As σ increases the curve extends for a greater distance on either side of the origin before becoming effectively in contact with the axis and the probability of zero error decreases. Thus when σ becomes large there is a greater relative probability of large errors. Conversely, for a small value of σ the probability of a zero error increases and large errors are unlikely to occur. The quantity σ is therefore a measure of the extent of possible errors and is known as the *standard error*, and affords a test of the relative accuracy of different series of observations. The value σ is also known as the *mean square error* or the *root mean square*. Since the probability of the occurrence of an error between E and $E + \delta E$ is $f'(E) \delta E$, and the total number of errors is n , the sum of the squares of the errors in the small integral will be $nE^2 f'(E) \delta E$ and the sum of the squares of the errors between the limits of error $+\infty$ and $-\infty$ when n is large is

$$\begin{aligned} \Sigma(E^2) &= n \int_{-\infty}^{+\infty} E^2 f'(E) dE \\ &= n \int_{-\infty}^{+\infty} \frac{E^2}{\sqrt{2\pi} \cdot \sigma} e^{-E^2/2\sigma^2} dE \\ &= n \sigma^2 \end{aligned}$$

so that

$$\sigma^2 = \frac{\Sigma(E^2)}{n}$$

making σ the root mean square as previously stated.

The *probable error* is the quantity most used in this country to compare series of observations. This term is very misleading, and it does not imply that this error is more likely to occur than any other; indeed it has been shown that zero error occurs more frequently than any other. In fact, the probable error is such that there are as many errors numerically greater as numerically less than it, or in other words the probability that the error will fall between the positive and negative values of the probable error is one half, so that

$$\frac{1}{\sqrt{2\pi} \cdot \sigma} \int_{-p.e.}^{+p.e.} e^{-E^2/2\sigma^2} dE = \frac{1}{2}$$

The solution of this equation is that the *p.e.* = 0.6745σ . If the observed values all have the same quality, σ will be constant and the product of all the probabilities $e^{-E^2/2\sigma^2}$ is $e^{-\Sigma(E^2)/2\sigma^2}$.

When $\Sigma(E^2)$ is a minimum the maximum value of the product occurs, so that, as was found previously, if the observations of a quantity have equal precision the most probable value is when the sum of the squares of the errors is a minimum.

If, however, all the values have not the same quality, then σ varies for different observations and in this case the maximum value of the product occurs when $\Sigma(E^2/2\sigma^2)$ is a minimum. Thus the most probable value of the quantity is found by summing the values of $E^2/2\sigma^2$ to give a minimum value.

ERRORS IN RELATED QUANTITIES. The law of error, so far considered, deals only with the direct observation of a quantity. Frequently in surveying the quantity required is a linear function of two or more separately observed quantities. Suppose that a quantity Y is a function of two observed quantities X_1 and X_2 such that

$$Y = k_1 X_1 + k_2 X_2 \text{ where } k_1 \text{ and } k_2 \text{ are constants}$$

Let σ_1 and σ_2 be the respective standard errors of the measured quantities. Then the probability of the simultaneous occurrence of an error E_1 in X_1 and E_2 in X_2 is given by

$$\frac{1}{2\pi\sigma_1\sigma_2} \cdot e^{(-E_1^2/2\sigma_1^2 - E_2^2/2\sigma_2^2)} \cdot \delta E_1 \cdot \delta E_2$$

Now if these errors E_1 and E_2 in the measured quantities produce an error E in Y then

$$E = k_1 E_1 + k_2 E_2 \quad . \quad . \quad . \quad (12)$$

So that the probability of an error E in Y is

$$f'(E)\delta E = \frac{\delta E_2}{2\pi\sigma_1\sigma_2} \int_{-\infty}^{+\infty} e^{(-E_1^2/2\sigma_1^2 - E_2^2/2\sigma_2^2)} dE_1 \quad (13)$$

since the relationship (12) is always satisfied by taking any value of E_2 with all values of E_1 within the range of $+\infty$ and $-\infty$, and also since E_1 is independent of E_2

$\frac{dE}{dE_2} = k_2$ and equation (13) can be rewritten to give

$$\begin{aligned} f'(E)\delta E &= \frac{\delta E}{2\pi\sigma_1\sigma_2 k_2} \int_{-\infty}^{+\infty} e^{(-E_1^2/2\sigma_1^2 - E_2^2/2\sigma_2^2)} dE_1 \\ &= \frac{\delta E}{2\pi\sigma_1\sigma_2 k_2} \int_{-\infty}^{+\infty} e^{[-E_1^2/2\sigma_1^2 - (E - k_1 E_1)^2/2\sigma_2^2 k_1^2]} dE_1 \\ &= \frac{\delta E}{2\pi\sigma_1\sigma_2 k_2} \int_{-\infty}^{+\infty} e^{[-E^2/(2k_2^2\sigma_2^2 + 2k_1^2\sigma_1^2)]} e^{\alpha} dE_1 \end{aligned}$$

where

$$\begin{aligned} \alpha &= \frac{-E_1^2}{2\sigma_1^2} - \frac{E^2 - 2Ek_1E_1 + k_1^2E_1^2}{2\sigma_2^2 k_2^2} + \frac{E^2}{2k_2^2\sigma_2^2 + 2k_1^2\sigma_1^2} \\ &= \frac{-(k_1^2\sigma_1^2 + k_2^2\sigma_2^2)}{2\sigma_1^2\sigma_2^2 k_2^2} \left[E_1 - \frac{k_1\sigma_1 E}{k_1^2\sigma_1^2 + k_2^2\sigma_2^2} \right]^2 \end{aligned}$$

So that

$$f'(E)\delta E = \frac{1}{\sqrt{2(k_1^2\sigma_1^2 + k_2^2\sigma_2^2)}\pi} e^{-E^2/2(k_1^2\sigma_1^2 + k_2^2\sigma_2^2)}$$

So that the probability of error curve has the same form as for the single quantity, namely

$$f'(E) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-E^2/2\sigma^2}$$

where

$$\sigma^2 = k_1^2\sigma_1^2 + k_2^2\sigma_2^2$$

This proof can be expanded to include the general case where

$$Y = k_1X_1 + k_2X_2 + k_3X_3 + \dots + k_nX_n$$

in which case the curve has the same form and

$$\begin{aligned} \sigma^2 &= k_1^2\sigma_1^2 + k_2^2\sigma_2^2 + k_3^2\sigma_3^2 + \dots + k_n^2\sigma_n^2 \\ &= \Sigma(k^2\sigma^2) \end{aligned}$$

This result can also be used to extend the theory of observations on a single quantity, for if n equally precise observations $X_1, X_2 \dots X_n$ are made of a quantity then the arithmetic mean is

$$x_o = \frac{X_1 + X_2 + \dots + X_n}{n}$$

If σ_o is the standard error of the arithmetic mean and σ is the standard error of each measured value then

$$\sigma_o^2 = \Sigma(k^2\sigma^2) = \sigma^2/n$$

or

$$\sigma_o = \sigma/\sqrt{n}$$

It has been shown that the value of the standard error

$$\sigma = \frac{\Sigma(E^2)}{n}$$

It is, however, unusual for the values of E to be known, as the true value of x , the measured quantity, would then have to be known. It is more usual to have the arithmetic mean or most probable value x_o as in equations (4) and connecting these equations with equations (1) to give

$$X_1 = x - E_1 = x_o - r_1$$

$$X_2 = x - E_2 = x_o - r_2$$

$$\dots\dots\dots$$

$$X_n = x - E_n = x_o - r_n \quad \dots \quad (14)$$

and by addition, since $\Sigma(r) = 0$, $n x_o = nx - \Sigma(E)$, and substituting this value for x_o into equations (14)

$$nr_1 = (n-1)E_1 - E_2 - E_3 - \dots - E_n$$

$$nr_2 = -E_1 + (n-1)E_2 - E_3 - \dots - E_n$$

$$\dots\dots\dots$$

$$nr_n = -E_1 - E_2 - E_3 - \dots + (n-1)E_n$$

Square each of these equations

$$n^2r_1^2 = (n-1)^2E_1^2 \dots + E_2^2 + \dots - 2(n-1)E_1E_2$$

$$\dots\dots\dots$$

These squared equations can be added together, and when this is done the double products cancel one another, as errors of opposite sign have equal probabilities, so that

$$n^2\Sigma(r^2) = [(n-1)^2 + (n-1)]\Sigma(E^2)$$

$$\Sigma(r^2) = \frac{(n-1)}{n}\Sigma(E^2)$$

$$= (n-1)\sigma^2$$

and

$$\sigma^2 = \frac{\Sigma(r^2)}{(n-1)}$$

Now σ is the standard error of an observation, and it has been shown that the standard error σ_o of the arithmetic mean is such that

$$\sigma_o = \frac{\sigma}{\sqrt{n}}$$

so $\sigma_o^2 = \frac{\Sigma(r^2)}{n(n-1)}$, that is, the standard error of the arithmetic mean

$$\sigma_o = \sqrt{\frac{\Sigma(r^2)}{n(n-1)}}$$

Thus, when various series of observations have been reduced to arithmetic means, the standard errors and probable errors can be calculated and the relative reliability of the readings assessed.

ADJUSTMENT OF SURVEY OBSERVATIONS

The methods of adjustment of surveying observations based on the law of errors is best indicated by numerical examples, and the following examples should clarify the procedure to be adopted.

ADJUSTMENT OF FOUR OBSERVATIONS. From a series of levels taken on four stations A, B, C and D the following results were obtained. B was 5.00 feet above A; C was 3.00 feet above B; D was 3.00 feet above C and 11.10 feet above A. Find the probable heights of B, C and D above A assuming that each level difference has equal accuracy.

It is obvious that in this case there is a discrepancy of 0.10 feet, and the problem is how this should be divided between the various levels.

Let the true heights of B, C and D above A be b , c and d respectively. Then the errors of observation are:

$$\begin{aligned} E_1 &= 5.00 - b \\ E_2 &= 3.00 - (c - b) \\ E_3 &= 3.00 - (d - c) \\ E_4 &= 11.10 - d \end{aligned}$$

Now by the method of least squares

$$E_1^2 + E_2^2 + E_3^2 + E_4^2 \text{ should be a minimum}$$

$$\begin{aligned} \text{Now } \Sigma(E^2) &= 25.00 - 10b + b^2 + 9.00 + c^2 + b^2 - 6.00c + 6.00b - 2bc + 9.00 \\ &\quad + d^2 + c^2 - 6.00d + 6.00c - 2cd + 123.21 - 22.20d + d^2 \\ &= 166.21 - 4b + 2b^2 - 28.20d + 2d^2 - 2bc - 2cd \end{aligned}$$

In order to find the minimum value of this expression, differentiate with respect to b , c and d in turn and equate each differential to zero.

$$\begin{aligned} -4.00 + 4b - 2c &= 0 \\ +4c - 2b - 2d &= 0 \\ -28.20 + 4d - 2c &= 0 \end{aligned}$$

Here the problem resolves into three equations and three unknowns. The equations are known as the *normal equations* and solve to give $b = 5.025$ ft.,

$c = 8.05$ ft., and $d = 11.075$ ft. It will be seen in this example that although only simple numbers are involved a considerable amount of computation is necessary in squaring and adding the errors and in differentiating with respect to each of the quantities in turn to form the normal equations. However, as these equations all take a similar form, there is a quick method of obtaining the coefficients of the normal equations directly from the observation equations without the intermediate steps.

Let a, b, c , etc., be the most probable values of some quantities connected by the observation equations:

$$\text{Error}_1 = A_1a + B_1b + C_1c + D_1d + \dots + N_1$$

$$\text{Error}_2 = A_2a + B_2b + C_2c + D_2d + \dots + N_2$$

$$\text{Error}_3 = A_3a + B_3b + C_3c + D_3d + \dots + N_3$$

where $A_1, A_2, \dots, B_1, B_2, \dots, N_1, N_2, \dots$, are numerical coefficients.

The condition that the sum of the squares of the errors should be a minimum is given by

$$\Sigma(Aa + Bb + Cc + \dots + N)^2 = \text{a minimum}$$

Differentiate therefore with respect to a, b, c, d , etc., in turn and equate to zero to give the following normal equations:

$$\begin{aligned} A_1(A_1a + B_1b + C_1c + \dots + N_1) + A_2(A_2a + B_2b + C_2c + \dots + N_2) \\ + A_3(A_3a + B_3b + C_3c + \dots + N_3) + \dots = 0 \\ B_1(A_1a + B_1b + C_1c + \dots + N_1) + B_2(A_2a + B_2b + C_2c + \dots + N_2) \\ + B_3(A_3a + B_3b + C_3c + \dots + N_3) + \dots = 0 \\ C_1(A_1a + B_1b + C_1c + \dots + N_1) + C_2(A_2a + B_2b + C_2c + \dots + N_2) \\ + C_3(A_3a + B_3b + C_3c + \dots + N_3) + \dots = 0 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned}$$

which can be rewritten:

$$\begin{aligned} a\Sigma(A^2) + b\Sigma(AB) + c\Sigma(AC) + \dots + \Sigma(AN) &= 0 \\ a\Sigma(AB) + b\Sigma(B^2) + c\Sigma(BC) + \dots + \Sigma(BN) &= 0 \\ a\Sigma(AC) + b\Sigma(BC) + c\Sigma(C^2) + \dots + \Sigma(CN) &= 0 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned}$$

It will be seen that in the first equation the coefficient of a is obtained by summing the squares of the coefficients of a in all the errors. The coefficients of the other variables are obtained by summing the products of the coefficients of a and the coefficient of the variable concerned in each error. Similar arrangements of the coefficients give the other equations and the required coefficients can easily be picked out if the observation equations are arranged in table form.

a	b	c	d	N
A_1	B_1	C_1	D_1			N_1
A_2	B_2	C_2	D_2			N_2
A_3	B_3	C_3	D_3			N_3
A_4	B_4	C_4	D_4			N_4
..
..

The example can now be tabulated directly from the observation equations.

b	c	d	N
- 1	0	0	5.00
+ 1	- 1	0	3.00
0	+ 1	- 1	3.00
0	0	- 1	11.10

The normal equations derived are:

$$b(1 + 1) + c(-1) + d(0) + (-5.00 + 3.00) = 0$$

$$\text{or } 2b - c = 2.00$$

$$b(-1) + c(1 + 1) + d(-1) + (-3.00 + 3.00) = 0$$

$$\text{or } -b + 2c - d = 0$$

and

$$b(0) + c(-1) + d(1 + 1) + (-3.00 - 11.10) = 0$$

$$\text{or } -c + 2d = 14.10$$

the same as previously derived by differentiating.

They solve to give

$$b = 5.025 \text{ ft.}, c = 8.05 \text{ ft.}, d = 11.075 \text{ ft.}$$

ONE EQUATION OF CONDITION. The following results were obtained in a round of levels on four stations A, B, C and D, each length of levels having equal accuracy: B was 4.00 feet above A, C was 5.00 feet above B, D was 3.00 feet above C and 12.08 feet above A. It had previously been established that C was 9.03 feet above A.

Using the same notation as the previous example the observation equation for c is not subject to any correction and is known as an equation of condition. The errors are now:

$$E_1 = 4.00 - b$$

$$E_2 = 5.00 - (9.03 - b) = b - 4.03$$

$$E_3 = 3.00 - (d - 9.03) = 12.03 - d$$

$$E_4 = 12.08 - d$$

and

b	d	N
- 1	0	4.00
+ 1	0	- 4.03
0	- 1	+ 12.03
0	- 1	+ 12.08

So

$$2b - 8.03 = 0$$

So that

$$2d - 24.11 = 0$$

$$b = 4.015 \text{ ft. and } d = 12.055 \text{ ft.}$$

In the previous examples it has been assumed that each observation has equal accuracy or precision. This is not the usual case, for frequently one or more of the observations may be the mean of n

observations, in which case its inaccuracy is reduced in the ratio of $\frac{1}{\sqrt{n}}$ compared with a single observation, and therefore its observation equation should have more importance and be multiplied by \sqrt{n} . The effect of this is to multiply the normal equations concerned with this particular observation equation by n , and such an additional factor is called the *weight* of the equation. There are also many other cases where the observations should be weighted, sometimes by direct mathematical approach as in the case above, or in other cases by estimation where different conditions have appertained to parts of the work or where different observers have been used.

WEIGHTED OBSERVATIONS. If in the first example an additional level reading from D to B showed that D was 6.03 feet above B and the accuracy of the observations was as follows, a new tabulation results.

AB and CD weight of 1
BC and DA weight of 2
DB weight of 3

If the weights are put in a column headed w the table now becomes:

w	b	c	d	N
1	- 1	0	0	5.00
2	+ 1	- 1	0	3.00
1	0	+ 1	- 1	3.00
2	0	0	- 1	11.10
3	+ 1	0	- 1	6.03

The normal equations derived are:

$$b(1 \times 1 + 2 \times 1 + 3 \times 1) + c(2 \times -1) + d(3 \times -1) + (1 \times -5.00 + 2 \times 3.0 + 3 \times 6.03) = 0$$

$$\text{or} \quad 6b - 2c - 3d + 19.09 = 0$$

$$\text{or} \quad b(2 \times -1) + c(2 \times 1 + 1 \times 1) + d(1 \times -1) + (2 \times -3.0 + 1 \times 3.0) = 0$$

$$\text{or} \quad -2b + 3c - d - 3.00 = 0$$

and

$$b(3 \times -1) + c(1 \times -1) + d(1 \times 1 + 2 \times 1 + 3 \times 1) + (1 \times -3.00 + 2 \times -11.10 + 3 \times -6.03) = 0$$

$$\text{or} \quad -3b - c + 6d - 43.29 = 0$$

$$\text{from which} \quad b = 5.04 \text{ ft.}, c = 8.05 \text{ ft.}, d = 11.08 \text{ ft.}$$

The method of normal equations is not the only method used to obtain the most probable values. Another method known as the *method of correlates* is particularly useful when there are a number of equations of conditions, as in this case it gives a quicker solution.

GENERAL SOLUTION FOR WEIGHTED OBSERVATIONS. Consider the first example on page 11, where B was 5.00 ft. above A, C was 3.00 ft. above B, D was 3.00 ft. above C and 11.10 ft. above A. The rises add to 11.00 and the fall to 11.10 so a correction on the round is required of a rise of 0.10 ft.

Take the required corrections to the four lengths AB, BC, CD, DA to be e_1 , e_2 , e_3 and e_4 respectively. Also to make it a general example take the respective weights as w_1 , w_2 , w_3 and w_4 .

Then the total correction = rise of 0.10 = $e_1 + e_2 + e_3 + e_4$, which is an equation of condition.

To satisfy the condition of least squares $w_1e_1^2 + w_2e_2^2 + w_3e_3^2 + w_4e_4^2$ has to be a minimum value, so

$$w_1e_1\delta e_1 + w_2e_2\delta e_2 + w_3e_3\delta e_3 + w_4e_4\delta e_4 = 0 \quad . \quad . \quad . \quad (a)$$

$$\text{also} \quad \delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 \text{ must be zero} \quad . \quad . \quad . \quad (b)$$

Multiply (b) by λ and subtract from (a):

$$(w_1e_1 - \lambda)\delta e_1 + (w_2e_2 - \lambda)\delta e_2 + (w_3e_3 - \lambda)\delta e_3 + (w_4e_4 - \lambda)\delta e_4 = 0$$

Since the small increments of e_1 , e_2 , e_3 and e_4 are independent, then each of the coefficients of these increments must equate to zero independently, so that

$$\lambda = w_1e_1 = w_2e_2 = w_3e_3 = w_4e_4$$

That is, the corrections to be applied to the observations are proportional to the weights.

To revert to the numbers in the example,

$$\frac{\lambda}{w_1} + \frac{\lambda}{w_2} + \frac{\lambda}{w_3} + \frac{\lambda}{w_4} = 0.10 \text{ and } w_1 = w_2 = w_3 = w_4 = 1$$

$$\text{so that} \quad \lambda = \frac{0.10}{4} = 0.025$$

and therefore $e_1 = e_2 = e_3 = e_4 = 0.025$ (rise) so that the corrected level differences are 5.025 ft., 3.025 ft., 3.025 ft. and 11.075 ft., making $b = 5.025$ ft., $c = 8.050$ ft. and $d = 11.075$ ft. as before.

METHOD OF CORRELATES. Solve the example for weighted observations on page 14 by the method of correlates, but again in order to make it a general example take the weights of the various lengths as AB — w_1 , BC — w_2 , CD — w_3 , DA — w_4 and BD — w_5 .

Let the respective corrections be e_1 , e_2 , e_3 , e_4 and e_5 .

The observations are AB — 5.00 ft., BC — 3.00 ft., CD — 3.00 ft., DA — 11.10 ft. and BD — 6.03 ft. If the two triangles ABD and BCD are taken, since BD is common to both it is necessary to consider the rises and falls in a definite direction. Thus take AB — rise, BD — rise, DA — fall, and BD — rise, CD — fall, BC — fall.

$$e_1 + e_3 + e_4 = +0.07$$

$$e_2 + e_3 + e_5 = -0.03$$

To satisfy the least squares condition

$$w_1e_1\delta e_1 + w_2e_2\delta e_2 + \dots = 0 \quad . \quad . \quad . \quad (a)$$

$$\text{also} \quad \delta e_1 + \delta e_3 + \delta e_4 = 0 \quad . \quad . \quad . \quad (b)$$

$$\text{and} \quad \delta e_2 + \delta e_3 + \delta e_5 = 0 \quad . \quad . \quad . \quad (c)$$

Multiply (b) by λ_1 and (c) by λ_2 and subtract the two products from (a)

$$(w_1 e_1 - \lambda_1) \delta e_1 + (w_2 e_2 - \lambda_2) \delta e_2 + (w_3 e_3 - \lambda_2) \delta e_3 + (w_4 e_4 - \lambda_1) \delta e_4 + (w_5 e_5 - \lambda_1 - \lambda_2) \delta e_5 = 0$$

Equating the coefficients of all the increments to zero in turn

$$e_1 = \frac{\lambda_1}{w_1}, e_2 = \frac{\lambda_2}{w_2}, e_3 = \frac{\lambda_2}{w_3}, e_4 = \frac{\lambda_1}{w_4} \text{ and } e_5 = \frac{\lambda_1 + \lambda_2}{w_5}$$

So that

$$\frac{\lambda_1}{w_1} + \frac{\lambda_1 + \lambda_2}{w_5} + \frac{\lambda_1}{w_4} = 0.07$$

and

$$\frac{\lambda_2}{w_2} + \frac{\lambda_2}{w_3} + \frac{\lambda_1 + \lambda_2}{w_5} = -0.03$$

The solution of these simultaneous equations will give the required corrections.

In the example $w_1 = 1; w_2 = 2; w_3 = 1; w_4 = 2; w_5 = 3$

so

$$\begin{aligned} 11\lambda_1 + 2\lambda_2 &= 0.42 \\ 2\lambda_1 + 11\lambda_2 &= -0.18 \\ \lambda_2 &= -0.0241 \\ \lambda_1 &= +0.0426 \end{aligned}$$

so that $e_1 = +0.043$ ft., $e_2 = -0.012$ ft., $e_3 = -0.024$ ft., $e_4 = +0.021$ ft., and $e_5 = +0.006$ ft.

These corrections make the level differences

AB — 5.04 ft., AC — 8.05 ft., AD — 11.08 ft., as before.

PROBABLE ERROR OF A SINGLE OBSERVATION. Find the probable error of a single observation and the probable error of the arithmetic mean of the following six observations of an angle. Assume each observation to be of equal weight.

Observation	r''	$(r'')^2$
50° 2' 3"	1.33	1.78
50° 2' 6"	1.67	2.78
50° 2' 7"	2.67	7.11
50° 2' 5"	0.67	0.44
50° 2' 2"	2.33	5.44
50° 2' 3"	1.33	1.78
6)300° 12' 26"	Σr^2	= 19.33
50° 2' 4".33		

The probable error of the single observation is

$$0.6745 \sqrt{\frac{19.33}{5}} = \pm 1''.32$$

The probable error of the arithmetic mean is

$$0.6745 \sqrt{\frac{19.33}{6 \times 5}} = \pm 0''.54$$

Find the probable error of the weighted mean in the previous example if the first three observations have weights of 2, 3 and 4 respectively.

It is only necessary to include the seconds in the tabulation.

Observation	Weight w	Weighted observation	r''	$(r'')^2$	wr^2
3"	2	6"	2.17	4.71	9.42
6"	3	18"	0.83	0.69	2.07
7"	4	28"	1.83	3.35	13.40
5"	1	5"	0.17	0.03	0.03
2"	1	2"	3.17	10.05	10.05
3"	1	3"	2.17	4.71	4.71
		12)62".0 5.17		$\Sigma(wr^2) =$	39.68

The probable error of the weighted mean

$$\begin{aligned}
 &= 0.6745 \sqrt{\frac{\Sigma(wr^2)}{\Sigma(w) \times (n-1)}} \\
 &= 0.6745 \sqrt{\frac{39.68}{12 \times 5}} \\
 &= \pm 0''.55
 \end{aligned}$$

CHAPTER 2

MEASUREMENT OF BASE LINES

IN A regular triangulated survey the measurement of one side of the triangulation is sufficient for the calculation of all the other sides, but in practice, even when the angular measurement is being made with the highest precision, it is customary to measure check bases at regular intervals.

Obviously, since all distances in the framework are dependent on it, the base (or bases) must be measured with exactitude; but the actual precision with which the measurement is made can be decided only after considering the degree of accuracy required for the survey as a whole which, in turn, is dependent on the purpose for which the survey is being made, the time, the staff and the funds available.

STANDARDS OF LENGTH. Before proceeding with the methods of making precise linear measurements, we must consider the units in which the measurements will be made.

The legal standards of length vary from country to country, but for geodetic purposes a fundamental international standard was considered desirable, and for this purpose the 'Convention of the Metre' was approved by several European nations, and the Bureau International des Poids et Mesures was set up at Paris in 1875. Instead of re-establishing the toise (approximately 6·4 feet) which was the standard to which all early European (except the British) geodetic work was referred, the Bureau established the 'International Metre' which approximates closely to the older 'Metre of the Archives' or legal metre of France. The originals are marked on three platinum-iridium bars deposited at the Pavillon de Breteuil, Sèvres, and copies, which were issued in 1883, are kept by the various national surveys.

In Great Britain the legal unit is the Imperial Yard and is defined by Act of Parliament as the distance between marks engraved on gold plugs set in a bronze bar when at a temperature of 62° F. The standard is kept at the Exchequer, but there are several official copies.

For many years the standard used by the Ordnance Survey was the ten-foot iron bar known as OI_1 (Ordnance Intermediate Bar). This is a strong iron bar in the form of a double T with terminal marks in the centre line on silver tongues let into the surface.

Because of the very small secular changes in their dimensions, fundamental standards of length made of metal are not altogether satisfactory, and the tendency now is to use the wave length of light as a standard. Michelson, the American physicist, found, in 1893, that the standard metre was equivalent to $1553\ 163\cdot5$ wavelengths of the light corresponding to the red line of the spectrum of cadmium vapour, at 15° C. MM. Benoit, Fabry and Perot, in 1906, obtained a value of $1553\ 164\cdot13$ which confirmed this.

MEASURING EQUIPMENT

Five types of base measuring equipment should be mentioned:

- (a) Rigid bars.
- (b) Flexible apparatus; chains, wires and tapes.
- (c) Electronic apparatus, such as Radar.
- (d) Optical equipment for subtense or tacheometry.
- (e) Sound ranging.

Of the above (a) is virtually obsolete and is chiefly of historic interest; (d) is of insufficient accuracy to be considered in this chapter and the method is described in Volume I; (e) is only of use for hydrographic surveys when the sound waves are propagated through water (the speed of sound through air is too much at the mercy of atmospheric conditions to be of any use save in the crudest measurements of distance). Consequently, it is only (b) and (c) which need be considered in any detail, though it should be mentioned here that the electronic methods are still (1951) very much in the experimental stage and decidedly limited in scope.

The history of precise base measurement, as far as this country is concerned, dates back to 1784, when General Roy measured a base on Hounslow Heath for the purpose of making the geodetic connection between Greenwich and Paris observatories suggested by M. Cassini de Thury (the third of the Cassinis).

Delay in the manufacture of the theodolite with which the angular measurements of the triangulation were to be made enabled the base to be measured several times and by different methods; first experimentally with a 100-foot steel chain and then with deal rods, tipped with bell metal and each $20\frac{1}{4}$ feet long. The

accuracy of this measurement however was suspected because of the variation in length of the rods due to changes in humidity; so another measurement was made with glass tubes, also about 20 feet long. The tubes were calibrated for coefficient of expansion, and their temperatures were taken during the measurement by two thermometers in contact with them. Finally in 1791, when the Ordnance Survey had been established formally, the base was re-measured with a steel chain (made by the famous Jesse Ramsden). The chain was laid out in a succession of deal coffers, mounted on trestles, and was stretched by a weight of 28 lb.; the temperature of each chain length was taken as the mean reading of five thermometers. The greatest care was used to secure correct alignment and to adjust for differences of level, and finally the measurement was reduced to mean sea level.

The history of this Hounslow base is very similar to that of all geodetic linear measurement over the next century; rigid bars being used until supplemented by flexible apparatus towards the end of the 19th century.

RIGID BARS. The rigid bars used for precise measurement fall into three main categories according to the method adopted for allowing for the fluctuations in length of the apparatus due to temperature changes during the measurement. These categories are (*a*) bi-metallic compensating bars such as Colby's apparatus in which the two bars actuated a compensating mechanism (somewhat similar to the compensating wheel of a watch) to give a constant length; (*b*) bi-metallic non-compensating bars such as Borda's Rods in which a compound bar, in this case of platinum and copper, were rigidly secured at one end and, being free to slide over each other, formed their own bi-metallic thermometer which enabled the temperature to be read off graduations engraved on their free ends after calibration in the laboratory; (*c*) mono-metallic bars kept at a constant temperature, such as Woodward's iced bar, in which a steel bar about 5 metres long resting in a sliding trough kept full of crushed ice was used for the measurement.

The methods of using the above apparatus can be further subdivided into those in which the ends of the bars were brought into successive contact; or those in which the effective lengths of the bars were engraved on the bars away from the ends and the

terminal marks were observed under a microscope or pair of microscopes which served as the 'point of contact' for the successive bar.

Before leaving the subject of base measurement with rigid bars it should be mentioned that opinion differed as to the ideal length of a base. The Lough Foyle base was 7.89 miles and the one on Salisbury Plain 6.93 miles. In Germany on the other hand, at about the same time, bases of 860 metres (0.53 miles) and 935 toises (1.13 miles) were used.

JÄDERIN WIRES. About 1883 Professor Jäderin of Stockholm devised the method of measuring a base by means of wires stretched between tripods, which has now been adopted almost universally for both geodetic and topographical bases.

The difficulty of obtaining the exact temperature of a bar which seldom agrees with that of the surrounding air is indicated by the variety of the rigid bar apparatus mentioned above, and this difficulty is emphasized with the longer wires; Jäderin therefore made use of two wires of steel and brass to form a bi-metallic thermometer. A graduated scale was attached to one end of each wire, and the measurement from tripod to tripod was made in a similar way to the catenary method shortly to be described. Two separate measurements, under the same tension, one with each wire, were made of each span of the base, the difference between the two being used to indicate the temperature of the wires and hence their correct length.

INVAR. The two-wire method of obtaining the actual temperature of the wires was not altogether satisfactory, and research was made to find an alloy with as small a coefficient of expansion as possible. This led to the discovery by Dr. Guillaume in 1896 of the alloy, 64% steel and 36% nickel, which he named *invar* (invariable), with a very small coefficient of expansion indeed, which rarely exceeds .000 0005 per 1° F. (coeff. of steel, c. .000 00625 per 1° F.). For over forty years now (1951) all important base lines have been measured with wires or tapes of invar.

Invar, however, in spite of its low expansion, is not altogether satisfactory for precise measurement. In the first place the coefficient of expansion differs with different rollings and, in fact, differs in tapes from the same rolling, and in extreme cases has been found to differ throughout the length of the same tape. For this reason it

is essential that the coefficient must be determined for each tape separately during standardization.

Secondly, invar has, apparently, some molecular instability which manifests itself in changes of length both permanent and temporary. The permanent change is a slow increase of length which persists for years though at a decreasing rate. This increase is reduced by annealing the wires, keeping them at 100° C. for several days and then allowing them to cool gradually to 25° C. over a period of about 90 days. After a period of about two years after annealing the increase does not exceed about 1μ a year. The temporary change occurs with a change of temperature; when a tape which has been in a stable temperature for some time is subjected to a change in temperature it expands or contracts according to its coefficient of expansion, but on continued exposure to the changed temperature a slow creep in the reverse direction (contraction after an expansion due to raised temperature and vice versa) is exhibited. Dr. Guillaume gives the formula for this creep as $\delta L = 0.00325 \times 10^{-6} L(T_1^2 - T_2^2)$, where T_1 and T_2 are expressed in $^{\circ}$ C.; the creep develops very slowly at atmospheric conditions, a period of several months being required before equilibrium is reached at the new temperature.

Thirdly, invar, being considerably softer than steel, is much more likely to kink, and very careful handling is necessary especially when unrolling a tape from its drum and rolling it back again. Tension, during measurement, must be very slowly and carefully applied. The tapes should be wound on large drums (Hotine* recommends a drum 4 feet in diameter for a tape $\frac{1}{2}$ in. by $\frac{1}{16}$ in. and for the Ridgeway and Caithness base measurements in 1951 and 1952 drums $4\frac{1}{2}$ feet diameter were used for the tapes $\frac{1}{8} \times \frac{1}{16}$ in.) to avoid the stresses due to the comparatively sharp bending of the tape. It has further been recommended (E. H. Thompson, *Empire Survey Review*, Vol. X, No. 78, p. 379) that, if the tapes are wound on metal drums, the drums should be covered with an elastic blanket to avoid the stresses caused by the drum expanding through a rise in temperature after the tape was wound.

Although invar tapes are likely to be used for some time to come, the recent experiments at the National Physical Laboratory (N.P.L.) in accurate determination of temperature of a steel tape

* 'The General Theory of Tape Suspension in Base Measurement' by Major (now Brigadier, M. Hotine, *R.E. Empire Survey Review*, Vol. V, No. 31.

by measuring its electrical resistance may lead, in view of the disadvantages of invar mentioned above, to the use of steel tapes for precise base measurement, provided the laboratory apparatus involved can be adapted for use in the field. (See 'Standardization of Steel Surveying Tapes' by J. S. Clark and L. O. C. Johnson, *Empire Survey Review*, Vol. XI, No. 81.)

BASE MEASUREMENT WITH FLEXIBLE APPARATUS. Two methods of using flexible apparatus for precise measurement may be adopted: (a) suspended in catenary (Jäderin's method) and (b) on the flat.

There are so many advantages in favour of the catenary method that it has been almost universally adopted for all geodetic base lines, though measurements on the flat are far from obsolete for many precise purposes. The principal advantages of the catenary method are:

(i) Far less preparation of the site is necessary since no leveling or smoothing of the ground is required; consequently longer bases can be measured for the same expenditure of time and labour.

(ii) Steeper slopes over much rougher ground can be accepted, thereby enabling the base terminals to be situated on elevated ground with, in consequence, improved accuracy in the base extension.

(iii) The actual measurement, made at a convenient height above the ground, can be carried out with considerably more comfort than a measurement on the flat. (It is true that measurements on the flat can be and have been made in troughs mounted on trestles, but this greatly increases the expense and labour of the measurement.)

WIRES AND TAPES. Opinions have differed about the merits of wires and tapes. The apparatus originally designed by Dr. Guillaume and M. Carpentier used wires 24 m. by 1.65 mm. diameter, but most modern opinion favours tapes. Wires expose less surface to the wind, but otherwise all the advantages seem to be with the tapes; twists can be spotted at once; terminal graduations can be engraved on the tape itself instead of on a *Réglette* specially attached to the wire; tapes have less tendency to coil up on themselves and consequently are more easily handled.

Invar tapes for simple suspension are usually either 24 m. by 3 mm. by 0.5 mm. or 100 ft. by $\frac{1}{8}$ in. by $\frac{1}{16}$ in. 100 m. and 300 ft.

tapes have been used extensively, but when these longer tapes are used they are usually supported at one or more equally spaced intervals.

The tapes are usually graduated only for a distance of 5 cm. or 0.2 ft. on either side of the terminal marks, the usual divisions of the graduation being 1 mm., 0.002 ft. or $\frac{1}{32}$ in. The figuring of the graduations is a matter of choice, but to obviate errors due to booking the wrong sign one of the two methods shown is desirable. In (a) all rear graduations are negative and all front graduations positive, in (b) all graduations are positive.

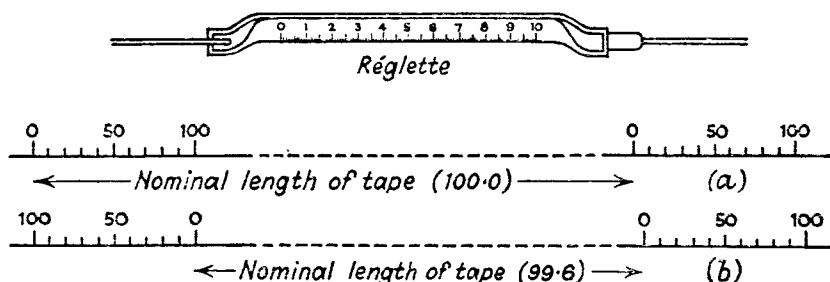


FIG. 2

For a geodetic base line two or three normal length field tapes are required with the same number of reference tapes. In addition a short, fully divided, tape must be included in the equipment and a long tape may be added with advantage.

MEASURING TRIPODS. Eight to ten measuring tripods are desirable if the measurement is to be carried out expeditiously. There is nothing peculiar about the design of the tripod, but rigidity and stability are essential. The measuring head consists of a small cylinder with the reference mark engraved over half a diameter on its head. The diameter perpendicular to the reference mark is stepped slightly to allow the tape to lie in the plane of the reference mark; the cut-away part is also slightly chamfered to allow the tape in catenary to fall away without fouling. A centring movement of at least 2 in. is desirable in the head, because the placing of the tripods is frequently left to unskilled labour. In the equipment manufactured by Messrs. Cooke, Troughton and Simms and Messrs. E. R.

Watts, a microscope on a hinged flap, which can be thrown clear when the tape is being inserted, is attached to the head for reading the tape. If such a head is not fitted, an ordinary magnifying glass

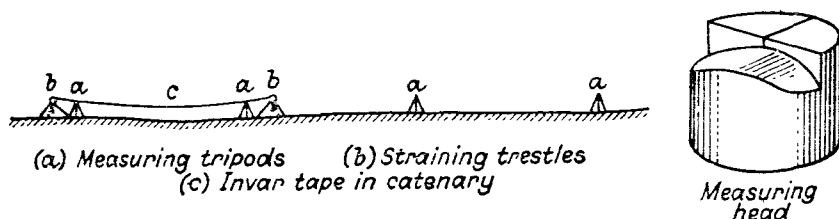


FIG. 3

must be used. Other equipment which can be fitted to the tripods without upsetting their centring are: (i) an optical plummet for transferring the positions of the heads to and from ground marks; (ii) an aligning and levelling telescope which can be used for aligning the tripods and, by means of a graduated arc and bubble, for obtaining the slopes for determining the slope correction (but, as will be mentioned later, the slope correction may be found by levelling alone); (iii) sighting vanes of a height equal to the height of the transit axis of the aligning telescope, thus avoiding height-of-instrument and height-of-target corrections when reducing the vertical angles to obtain the slopes.

STRAINING APPARATUS. Three methods of applying the correct and constant tension to the wire can be employed: (a) by means of weights slung over frictionless pulleys; (b) by means of a weight over a bell crank lever with a spirit level attachment to ensure that the crank lever is always horizontal; (c) by means of a spring balance extended by a straining lever.

Although (c) is used frequently for precise traverse work, (a) is usually preferred for geodetic base lines, and for this special trestles are necessary to carry the frictionless pulleys, which should be free to rotate about a vertical axis (thus ensuring that pulley and tape lie in the same vertical plane) and have slow-motion devices for both lateral and vertical setting. It is usual to have one leg of the trestle, the one pointing down the base, longer than the other two to resist the strain of the tape.

The weights are carried at the end of a cord or wire. Opinion now favours the latter. It is led over the pulley and carries a hook at the free end to which the tape may be hooked when the strain is to be applied. The tension should not be less than 20 times the weight of the tape. It is customary to use weights of 20 lb. with 100 ft. and 24 m. tapes of cross section $\frac{1}{8}$ in. by $\frac{1}{50}$ in. (Ordnance Survey used 15 lb. for their 24 m. tapes in 1951 and 1952.)

ADDITIONAL EQUIPMENT. A theodolite is required for aligning the measuring heads if no aligning telescope is carried, and two theodolites may be required for transferring the measuring heads to the ground mark at the end of each day's work.

A special light levelling staff, which can be set on the measuring head, and a level are desirable for obtaining the slopes of the measuring bays either instead of or as a check on the vertical arc of the aligning telescope.

A spacing tape for setting out the tripods; and pickets with movable heads, for marking the section terminals, are useful accessories.

Finally about six thermometers should be provided; at least two of these should have been standardized at a laboratory, and should be kept during the measurement entirely as standards. The field thermometers should be compared with the standards at frequent intervals, preferably in water at different temperatures.

FIELD WORK

SELECTION OF SITE. Many factors govern the selection of the base site. Clear and reasonably smooth ground is required; the terminals should be intervisible, and it is desirable that they should be sufficiently elevated to avoid grazing rays in the base extension. (This is rather a reversal of former practice when flat low-lying sites were looked for.) In small topographical surveys it does not much matter where the base is situated, but in primary work it is desirable to have the bases and bases of verification placed at fairly regular intervals. It is essential that the site should be chosen so that extension, by means of well-conditioned triangles, to one of the sides of the primary triangulation is as simple as possible. The latter consideration is also a factor in deciding the length of the base; generally speaking the length of the base should bear as large a ratio as possible to the average length of side in the

triangulation. No hard and fast rule can be laid down, but the ratio to be aimed at should be between $\frac{1}{2}$ and $\frac{1}{3}$. It should not be smaller than $\frac{1}{10}$.

BEACONING AND CLEARING. The terminals of the base must be marked with the same care and precision as are used for beaconing primary triangulation stations. Much depends on the nature of the ground, but it is customary to have a buried mark and a surface one centred carefully over it. The surface mark is usually in the form of a concrete pillar with a self-centring tribrach frame embedded in its top, on which it is possible to set up a theodolite or a measuring head.

Comparatively little clearing is necessary, but boulders and undergrowth which obstruct the line must be cleared away. On the Ridgeway base in 1937 the line ran straight through a wooden barn, but this difficulty was overcome by threading the tapes through holes cut in the walls. While the clearing is being done, alignment marks should be erected by means of theodolites set up on the base terminals. Ranging poles may be used for these alignment marks and time is ultimately saved if too many rather than too few are erected, since the correct positioning of the tripods, many of which are of necessity placed in dead ground, depends on the ability to see one or more of these marks.

ORGANIZATION. For expeditious working at least two parties are required. (a) A forward party who place the tripods in correct alignment and at approximately the correct distance apart ahead of the measurement. This party may be composed largely of unskilled labour, but a surveyor and, if possible, a recorder should be attached for working the alignment telescope and obtaining the slope between adjacent measuring heads. (b) A measuring party of two observers and a recorder with labourers for transporting the apparatus. If the personnel is available, a third, levelling, party consisting of a leveller and staff holder can be added.

PLACING TRIPODS. The first tripods are aligned by a theodolite centred over one of the alignment marks previously set out; thereafter they can be aligned by this method or by using the aligning telescope. Their distance apart is determined by the spacing tape. When placed, the measuring head is set level with its cut-away

portion in the line of the base and given a final adjustment for alignment. Fresh tripods are brought up from the rear, but at least one and preferably two are always left behind the measuring party in case they are needed for reference purposes. It should be noted here that once the combined parties have settled to the routine, the measuring party can usually work faster than the forward party, which poses the question of the number of tripods to be used. Too few means frequent irritating delays for the measurers; too many means an unnecessary long journey bringing the used tripods up to the front; ten appears to be a convenient number.

MEASUREMENTS. The first measurement between each pair of tripods is that of the slope between their heads. This, as has been stated, may be made either by the aligning telescope, in which case both fore and back readings are required between each pair, or by a level set up to the side of the line used in conjunction with a special short levelling staff fitted with a rubber pad which can be held on the tripod heads. Three or four tripods may be levelled together, readings being estimated to 0.001 ft. (If the party is strong enough both methods may be used as a check.)

While the slope is being determined, the tapes, weights and straining trestles are brought forward from the preceding span. (Various methods of carrying the tapes, of which there will usually be at least two, are adopted. If there is much undergrowth, they may be hooked to two ranging poles and carried well clear of the ground; if the ground is relatively smooth, the method adopted in the Sudan is convenient, in which the hooks for carrying the tapes were screwed into boards slung over the shoulders of two chainmen. Whichever method is adopted, it is advisable to insert spiral springs between the hooks and the tapes.) The straining trestles are placed at their proper distance from the measuring heads and as nearly as possible in the correct line. The tape is then hooked, by the observers, to the swivel hooks of the wires over the pulleys, and at the executive order the men tending the weights hook them to the other ends of the pulley wires and lower them, very gently, to take the strain on the tape. A final adjustment is then made to the pulley by means of the lateral and vertical slow motion devices, so that the graduated part of the tape lies in gentle contact with, and in the same plane as, the reference mark of the measuring head. A check is made to ensure that there is no twist in the tape.

Simultaneous readings of both ends of the tape are made by the observer using either the microscopes attached to the measuring heads or magnifying glasses. Readings are estimated to $\frac{1}{10}$ of the graduation divisions, i.e. usually to 0.0002 ft. or 0.1 mm. An even number of readings in pairs are taken, usually 6 or 8, the tape being displaced slightly after each pair, by a slight lifting of one of the weights. This lifting must be done gently, first from one end and then from the other. Accuracy is improved if the pulley can be rotated through approximately equal angles between each pair of readings thus reducing errors of eccentricity. The maximum range of the sums (or differences) should not exceed 0.3 of a division. Eight column booking is convenient.

TABLE 1
BOOKING OF BASE LINE MEASUREMENT
(IN CATENARY)

Span No.	Tape No.	Readings in mm. $\times 10$			Mean	Temp. °F.	Remarks
		Rear	Fore	Sum			
11	3	382	723	110.5	110.42	73.4	
		698	406	.4			
		503	600	.3			
		814	291	.5			
		609	495	.4			
		921	183	.4			

(Note: Tape graduated outwards at each end)

Temperatures are read by the recorder. An error of 1° F. in the temperature of an invar tape leads to an uncertainty of the length of only 1/2,000,000 as opposed to about 1/160,000 for a steel tape; nevertheless, in view of the great difficulty in obtaining the true temperature of the tape, every effort must be made to reduce the uncertainty. Two or three thermometers, which should be read to 0.2° F., should be used, one at each end of the tape and the third, if used, in the middle; they should be enclosed in a metal guard which should be of a similar colour and brightness to the tape itself. They should be held close to the tape, almost horizontally but with the bulb very slightly lower than the top.

When the measurement is complete the weights are lifted simultaneously and unhooked, but before moving on to the next bay the process is repeated with the other field tapes being used for the measurement.

To avoid any chance of disturbing the position of the tripod it is essential that the party when moving forward should keep well clear of the tripod they are passing. In the recent (1952) Ordnance Survey base measurements the fore end observer remained at his tripod and kept passers-by clear, and, since he himself remained still, movement in the immediate vicinity of the tripod was reduced to a minimum. A routine of this sort means that the observers change ends automatically, fore to rear, or vice versa, at each bay; but if this 'leap-frogging' is not carried out, the observers should change ends in some systematic way, either at regular intervals during one complete measurement (say every ten bays) or after a completed measurement so that, if for one measurement observer A was in the rear, for the next measurement, in the other direction, A would be in the front. More important, perhaps, is that the observers should change sides at definite intervals, reading on the north (or east) side for one section and on the south (or west) side for the next. The organization should be such that sections which were read from the north side in the first measurement should be read from the south in the next.

At the end of each section, usually a day's work, the position of the last measuring head is transferred to a ground mark, either by two theodolites or by the transferring head. Each section must be measured at least once in each direction; a third or fourth measurement being made if serious discrepancies occur.

It is impossible to make accurate measurements in a wind, and a long canvas screen, supported on poles, held to windward is almost a necessary piece of equipment. Experience in the Sudan has shown that 7 feet is the maximum height of screen which can be held with safety in winds exceeding 15 m.p.h.

STANDARDIZATION OF FIELD TAPES. It is assumed that at least two but preferably three standard tapes which have recently been standardized at the N.P.L. are carried in the equipment and that they will be re-standardized there as soon as possible after the measurement has been completed. The utmost refinement is employed in the standardization at the N.P.L., where the tapes are compared with the

standard bars under different conditions of temperature. The N.P.L. Class A certificate guarantees the accuracy of the result to $1/1,000,000$. It is advisable that the straining equipment actually used in the field should also be used in the N.P.L. standardization.

In addition the expansion coefficients of the field tapes should be determined at the N.P.L., though their actual lengths are determined by the field standardizations.

Opinions differ as to the best methods of field standardization and how frequently it should be done. One method is to lay out a special reference base in some convenient position adjacent to the middle of the base line. Such a reference base may be of one tape length, or of several, with terminals marked on metal plugs let into concrete pillars. Another method is to set up a reference base when required between ordinary measuring heads on their tripods. It is as well to enclose these reference bases with canvas screens while the standardizations are being made. Yet another method is to use one of the spans of the base line, measuring the distance between the heads several times with the standard tapes and the field tapes. If the last method is adopted the standard comparisons may be made before and after each day's work.

It is, on the whole, undesirable to use the standard tapes more than necessary, and, since the constant inter-comparison of the field tapes during the ordinary process of the measurement will give immediate indication of any erratic behaviour on their part, field standardization has frequently been carried out only before and after each completed measurement. Two geodetic bases were measured in the Sudan in 1936 and 1939. In 1936 erratic behaviour of the field tapes was disclosed, and so standardization was carried out at intervals of three days or so. In 1939 uniformity of behaviour was displayed by the field tapes during the measurements; consequently comparison for standardization was made only three times – before the outward measurement, between measurements and after completion of the return measurement.

BASES FOR MINOR TRIANGULATION. The field work described above is for bases of geodetic or primary triangulation where the highest precision is sought; for minor triangulation certain relaxations in the procedure, with consequent slightly lowered accuracy, are permissible. Frequently only one field tape and one standard tape will be employed and slightly rougher standards of alignment and

slope measurement can be accepted. Progress may be speeded if long tapes of 300 ft. or 100 m. are employed, but with them the accuracy of field standardization and temperature determination is definitely lowered while in many cases protection from the wind is more difficult. In this connection, and particularly as regards protection from the wind, it is interesting to note that D. F. Munsey, in his articles on 'Base measurement in the Anglo-Egyptian Sudan',* came to the conclusion 'that in windy country, the geodetic apparatus is not best suited to work of lower accuracy. It would seem that the old-fashioned method of measurement to stakes is more appropriate. An important advantage of stakes over tripods is that the tape is generally closer to the ground and the screening is therefore more effective. Furthermore the size of party would be considerably reduced and only two surveyors would be necessary. Windy weather would be spent in staking and levelling and the calmer spells devoted to taping. Bad weather conditions demand flexibility in the arrangements and, where the work is not required to be of the highest precision, it may be carried out more economically by a smaller party using simpler methods'.

FLEXIBLE APPARATUS LAID FLAT. Although it is unlikely that any geodetic bases will ever again be measured on the flat, nevertheless precise measurement for engineering work may so have to be carried out; e.g. one of the three bases for the Sydney Bridge triangulation was thus measured in 1927.† Flat sites with slopes of less than 3° are advisable for this type of measurement.

The base terminals should be marked with ground marks and the ground between them cleared and smoothed off very carefully. The latter may involve a considerable amount of cutting and embankment. Along the line of the cleared base and at intervals corresponding to the length of the field tapes, sheets of zinc about 18 in. long by 4 in. wide are nailed to the heads of pickets which are driven down flush with the ground. Lines in the correct line of the base are scribed as centre lines down each plate. It is convenient to measure the slopes between the centres of the plates while this operation is being carried out. Levelling is possibly the most satisfactory method, but a theodolite may be used set up at alternate plates and sighting on to a target, set to the theodolite's height of collimation.

* Empire Survey Review, Vol. X, No. 74, p. 160.

† See Chapter 12.

During the measurement, with the tape in contact with the ground, the possibility of obtaining the true temperature of the tape is more doubtful than ever. Consequently it is advisable that the measurement and standardizations should be carried out in cloudy weather or at night, when temperature conditions are as stable as possible.

Should this type of measurement be carried out with tapes fitted with the ordinary 'chain' handle, in which the length is determined between the flats of the handles, an additional straining handle must be made by which the required tension may be put on the tapes during measurement leaving the flats of the handles clear.

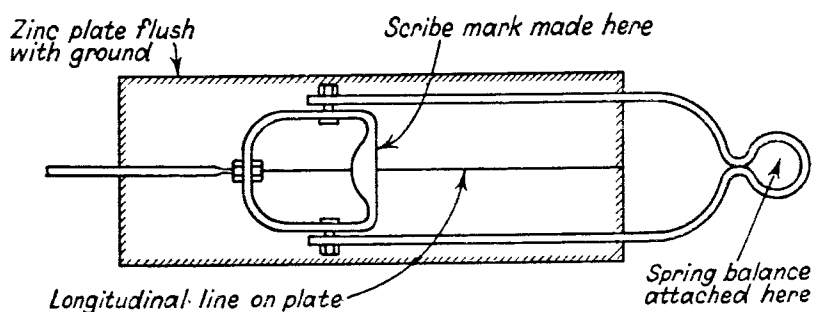


FIG. 4

Measurement on the flat. A party of three surveyors and about four chainmen are required. At each bay the field tape is held steady on the rear zinc plate by means of a chain arrow rove through the ring of the straining handle and stuck into the ground. The spring balance is attached to the fore straining handle and the correct tension applied after the alignment of the tape has been checked by the rear surveyor. Thermometers carried by the surveyors are laid at each end and the middle of the tape with their bulbs adjacent to it. On a signal from the fore surveyor, when the tension is correct, lines are scribed across the longitudinal line on the zinc plate with a scratcher, using the flat of the handle as a straight edge. The scribed lines on the plates are labelled (say) r_3 and f_3 - 'rear' and 'fore' with the number of the span - and the thermometers are read and booked. If a second field tape is being used, a repetition is made with it; otherwise the tapes are moved

forward to the next bay and the process is repeated. No attempt is made to obtain coincidence between the rear end of one span and the fore end of the preceding one, but the small space between the extremities of the spans is measured by the third surveyor with dividers and a fine scale. (If the third surveyor is not available these small measurements must be left until the measurement of the section is finished when the senior surveyor can go back to make them.) Booking errors may be reduced if all these small measurements are positive.

A convenient method of booking is:

TABLE 2
BOOKING OF BASE LINE MEASUREMENT
(ON THE FLAT)

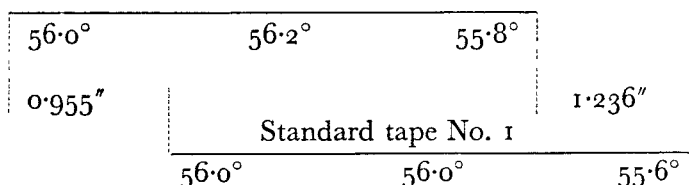
Rear	Temp. °F.			Small diffs. in inches		Slope	Remarks
	Middle	Fore		+	-		
		55.2	$\triangle B$ 50 32	3.35			North terminal
54.0	54.8	55.4	50 00	1.37			
54.6		54.4	49 00		0.98		
55.0	54.6	54.9	48 00				
.
		55.2	2 00	0.63			South terminal
55.4	55.2	55.3	1 00	0.32			
55.4	55.4		0 00	1.22			
			$\triangle A$				
<i>Mean</i>				<i>Sum</i>	<i>Sum</i>		
				<i>Diff.</i>			

Comparisons with the standard tapes are made at regular intervals (as for catenary measurements) and may be carried out in any

convenient bay of the main measurement. The comparisons are best booked graphically, thus:

Comparison before first measurement

Field tape No. 3



CALCULATING LENGTH OF BASE

CORRECTIONS. The corrections to be applied to precise measurements are:

- (i) Standard
- (ii) Temperature
- (iii) Tension (modified by correction due to change in gravity)
- (iv) Catenary or sag
- (v) Slope
- (vi) End readings (corrected for inclination of end scales)
- (vii) Alignment
- (viii) Height above mean sea level.

Most of these corrections are described in Vol. I; and in many cases some of them (usually (iii) and (vii)) may be nil.

(i) Comparison with the standard tapes will give a 'normal length' for each field tape at a certain temperature. If this length is l it will have to have applied to it a small temperature correction c depending on the difference between the average temperature of the field tape during the base (or section) measurement and its temperature when the comparison was made. If there are n spans in the base (or section) the length will be $n(l \pm c)$.

- (ii) If a = coefficient of expansion

L = measured distance

T_s = temperature of standardization

T_m = mean temperature during measurement

then the temperature correction is $+a(T_m - T_s)L$ and is applied to the standard tape for the purpose of calculating the true length of the test bay. A temperature correction must also be applied to

the field tape to find its mean length during the measurement from the data obtained during its comparison in the test bay.

In all cases it is important that the thermometer readings are corrected for their scale errors before being used in the calculations.

(iii) A correction for tension is required only if the tension applied during measurement is different from that applied during standardization. Such an alteration in tension has two effects, first, to alter the length of the tape, and, second, to change the shape of the catenary (see (iv) below).

If F_s and F_m = tension applied during standardization and measurement

A = cross-sectional area of tape

E = modulus of elasticity

L = measured distance

then the correction to be applied to find the true length of the tape

$$\text{is} \quad + \frac{(F_m - F_s)L}{AE}$$

For geodetic work, rather than adopting this general formula it is better that the tape in question should be standardized at the N.P.L. under widely different tensions.

The acceleration due to gravity, g , changes with latitude and elevation and in refined work allowance is made for the effect of this change of g between the laboratory where standardization was carried out and the site of the field measurement. (a) When the tension is applied by weights the shape of the catenary is not affected by a change in g since the changes in pull of the weights and the weight of the tape are proportional; however, the change in the pull of the weights is the equivalent of a change in tension and must be allowed for accordingly. The N.P.L. certificate of standardization may give the necessary data in the form

'0.000 04 feet per 100 feet per 1 cm/sec² change in g '

otherwise the change in the nominal value of the tension δF may be computed from $\frac{\delta F}{F} = \frac{g_F - g_s}{g_s}$ where g_s and g_F are the accelerations due to gravity at the place of standardization and at the site of the measurement respectively.

(b) When the tension is applied by spring balance the change in g does not affect the tension but does affect the weight of the tape

and so causes an increase or a decrease in sag, the 'W' in the sag formula (see below) being altered in proportion to the alteration in g .

When the value of g is unknown it may be calculated from

$$g = \left(1 - \frac{h}{\cdot 8R}\right) \times g_0 (1 + 0.0053 \sin^2 \phi)^*$$

where g is the acceleration due to gravity at an elevation h in latitude ϕ and g_0 is the value at sea level on the equator, which may be taken as 978.049 cm/sec² or 32.088 ft/sec², and R is the mean radius of the earth (say) 2,089,000 ft.

(iv) The catenary, or sag, correction is the difference between the length of the tape on the flat and its chord length when suspended between two points at the same level. Normally, for geodetic work, the tapes will have been standardized at the N.P.L. when suspended in catenary under the tension used in the field, so that no sag correction is necessary: but if a tape is used which was standardized on the flat, the correction to be used is

$$(D - L) = -\frac{L^3 w^2}{24 F^2} \quad \left(= -L \frac{W^2}{24 F^2} \right) \quad . \quad . \quad (a)$$

$$= -\frac{D^3 w^2}{24 F^2} \quad \left(= -D \frac{W^2}{24 F^2} \right) \quad . \quad . \quad (b)$$

where

- w = weight of tape per unit length
- (W = weight of tape between supports)
- F = applied tension in units of W
- L = true length of wire between supports
- D = chord distance of wire between supports

Both formulae are approximate but are sufficiently accurate provided F is not less than $20W$.

A sag correction must be applied to tapes which have been standardized in catenary should the tension applied in the field be different from that applied during the standardization at the N.P.L. (see (iii) above). In this case L must be calculated from formula (b) using F_s , the tension of standardization; and then a new Dm , the

* The first part of the above expression is a combination of the correction for the height above sea level $= -g \frac{2h}{R}$ and for the density of the strata between sea level and the

station $= +g \frac{2}{R}$
u.-p

chord length under the tension of measurement, Fm , calculated from formula (a). When $(Fm - Fs)$ is small, the new sag correction Cm may be found from $Cm = Cs - 2Cs \frac{(Fm - Fs)}{Fs}$

A further sag correction is required when the measurement is being made on a slope, i.e. when the catenary supports are at different levels (see (v) below); this is sometimes known as the correction for deformation of the catenary.

If C = the normal catenary correction

Cd = the deformed catenary correction

θ = the angle of slope

then

$Cd = C \cos^2 \theta \left(1 + \frac{W}{F} \sin \theta \right)$ when tension is applied at upper end

or $= C \cos^2 \theta \left(1 - \frac{W}{F} \sin \theta \right)$ when it is applied to the lower end.

$\frac{W}{F}$ is the ratio weight/tension, and it should be noted that, provided this ratio is not more than $\frac{1}{250}$, the error involved through using a simplified formula $Cd = C \cos^2 \theta$ is less than $\frac{1}{800000}$ for slopes not exceeding 30° .

Note. It must be realized that the correction Cd , when applied, gives the chord distance between the supports and that a further slope correction is necessary to give the true horizontal distance.

(v) If θ = the angle of slope and l = the measured length, the slope correction $= -1(l - \cos \theta) = -l \text{ versine } \theta$.

As mentioned in (iv) above, when the measurement is made in catenary there is a further correction because of the deformation of the catenary which must be made to obtain the correct l for the ordinary slope correction given above.

When applying the slope correction it is customary to make use of a table of natural versines assuming a constant length for each span. With a tape 100 ft. long and graduated for 0.4 ft. at either end, the actual span length being corrected may possibly be 0.4 ft. longer or shorter than the standard length and the slope correction for this excess, or deficiency, may be significant. (E.g. for a slope of 3° , the correction for 0.4 ft. is 0.00055.) A second slope correction must therefore be applied when either the slope or the difference from the standard length warrants it.

Should the slopes have been determined by levelling (provided they do not exceed 1 in 25), the formula, total correction = $\frac{\Sigma h^2}{2l}$, may be used without appreciable error; where h = the difference in height between measuring heads and l = the uniform chord length of the spans.

(vi) The algebraic sum of all the end readings is usually very small and is usually applied uncorrected for temperature to the length of the base (or section) after corrections (i) to (v) have been made. Strictly, however, a small correction is necessary in catenary measurement, because of the inclination of the end scales to the horizontal. This angle of inclination ' i ' is frequently given in the N.P.L. certificate of standardization. If it is not it may be found from

$\sin^{-1} \frac{W}{2F}$ where W = weight of unsupported tape and F = tension

applied in the same units; then correction for inclination of end scales = $-\Sigma r \text{ versine } i$ where Σr = algebraic sum of end readings. (The

correction given by David Clark is $-\Sigma r \times \frac{C}{l}$ in which C = the

ordinary sag correction and l = the standardized length of the tape.)

(vii) Corrections for alignment are seldom necessary, and, when they are, the deviations from the direct line are usually small enough for the slope formulae to be used, where θ = the deviation of the course from the direct line, l = the length of the deviation, and h = the perpendicular offset from the direct line. (A case occurred during the measurement of the Lossiemouth base in 1909, when mist during the setting up of one section of the tripods was responsible for its deviating about 5 feet from the direct line.)

(viii) When all other corrections have been made, the length of the measured base must be reduced to mean sea level, using an average elevation and a mean radius of the earth as data.

Fig. 5 shows the problem:

s = base as measured

b = base reduced to M.S.L.

H = average elevation of base above M.S.L.

R = radius of earth at latitude and azimuth of base

Obviously

$$b : s :: R : (R + H)$$

correction to $s = -(s - b) = -s \left(1 - \frac{R}{R + H} \right) = -s \frac{H}{R + H}$

and since H is small compared with R , $= -s \frac{H}{R}$ approx.

A further approximation may be made by letting

$$\begin{aligned} R &= \text{mean radius of earth} \\ &= 20,890,000 \text{ ft.} \end{aligned}$$

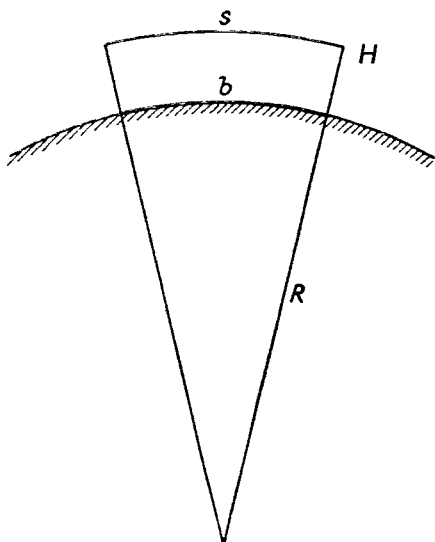


FIG. 5

Correction for error of spring balance. A further correction, not previously mentioned, may have to be made. If a spring balance is employed for applying the tension, a correction may have to be applied for a scale error due to the fact that the horizontal tension may differ from that recorded when the balance is held vertically. The simplest way to deal with this error is by using the same balance for the standardization at the N.P.L. as that used in the field; otherwise the scale error must be determined by experiment.

GAPS IN BASE. Although gaps in the measured line of a base are avoided when possible, it sometimes happens that the best site for a base does contain short sections over which it is impossible to measure with the necessary degree of accuracy. Such a gap occurred

in one section of the Ridgeway base in 1938, where a ravine about 400 feet wide had to be bridged by triangulation, the triangulation being carried out 'both from single triangles and from quadrilaterals with offset bases on both sides. . . . Finally the gap was negotiated with 24 metre tapes in order to gain experience of measurement on very steep slopes amounting in some cases to 40°. The result of this direct measurement agreed well, although possibly fortuitously, with the triangulated measures'.*

ACCURACY OF BASE MEASUREMENT. The sources of error in base measurement are, firstly, the constant error of standardization of the apparatus, and secondly, the accidental errors in the field of reading, levelling, temperature, etc.

Errors in the second group tend to cancel out and may be reduced by repetition of the measurement, but obviously some error will remain, and consequently the accuracy of the base measurement must always be less than the accuracy of the standardization.

The N.P.L. Class A Certificate guarantees standardization to an accuracy of 1/1,000,000, and from this David Clark concludes that the true probable error of a base measurement usually lies somewhere about 1/500,000.

MEASUREMENT BY ELECTRONIC METHODS

It has already been mentioned that the use of radio for precise measurement is still (1951) in the experimental state, and the remarks that follow are almost entirely extracted from articles contributed to the Empire Survey Review† by J. Warner of the Commonwealth Scientific and Industrial Research Organization, Australia, and Lieut.-Col. C. A. Hart, R.E.

Most of the experimental work has been carried out with radar equipment, and the account published by Mr. Warner is of trials carried out in Australia where 'the results of a large number of radar measurements of six distances, varying from 160 to 310 miles in length, indicated that an accuracy of 7 parts in 10^5 can be achieved. Equipment errors constitute the immediate limits to accuracy, but reasonable modifications would yield a figure of 2

* Empire Survey Review, Vol. V, No. 34, p. 223.

† Empire Survey Review, Vol. X, No. 78. 'The Application of Radar to Surveying'. J. Warner.

Empire Survey Review, Vol. IX, No. 65. 'Modern Mapping—including the employment of radio techniques'. C. A. Hart.

parts in 10^5 . Radar measurements (of this length) can be completed in a fraction of the time required by normal ground survey methods . . .'

LIMITATIONS OF THE TECHNIQUES. A radar measurement of distance is calculated from the time taken by a pulsed radio signal to travel that distance and return and from a knowledge of the velocity of propagation of radio waves. The time interval is usually measured by comparison with the period of a crystal oscillator, the accuracy and stability of whose frequency is in the order of $1/10^8$; but the velocity of propagation of electromagnetic waves *in vacuo* is at present known only to about $1.3/10^5$, and this uncertainty is further increased because the velocity varies with the refractive index of the medium in which the propagation occurs. In the case of the atmosphere this varies with air temperature, pressure and water vapour content; extreme changes of atmospheric conditions from normal may cause changes in the time of travel of the order of $4/10^5$. It can be seen, therefore, that the precision of the measurement depends chiefly on the accuracy with which the refractive index can be calculated from the meteorological conditions. With the accuracy of recording these conditions at present prevailing, Warner puts the accuracy of the resulting refractive index at $2/10^5$. However, it is impracticable to measure temperature, pressure and humidity at every point along a radio path of 200 miles or so, and so $2/10^5$ must be taken as the practical upper limit of accuracy of measurement under present conditions.

TABLE 3
RESULTS OF EXPERIMENTS IN MEASUREMENT
OF LONG DISTANCES BY RADAR

Ground survey distance (miles)	Radar			
	Corrected distance (miles)	No. of observations	Error (miles)	Proportional error $/10^5$
158.812	158.824	6	± 0.012	7.6
192.815	192.828	17	± 0.013	6.7
239.723	239.726	30	± 0.003	1.3
187.234	187.220	30	$- 0.014$	7.5
260.005	260.026	19	± 0.021	8.1
311.301	311.334	20	± 0.033	10.6
Average			± 0.016	7.0

Hart states that the fundamental accuracy at present obtainable with radar is about ± 20 metres; it will be seen that the average error of the above experiments was ± 25.7 metres. After mentioning two experiments over ranges of 618 km. and 200 km., which gave errors of 3.3 and $1/10^5$, Hart concludes that 5 to $7/10^5$ is the limit of universal accuracy with our present knowledge.

METHOD OF MEASUREMENT. The quasi-optical characteristics of high-frequency transmission limits the range of radar to roughly the optical horizon which will seldom exceed about 60 miles. To increase the range the method adopted is to fly an aircraft several times across the approximate mid point of the base, using the mean of the minimum sum of the ranges as the data for calculating the geodetic distance. In the Australian experiments self-recording instruments were carried in the aircraft, which automatically plotted the sum of the two distances on a graphical record.

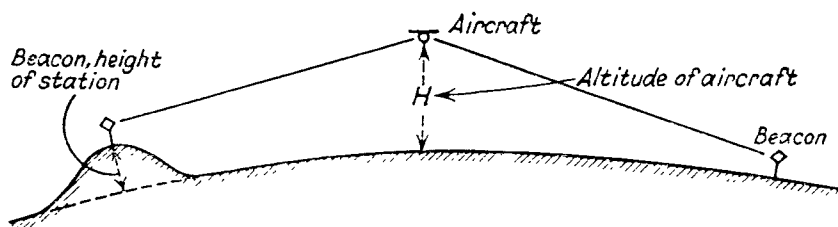


FIG. 6

It is obvious from the figure that a knowledge of the altitude of the aircraft and the heights of the stations is essential. The latter will probably be known to a high precision of accuracy; knowledge of the exact altitude of the aircraft is, however, less certain, and it is interesting to note that for a range of 200 miles between ground stations and an aircraft flying at 10,000 ft. an error of 100 ft. in altitude produces an error of about 6.2 ft. in the range, i.e. less than $1/10^5$.

CHAPTER 3

MINOR TRIANGULATION

FOR APPROXIMATELY three centuries the 'control' for national surveys has been provided by triangulation; but, as will be explained later, in (comparatively) recent years, in countries which are fundamentally unsuited for triangulation, the control has been provided in part or in whole by precise traversing. When electronic methods of measuring long lines (see Chapter 2) are perfected, it is possible that it may prove easier and cheaper to measure the distances rather than the directions between control points; the resulting figures when plotted will still form a series of triangles, but the technique of observing, computing and adjusting is so different from that of the conventional triangulation that the name *trilateration* has been given to it. Throughout this chapter the word triangulation will be taken to mean a system of triangles of which the angles are the measured quantities.

ORDERS OF TRIANGULATION. In the national surveys it is customary to organize the triangulation in stages. The country is first covered with a network, as in Great Britain, or a grid, as in India, of large triangles with sides of from 10 to 100 miles, or more; this is known as the primary or major triangulation, and all observations are made with the utmost precision. Obviously points 100 miles apart are of little use for 'controlling' the detail needed for large-scale mapping, and so the country between the primary stations is covered with a network of smaller triangles, with sides of from 5 to 25 miles, known as the secondary or minor triangulation, the observations for which will be carried out with slightly less precision than those for the primary triangulation. Finally, the secondary triangulation will be broken down into a tertiary triangulation, with sides of less than six miles, to provide the close control necessary for plotting the topographical detail by means of plane-tableing, traverse or chain surveying. In small countries, major and minor triangulation may suffice, and in smaller, such as the Island of Malta, a minor triangulation provides all the control necessary.

Yet another type should be mentioned, and that is the triangulation undertaken to determine the size and shape of the earth; this is geodetic triangulation. (Historically the first extended, calculated triangulation, of which we have record, was the geodetic triangulation made by Willebrord Snel van Roien [*Snellius*] in 1615 for measuring the arc of the meridian between Alkmaar and Bergen op Zoom; though there are records of plane-table triangulations dating back to the last half of the sixteenth century.) Geodetic triangulation usually has as its objective the measurement of as large an arc of a meridian as possible, and, since all measurements are carried out with the greatest refinements of precision, such a triangulation when passing through a country is naturally used as the basis and origin for its primary triangulation, so much so that the terms *geodetic triangulation* and *primary triangulation* have now become almost synonymous.

A further classification of triangulations is sometimes used, that of 'orders', arranged according to mean triangular error. The following grading has been suggested (Brit. Assoc. Report, 1913):

First Order	mean triangular error not to exceed	1"
Second Order	" " " " " "	5"
Third Order	" " " " " "	15"
Fourth Order	" " " " " "	30"

These gradings are not rigidly followed, and the term 'First Order', the most frequently used, usually denotes triangulation of geodetic standard.

Although many engineering surveys must be conducted with first order precision, such surveys (e.g. that for the Sydney Bridge) are usually of small extent; for any of larger extent, second or third order procedure usually suffices. Throughout this chapter the methods described are those for minor triangulation which will be found suitable for most engineering and topographical surveys.

For descriptive purposes triangulation may be divided into definite stages

- (a) Reconnaissance (selection of stations)
- (b) Beaconing and marking of stations
- (c) Observing
- (d) Computing and adjusting

RECONNAISSANCE

The necessity for careful reconnaissance before undertaking any survey, large or small, cannot be over-emphasized. The form of reconnaissance may vary between a complete plane-table sketch survey of the area and a few notes concerning the amenities, means of access and arcs of visibility at sites previously selected from air-photographs or previous records. The fullest use should be made of all existing data concerning the area, maps, photographs and literature which sometimes may be good enough to enable the bulk of the reconnaissance to be carried out in the office.

SITES FOR STATIONS. Many requirements must be considered when selecting the sites for the triangulation stations, the first of which is the average length of side of the triangulation. The nature of the country and its climate, and the purpose of the survey, will largely determine this; a balance being struck between long sides with comparatively few stations and short sides with a multiplicity of stations. The former may cover the ground more quickly and cheaply but risks long delays during spells of poor visibility and bad weather, while the latter contains much source for inaccuracy and accumulation of errors. The estimated time that will be taken in providing the denser secondary control from the main triangulation must be an important factor in making the final decision.

Once the general length of side is decided (and it must be realized that considerable departure from this average length will probably be necessary), the 'condition' of the triangles is all important. The ideal shape for an individual triangle is the equilateral, and it may be possible to arrange the triangulation as a net of interlacing hexagons, each composed of six (approximately) equilateral triangles. (See Fig. 7 which illustrates the south-east part of the triangulation of Malta.)

On the other hand if the triangulation is planned as a chain of braced quadrilaterals, the 45° right-angled triangle is ideal. (See Fig. 8.) Generally speaking, angles of less than 35° to 40° should not be permitted in the main triangulation. (It is, however, permissible to insert a wedge-shaped triangle with a small hinge angle when the general direction of the chain is being changed.)

In making the final choice of site attention must be paid to the following:

Intervisibility between surrounding stations. An all-round view is very desirable, but if this is unobtainable there must be a clear line of sight to adjacent stations (or lines of sight which may be cleared).

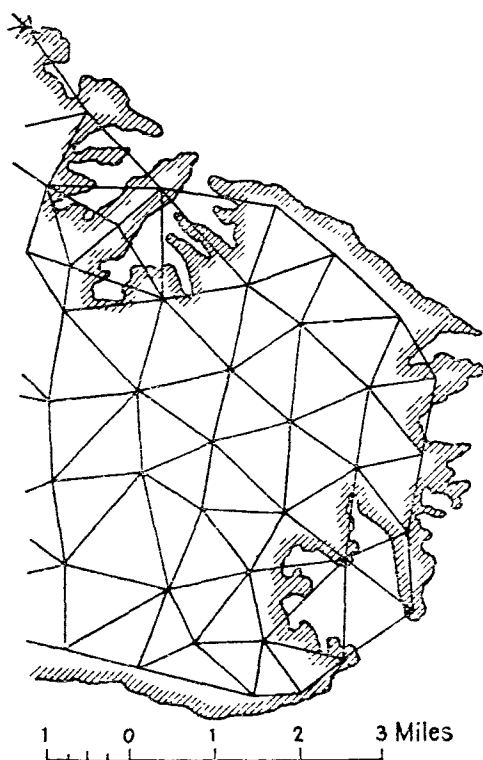


FIG. 7

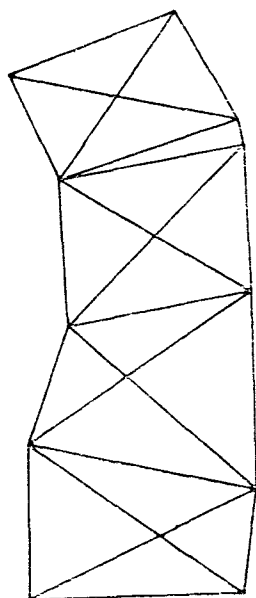


FIG. 8

In some cases intervisibility can be achieved, on an otherwise desirable site, only by the erection of some structure which will elevate the observer sufficiently for the line of sight to clear the intervening obstruction.

The vertical distance Q in Fig. 9, between a horizontal line of sight, as modified by terrestrial refraction, and the surface of the earth is given by $Q = \frac{S^2}{R} (\frac{1}{2} - \mu)$ where S = the length of the line of sight and R = the radius of the earth.

μ is the coefficient of refraction, and, taking an average value of 0.07 for this, Table 4 shows the value of Q in feet for values of

$AB (= S \text{ approx.})$ in miles, i.e. Q in feet $= 0.574 S^2$ in miles. (See page 99.)

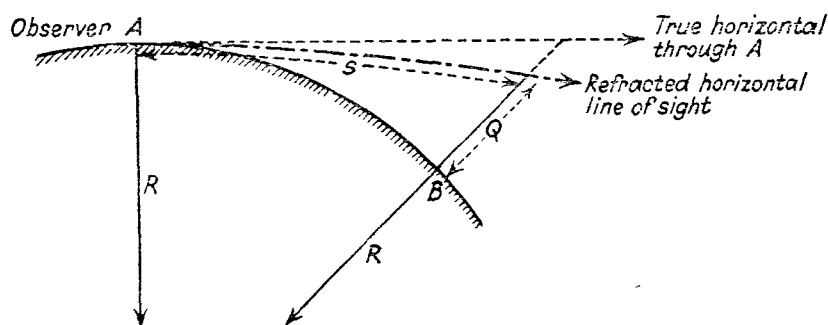


FIG. 9

TABLE 4
CURVATURE AND REFRACTION

Dist. miles	Q feet	Dist. miles	Q feet	Dist. miles	Q feet	Dist. miles	Q feet
1 ..	0.6	7 ..	28.1	13 ..	97.0	19 ..	207.2
2 ..	2.3	8 ..	36.7	14 ..	112.5	20 ..	229.5
3 ..	5.2	9 ..	46.5	15 ..	129.1	21 ..	253.1
4 ..	9.2	10 ..	57.4	16 ..	146.9	22 ..	277.7
5 ..	14.3	11 ..	69.4	17 ..	165.8	23 ..	303.6
6 ..	20.7	12 ..	82.6	18 ..	185.9	24 ..	330.5

From the table it can be seen that, for a line of sight of 20 miles (which for the purpose of this chapter may be taken as a maximum), if both stations are elevated 57.4 feet above datum level, or alternatively if one is 5 ft. and the other 166 ft. above datum level, the lines of sight will just clear the intervening ground if it, also, is at datum level.

Should the ground levels of the stations be below the line of sight, the above table may be used to determine the height of structure necessary to provide a clear line. It can be readily seen that, when the ground levels of the two stations are of equal height, it will be most economical of material to build structures at each station, but, when they are of unequal elevations, for economy the structure should be erected at the lower station.

When considering intervisibility two further factors must be taken into account. Firstly, to escape the disturbed air close to the ground grazing rays must be avoided and lines of sight should clear

intervening ground by at least 6 feet (10 feet for first order work), and, secondly, allowance must be made for the intervening topography. The two factors may be taken together by adding 10 ft. to the elevation of any peaks which offer obstruction.

Usually it will be possible to test intervisibility during the reconnaissance and by climbing a tree, etc., ascertain whether intervening ground will necessitate the erection of scaffolding from which to observe. If the height and position of any high intervening ground are known, it is possible to make a fairly accurate estimation of the height of scaffolding required, by use of the above table and a slide rule.

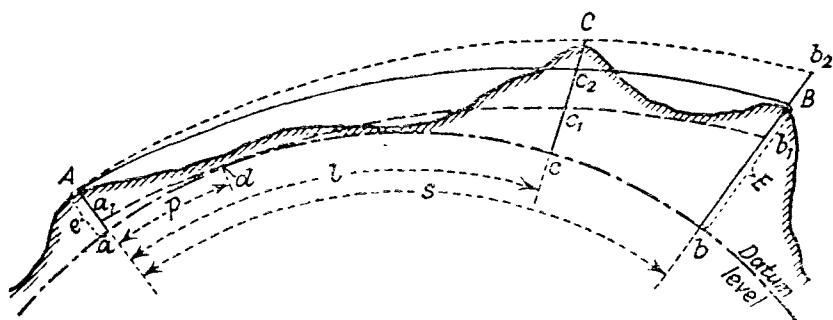


FIG. 10

In Figure 10 let A and B be two stations of heights e ft. and E ft. and C an intervening obstruction. a , b , and c are the projections of A, B, and C on the datum level. The firm line $A c_2 B$ is the direct line of sight between A and B, and the pecked line $a_1 d c_1 b_1$ is a line drawn parallel to this, tangential to the datum level at d ; so

$$Aa_1 = c_2 c_1 = Bb_1 = x \text{ (say)}$$

let the distances, $ab = s$ miles, $ac = l$ miles and $ad = p$ miles

From page 48 and using coefficient of refraction of $\cdot 07$:

$$e - x = \cdot 574 p^2$$

$$E - x = \cdot 574 (s - p)^2$$

By subtraction

$$E - e = \cdot 574 (s^2 - 2 sp)$$

i.e.

$$p = \frac{s}{2} - \frac{(E - e)}{1 \cdot 148 s}$$

The heights aa_1 , bb_1 and cc_1 may be found from the table opposite distances of p , $s - p$, and $l - p$ from which x may be found

$$= e - aa_1 = E - bb_1$$

This checks the working and enables $cc_2 = cc_1 + x$ to be calculated. To clear C by 10 feet the ray must be raised by $Cc_2 + 10$ and by simple proportion this will raise the ray at B by

$$Bb_2 = \frac{s}{l} \times (Cc_2 + 10)$$

Example

Two stations A and B, 19.6 miles apart, are 52 feet and 195 feet, respectively, above datum. An intervening peak C, situated 3.9 miles from B, is 169 feet above datum. Calculate height of scaffolding necessary for the ray AB to clear C by 10 feet.

- (1) *Test intervisibility between A and B (apart from obstruction C)*

From Table: distance for Q of 52 ft. = 9.5 miles

„ „ Q of 196 ft. = 18.4 „

Sum 27.9 miles exceeds 19.6 miles

∴ (but for C) A and B are intervisible.

(If they were not, scaffolding would be required at A, the lower station, to make them so. See p. 48.)

- (2) *Find height of scaffolding at B, the station nearer to C, so that ray AB may clear C by 10 ft.*

$e = 52$ ft. $E = 195$ ft. $s = 19.6$ miles. $l = 15.7$ miles

$$\begin{aligned} \text{In Fig. 10} \quad p &= \frac{s}{2} - \frac{(E - e)}{1.148s} = 9.8 - \frac{143}{1.148 \times 19.6} \\ &= 9.8 - 6.35 \quad (\text{slide rule}) \\ &= 3.45 \text{ miles} \end{aligned}$$

From table

Q for p , 3.45 miles = aa_1 = 7.0 ft. and $e - aa_1 = 45$ ft. = x

Q for $s - p$, 16.15 „ = bb_1 = 149.7 ft. „ $E - bb_1 = 45.3$ ft. = x

Q for $l - p$, 12.25 „ = cc_1 = 86.2 ft.

$cc_1 + x = 131.3$ ft.

To clear C by 10 ft. ray AB must be raised by $169 + 10 - 131.3 = 47.7$ ft.

Heights of theodolites and signals at A and B will supply 4.7 feet of this.

∴ height of scaffold required at B = $\frac{s}{l} \times 43$ feet

$$= \frac{19.6}{15.7} \times 43 = \underline{54 \text{ feet}}$$

Scaffolds may be made of different materials according to the local supply situation, but all should have this in common: the structure on which the theodolite stands must be rigid and must make provision for it to be centred over the ground mark and, further, must be entirely independent of the structure which supports the observer. Masonry or brick towers are undoubtedly the most satisfactory, but scarcity of local material and the cost will

usually make them prohibitive. Braced, lattice wooden towers are perhaps the most common, an inner three-cornered structure for the theodolite and an outer three- or four-cornered tower to support the observer's platform. The inner towers may be provided with tubes in which the plumb line can hang undisturbed by the wind, but with the efficient optical plumbing fitted to most modern theodolites this should usually be unnecessary. Portable steel towers are now frequently used. The *Bilby* tower used by the U.S.C. & G.S., which can raise the observer 100 to 130 feet above ground, with a beacon 10 feet higher, weighs 3 tons (max. weight of any one piece, 60 lb.) and can be erected in 5 hours.

GRAZING RAYS TO BE AVOIDED. The vertical clearance of the ray above the ground has been mentioned above but it is even more important that the ray should also have a lateral clearance of 10 feet from cliffs and shoulders of hills.

ACCESSIBILITY. If time is not to be wasted the site chosen should be reasonably accessible, so that advantage may be taken of temporary favourable observational conditions.

BEACONING AND MARKING OF STATIONS

BEACONING. Any object or device used to define for a distant observer the exact position of a station may be termed a signal, and such signals may be either opaque or luminous. Requirements for all signals are that they must be: clearly visible; so designed that they can be accurately bisected, both horizontally and vertically, under all conditions of light; stable and rigidly centred over the station mark.

For most accurate work, especially when sides are long (say 20 miles or more), it is better to use luminous signals; not only because they show up better and are more easily bisected but because it is usually preferable to observe at night owing to the more favourable atmospheric conditions. Even by day a heliotrope (or heliostat) reflecting the sun's rays or a powerful electric lamp will show up better than an opaque signal should the weather be at all hazy.

Heliotropes may be improvised from an ordinary small (shaving) mirror mounted on a horizontal axis and free to turn in azimuth; the old military heliograph has frequently been used with success. The manufactured variety is fitted with a telescope, rigidly

attached to which are sighting rings through which the reflected beam may be directed after the telescope has been sighted on the observer. The Admiralty hydrographic service use the Galton heliostat, which is fully described in the *Admiralty Manual of Hydrographic Surveying*.

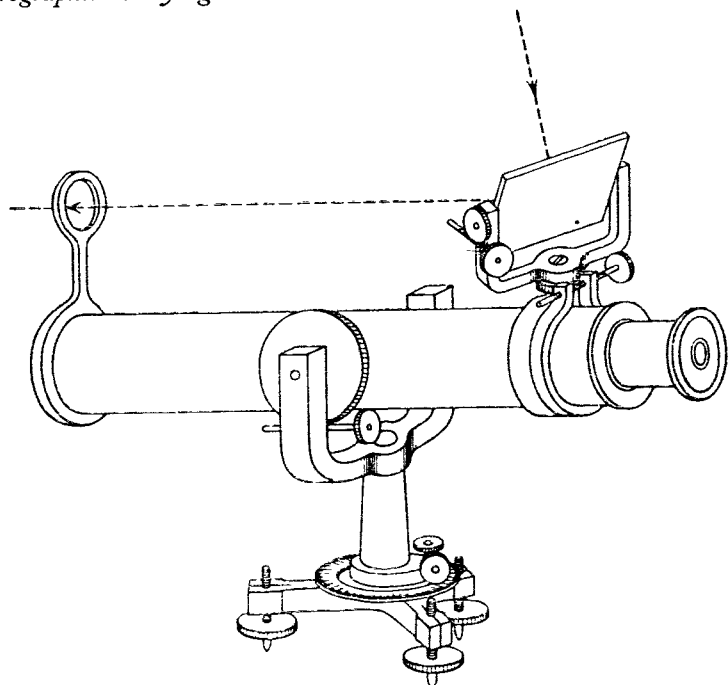


FIG. 11. Diagram of Heliostat

With all sun signals the mirror must be centred over (or in line with) the station mark, after which the reflected beam from the sun must be directed towards the observer. With the manufactured variety directing rings or targets are incorporated in the design. With the improvised variety a pole must be set up some little distance from the heliotrope with its tip (or a small target on it) exactly in line with the observer; the reflected beam may then be directed on to the pole end. No very great precision for this alignment is necessary, since the reflected beam diverges from the mirror at an angle of $32'$ equal to that subtended by the sun's diameter at the mirror. If as frequently happens the observer's station cannot be seen from the heliotrope, the required direction and elevation

must be calculated, possibly assisted by flashes from the observer. The heliotrope must be attended while observations are in progress, so that adjustments may be made every minute or so to keep the reflected beam pointed in the required direction. When the angle subtended at the heliotrope between the observer and the sun becomes too obtuse, a second mirror must be introduced with which to reflect the sun's rays directly on to the heliotrope mirror.

Various forms of lamps have been used for night observations. Lime light was used in the original Ordnance triangulation, and an electric lamp manufactured by Messrs. Cooke, Troughton and Simms was used in the re-triangulation of Great Britain. Current was supplied from a 6-volt dry battery or accumulator, and bulbs of 24, 12, 6, and 3 watts were interchanged according to the various conditions of distance and visibility. Oil, acetylene and petrol vapour lamps have been used in other countries. With all lamps careful centring of the light or, alternatively, careful alignment in the direction of the observer is essential.

For sides up to 20 miles opaque signals are generally satisfactory; they may be of many types but must satisfy the requirements already mentioned. The signal should subtend an angle of about 2 sec. at the observer, i.e. its width should be about $\frac{1}{2}$ inch per mile of the length of the ray. A single pole may be used for rays up to about 3 miles (provided it has a top mark to give it prominence), but beyond that range the width of the pole must be increased by securing slats or targets to the pole. These must be placed symmetrically and ideally should face the observer; but, as most beacons have to be viewed from many directions, the targets should be placed in pairs at right angles to and one above the other. So that the theodolite may be set up under the beacon the central pole is normally supported on a tripod or quadripod; the latter is better since it is symmetrical about the centre from all directions. The top of the quadripod may be slatted, and in some cases this suffices and no central pole is required.

Opaque signals will usually show 'phase' when illuminated by the sun, and this must be guarded against or allowed for; e.g. two sides of a four-sided pyramid will be in sunlight and two in the shade. An observer from any distance will see the illuminated face so much more plainly than that in shadow that he will probably bisect this face with his cross-wires and so obtain a very erroneous reading. It can be seen that the crossed targets shown in Fig. 12

will exhibit no such phase. Phase error can be seen more plainly in the case of a cylindrical mark as shown in Fig. 13.

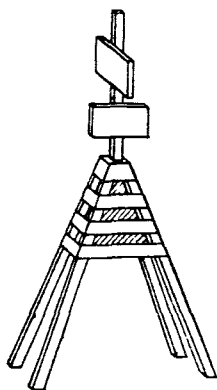


FIG. 12

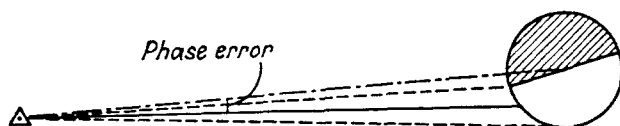


FIG. 13

If it is to be conspicuous the height of a beacon in feet should about equal the distance it has to be seen in miles. In other words most beacons for the work being considered would be from 10 to 20 feet high.

As far as centring the beacons over the station mark is concerned, it is usually easier to erect the beacon first and centre the mark under it.

Towers have already been mentioned. With them the signal is incorporated on the outer, observer's, structure.

MARKING THE STATION. Permanence and easy recovery are essential. On solid rock a hole should be drilled and a brass screw or something similar cemented in; but, on ground which can be dug, two marks should be used, a buried mark and a surface mark. The buried mark should be cut on a stone which should be covered with earth, and a similarly marked stone should then be inserted above

it at ground level, and the whole surmounted by a cairn, or better still by an observing pillar of the type used by the Ordnance Survey. The buried mark should not be referred to unless there is reason to suspect that the surface mark has been disturbed. In addition three or four witness marks, broad arrows cut on rock, bolts set in cement, etc., should be placed at some distance from the station and carefully fixed, by azimuth and distance, from it; by these the station may be recovered.

A full description of each station must be prepared, with its name, number, general locality, position of witness marks and, in addition, its position with reference to any conspicuous permanent or semi-permanent objects, natural or otherwise (e.g. 3.3 feet north of line joining St. Paul's church spire with St. Elmo Light House). The description should be amplified by sketches and/or photographs. In addition information concerning local facilities, labour, transport, subsistence, etc., should be given which may assist subsequent occupiers of the station.

Obviously many minor triangulation schemes undertaken by civil engineers will cover such a small area and be of such a temporary character that much of what has been written above will be redundant; nevertheless stations should always be marked so that their recovery may be assured for a period longer than appears foreseeably necessary.

OBSERVING

Theodolites, their methods of reading and adjustment, have been described in Volume I, and for the work envisaged in this chapter it is assumed that optical theodolites of the double reading type such as the Cooke, Troughton and Simms ($3\frac{1}{2}''$) 'Tavistock', the Watts 'Microptic No. 2' or the Wild 'T2', all reading to seconds of arc, or a 5-inch micrometer theodolite, reading directly to 10 secs. and by estimation to 1 sec., will be available.

ADJUSTMENTS OF THEODOLITE. No extra adjustments are required for the precise observations being considered, but special attention should be paid to the following:

Loose joints. The test is to intersect some clearly defined object and clamp the horizontal circle; then, while looking through the telescope, gently press the eye end sideways. This will throw the vertical cross-wire off the object, but it should return when the

pressure is removed. If it does not, suspicion should fall on loose foot-screws or loose joints between the wood and metal parts of the tripod.

Centring. Many modern instruments have either built-in optical plumbing or a separate optical plummet. These sometimes have a small error, so that even after careful levelling the cross-wires of the plummet will describe a small circle round the plumb point when the theodolite is rotated. This error, if small, is of no account, provided the centring procedure includes a complete rotation of the theodolite while the plumb point is being observed.

Run of micrometers. In the 5" micrometer theodolite the 'run' should be adjusted if it exceeds 10 seconds for a traverse of the webs across a 10-minute gap, but if it is less than that it should be allowed for during the observations, by taking two settings of the micrometer drum for each reading of the theodolite, one when the webs have been centred over the main scale division behind the mean setting and one when it is centred over the main scale division in advance of the mean setting. The settings will be booked in the angle book as '*b*' and '*f*'. Difference between the two settings indicates the run, so that '*r*', the run developed in '*D*' (which is the distance between two main scale graduations), is equal to $(b - f)$ and *m*, the mean reading $= \frac{b + f}{2}$, then the necessary correction to

m is expressed with sufficient accuracy as $\frac{r}{2} - \frac{mr}{D}$

(It will be noted that: (i) The correction has the same sign as *r* for values of *m* up to $\frac{D}{2}$ and the opposite sign when *m* exceeds $\frac{D}{2}$;

(ii) It is unnecessary to correct each micrometer separately; the two back readings are meaned and the two fore readings, and the correction is obtained from those means and applied to the mean of the means.)

Stability. Particular attention must be paid to the stability of the instrument, and, when it is being used on its own tripod, if the ground is at all soft, stakes should be first firmly driven in to support the tripod feet. The instrument should be kept shaded from the sun either by a tent or a large umbrella to prevent twist as one side of the tripod and theodolite becomes more heated than the other.

TIMES TO OBSERVE. In precise work probably the worst source of error in horizontal angles is refraction out of the vertical plane, and this is likely to be least in the couple of hours or so after sunrise or before sunset; these hours also are usually the best for heliotrope work. In the heat of the day the shimmer is often so bad that rays which pass close to the ground are impossible to observe; in cloudy weather, on the other hand, it is often possible to observe all day. If hazy weather is predominant, probably all rays longer than (say) 10 miles will have to be observed to luminous signals and so will be easier seen at night. Normal observing hours for the re-triangulation of Great Britain (1935-38) were for the five hours after sunset.*

For vertical angles the traditional times for observing are in the heat of the day, near to 3 p.m. when terrestrial refraction is most stable; but during the measurement of the East African Arc of Meridian it was found that observations made between 10.30 and 11 p.m. gave equally good results,† and consequently the permitted hours during the re-triangulation of Great Britain were noon to 3 p.m. and 'any time over three hours after sunset'.*

METHODS OF OBSERVING. Bomford in his *Geodesy*‡ gives four methods of observing horizontal angles for primary triangulation and deliberately excludes 'the old system of measuring angles by "repetition"'. Any method adopted must be designed to reduce to a minimum the effect of instrumental and observational errors. Errors of collimation and dislevelment of the horizontal axis are eliminated by observing half the angles on one face and half on the other. To eliminate errors of twist half the observations should be taken swinging right and half swinging left. Errors of eccentricity are eliminated by reading symmetrically placed micrometers or verniers. (This is done automatically in the case of the optical instruments mentioned on p. 55.) The effect of errors of graduation is minimized by changing zero and so using a different part of the circle for each 'set' (of a pair of *face right* and *face left* measurements). Errors of bisection of the target are reduced by increasing the number of observations; and errors of reading, by re-setting and reading the micrometer twice or thrice at each pointing.

(*Note.* In some geodetic theodolites such as the Wild 'T3' a second reading of the micrometer is an essential part of the reading routine.)

* Empire Survey Review, Vol. IV, p. 400.

† Empire Survey Review, Vol. III, p. 78.

‡ *Geodesy*, Brigadier G. Bomford, p. 21.

Method of rounds. This is undoubtedly the most satisfactory method and that which is most economical in time and labour; it is alternatively known as the *reiteration* method or the *direction* method (though a more narrow and specialized meaning of this term is described later). In this method the directions of all stations to be observed are measured with reference to one of the stations which has been arbitrarily selected as the referring object, the R.O.

The R.O. should normally be one of the triangulation stations to be observed, and should be the one that is most likely always to be visible. If all the stations are distant and equally liable to be obscured by haze, an extraneous R.O. may be selected, or established, at a distance of about two miles from the observer and at about the same elevation.

Observing routine. Suppose four stations labelled consecutively as A, B, C, and D are being observed, A having been selected as the R.O. After setting up, rotate the theodolite two or three times in azimuth to work in the bearings and then in the *face left* position set the optical micrometer (or the 'A' micrometer in older types) to read about 0° , and point at the R.O. Read all micrometers and book, throw off the micrometer and re-read, and then, swinging right, point successively to stations B, C and D and carry out the same reading procedure. It is important for precise work to preserve the direction of the swing and see that each succeeding station is not overshot either in the main setting or with the slow-motion screw. After observing D, move on slightly and change face to *face right*; then, swinging left, intersect D, C, B and A in that order. The final reading will, of course, be about 180° , and the measures so far made will constitute one *set* (of two *rounds*, one face on each). The above is frequently modified by 'closing the horizon' on each round. The procedure then is: Face left, swing right to A, B, C, D, and again to A; overshoot slightly, change to *face right* and swing left to A, D, C, B, A. On each round, therefore, there will be a double measure of the R.O., the mean of which is accepted. This modification is recommended to beginners (in precise observation) as it ensures detection of disturbance or excessive twist in the instrument during the round, but, once the observer has confidence in himself and his instrument, this modification may be dispensed with and only resorted to when the observer has reason to think that disturbance has taken place.

After the first set, the zero is altered by a pre-arranged amount and another set of two rounds observed, the only difference being that, since the theodolite will already be in the *face right* position, the first round of the set will be made swinging left. The number of zeros or sets used must depend on the precision required and the character of the instrument. For the work contemplated in this chapter a series of four zeros, F.L. $0^{\circ} 0' 0''$, F.R. $90^{\circ} 01' 15''$, F.L. $315^{\circ} 02' 30''$ and F.R. $45^{\circ} 03' 45''$, which will give eight measures of each direction, should suffice. If greater precision is required, either a complete second series with zeros $22^{\circ} 30' 10''$, $112^{\circ} 31' 25''$, $337^{\circ} 32' 40''$ and $67^{\circ} 33' 55''$ should be added, or, alternatively, a swing right and a swing left may be observed on each face of the first series only.

One or two points in the above are worth noting: (i) The minutes and seconds are altered for each zero to eliminate errors of run in the micrometer, but, when setting the micrometer for the zero at each change, a setting within about $5''$ of that specified suffices. (ii) It is generally considered undesirable to shift zeros by intervals of 60° because the manufacturers' method of dividing circles is frequently in repetition arcs of 60° ; consequently a faulty interval of graduation is liable to be repeated at regular intervals 6 times round the arc. (iii) The abstract measures must be examined before leaving a station and any measure which differs from the mean by more than $5''^*$ should be discarded and that measure repeated. (iv) In the re-triangulation of Great Britain the same precision in angular measurement is being demanded for the secondary triangulation as for the primary, except that only the eight zeros given above are observed; for the long rays of the primary triangulation a second complete series of double-face reiterations on eight zeros was added of which the whole degrees were 11° , 101° , 56° , 146° , 33° , 123° , 78° and 168° . (v) An unskilled observer is unlikely to increase the accuracy of his measures by adding yet another series of four zeros.

Broken rounds. In hazy weather in particular, but sometimes for other reasons, it may be impossible to see one of the stations to be observed. In that case the round should be completed without it and the missing station observed when it becomes visible. This missing observation may be made by measuring the angle between the

* For the optical theodolites reading to seconds; but a tolerance of $10''$ can be allowed for the older micrometer theodolites.

R.O. and the missing station, that is measuring its direction. (*Note.* The specialized meaning of the complete *method of directions* [see p. 58] is to measure separately and in turn the angle between the R.O. and each station to be observed.) Or it may be made by measuring the angle between the missing station and any other station, preferably an adjacent one which happens to be visible, i.e. the *method of angles*. When weather conditions are such that broken rounds are likely to be the rule rather than the exception it may be better to adopt either the *method of directions* (in its narrow meaning) or the *method of angles* in the first place. In either case the observing is done by *reiteration*, and the requisite number of pointings on the specified zeros must be made.

HINTS ON OBSERVING. Great concentration is necessary, but provided that is present the observations should be made quickly, though without hurry. Dithering about on both sides of the mark leads to inaccuracy and should be avoided. Speed can only be achieved if the telescope is in perfect focus with parallax completely eliminated; focus should not be altered during a round. As a further guard against parallax always look straight into all eye-pieces. Many vertical cross-wires are half single and half double; under most conditions it is better to use the double wires but, whichever is chosen, the same half should be used for the whole round. In all cases it is best to observe slightly off the horizontal wire. Consistency, and therefore accuracy, of reading the micrometers can only be achieved under similar light conditions; it is therefore better always to use the artificial illumination of the circles. With practice, consecutive readings of the micrometers for the same pointing should not differ by more than about two seconds. Clamping screws should be tightened only enough to make the slow-motion screw work; during horizontal angle observations the vertical circle should be lightly clamped, and the clamp should not be touched during the whole round. Secondary stations and intersected points should not be observed with the main stations but in a different series, usually on only half the number of zeros used for the main stations.

VERTICAL ANGLES. When these are required, they should be observed after the completion of the horizontal angles during the periods specified on page 57. They should be observed in sets, a

face left immediately followed by a *face right* in each set. Two or three sets, preferably in the form of reciprocal observations, should be made over a period of not less than one hour (to minimize short-term vagaries in refraction). Ideally, simultaneous reciprocal observations between the two stations should be made, but this is seldom practicable; the 'reciprocal' observations should, however, be made at the same period of day or night even if they cannot be made on the same day.

BOOKING. In any precise work a booker should be employed. The observer is then able to concentrate entirely on his observation, and the work is not only much expedited but more accurately observed. In any case the booking must be clear, consistent and permanent.

The lay-out may take many forms but should provide spaces for the booker to mean and abstract the readings as they are made, so that faulty observations may be repeated without undue delay. Full details of the set-up, height of instrument, height of signal, weather (chiefly wind and visibility), etc., must be included so that the value of the observations may be properly assessed at headquarters.

The degrees and minutes of each pointing need be read only on the first two rounds of angles; thereafter the seconds only are required.

A suitable form for observing with a Tavistock theodolite is shown overleaf.

A sound rule is that all bookings should be in ink and no erasures allowed.

The description of the station may conveniently be made in the angle book.

SATELLITE STATIONS. Although these should be avoided in precise work, it is sometimes impossible to do so, particularly when church spires are being used as stations or on the occasions when it is impossible to set up the light signal directly in line with the observing station.

Two cases arise: (i) When the satellite station is at the observer's end of the ray and (ii) when it is at the station being observed. In each case the problem is simplified if the necessary correction is made to each 'direction', the final corrected angle between two stations being obtained by the difference between their (corrected) directions in the usual way.

TABLE 5
BOOKING OF THEODOLITE ANGLES

<i>Survey and General Locality</i>								
<i>OBSERVING STATION</i>						<i>Date</i>		
<i>Instrument No.</i>			<i>Ht. of Inst.</i>			<i>Observer</i>		
<i>Weather</i>			<i>Ht. of Beacon</i>			<i>Recorder</i>		
<i>Station Observed</i>	<i>Set</i>	<i>Face Swing</i>	<i>Time</i>	<i>Reading</i>		<i>Mean "</i>	<i>Direction Abstract</i>	<i>Remarks</i>
P Δ (R.O.)	1	L	5.07	360 00	05.2 06.2	05.7	360 00 00	
		R	5.22	180 00	21.3 20.5		00 00	
	2	R	5.44	090 01	18.4 17.6	18.0	00 00	
		L						
		L	6.00	270 01	33.5 33.9		00 00	
		R						
	
	
Q Δ	1	L	5.12	051 27	19.3 20.5	19.9	051 27 14.2	
		R	5.16	231 27	35.2 36.7		15.1	
	2	R	5.38	28	35.6 33.8	34.7	16.7	
		L						
		L	6.05	28	49.7 48.7		15.5	
		R						
	
	

In Figure 14 O is the observer and T is the true station. Let the distance $OT = d$ and the distance from T to the station being observed be s_a, s_b and s_c , etc.

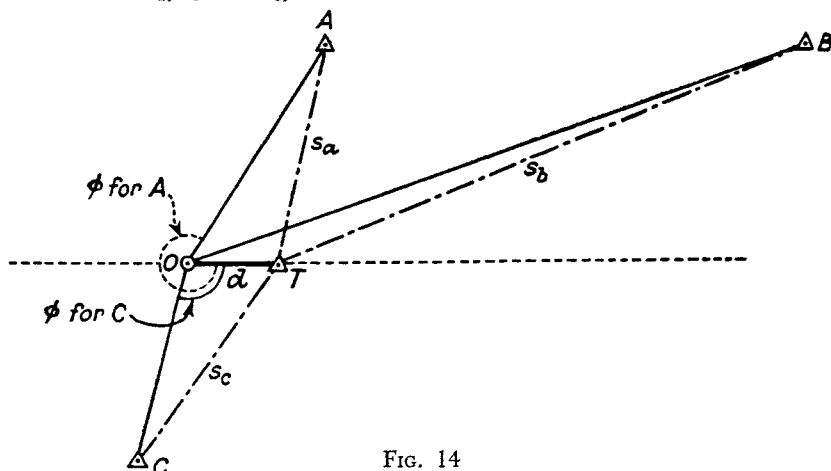


FIG. 14

For each ray the direction as observed from O may be reduced to the direction that would have been observed from T by applying the angles OAT, OBT, OCT , etc., and the values of these angles are $\sin^{-1} \frac{d \sin AOT}{s_a}, \frac{d \sin BOT}{s_b}, \frac{d \sin COT}{s_c}$. The angles AOT, BOT, COT , etc., are in each case the difference between the direction of the ray being corrected and the direction of the ray to the true station; therefore *from the theodolite direction it is desired to correct subtract the direction to the true station - call the resulting angle ϕ : then the satellite correction to each direction to reduce it to centre is given by*

$$\sin^{-1} \frac{d \sin \phi}{s}$$

where $d =$ distance from satellite to true station
 $s =$,, ,, true station to station observed } expressed in same units

when ϕ is less than 180° correction is positive

,, ,, ,, greater ,, ,, ,, negative

When d is very small the correction can be obtained directly in seconds by using the formula

$$\text{reduction} = \frac{k \sin \phi}{s} \text{ where } k = d \operatorname{cosec} 1''$$

In case (ii), when the correction is necessitated by the signal being out of line, the angle ϕ must be measured at the true station being observed.

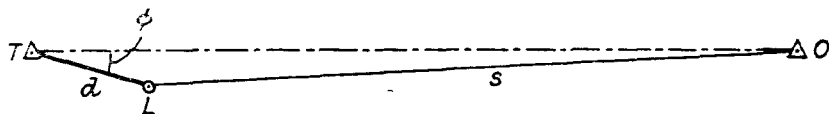


FIG. 15

In Fig. 15 O is the observer, L the signal being observed and T the true station required. The angle $OTL = \phi$ is observed at T and TOL the required correction $= \sin^{-1} \frac{d \sin \phi}{s}$ where, as before, d = distance TL, satellite to true, but $s = OL$, observer to satellite.

The correction is positive when at T, L is to the right of O
 „ „ „ negative „ „ T, L is to the left of O
 d must be measured to the nearest $\frac{1}{2}$ inch if accuracy is to be maintained ($\frac{1}{2}$ inch subtends 0.5 sec. at $3\frac{1}{4}$ miles); ϕ must be measured with similar accuracy, i.e. T must be bisected from O (case (i)) or L must be bisected from T (case (ii)) to within $\frac{1}{2}$ inch. This accuracy is frequently hard to attain, and ingenuity may have to be exercised particularly when O and T are at different elevations. For the same accuracy of 0.5 sec. s must be known to 1 part in twice the correction in seconds (i.e. correction = 20", s must be accurate to 1:40). Frequently s may be measured from the rough plot of the triangulation, but when the correction is large a preliminary computation may be necessary.

COMPUTATION

In a regularly triangulated survey in which all the angles of each triangle have been observed and one or more sides or bases have been measured, the computation may be considered in stages,

- (i) adjustment of the angle measurements
- (ii) computation of the side lengths
- (iii) adjustment of the side lengths to accord with the different measured bases
- (iv) adjustment of the side lengths and azimuths to accord with the known positions of the terminal stations
- (v) calculation of the 'rectangular' co-ordinates for the particular projection being used for the finished plan or map
- (vi) calculation of geographical co-ordinates.

Of the above, (i), (iii) and (iv) are dealt with in Chapter 6, though it might be noted that (iii) and (iv) are frequently alternatives, and (vi) is usually unnecessary in the type of work being considered.

SPHERICAL EXCESS: LEGENDRE'S THEOREM. All triangles measured on the earth are spherical triangles (strictly, spheroidal triangles, but for the purposes under discussion the two are the same), and therefore the sum of the three angles of any triangle exceeds 180° by ϵ , the spherical excess. It might appear that the rules of plane trigonometry cannot be used for the calculation of the side lengths, but, in fact, ϵ is so small, 1 second per 76 square miles (approx. the area of an equilateral triangle with sides of 13 miles), that in most cases of 'minor triangulation' it may be ignored altogether. But even in the few cases in which it must be considered *if one-third of the spherical excess of the triangle is deducted from each angle, the triangle can be solved, in terms of the linear lengths of the sides, by the ordinary rules of plane trigonometry.* This is Legendre's Theorem, the proof of which will be found in most text-books on geodetic surveying. The theorem may be considered exact for triangles with sides up to 40 miles in length.

Calculating lengths of sides therefore will consist largely of solving triangles by means of the sine formula of plane trigonometry. For most of the work being considered 7-figure logarithms are commonly used, though 6-figure logarithms are sufficiently accurate. Machine computation is being used increasingly, but for this tables of natural sines to 8 places are necessary.

There is much repetition work, and the use of forms, such as shown overleaf, are considered a convenience by many. Such a form is self-explanatory, and the only comment necessary is that, for the avoidance of gross errors, it is advisable to draw a rough diagram for every triangle being solved in addition to the diagram of the whole triangulation.

RECTANGULAR CO-ORDINATES AND PLOTTING. Before the ground detail can be surveyed the triangulated control points must be plotted on paper at the correct scale, and further computation is necessary to enable this to be done. This may take the form of computing the length of the *long side*, i.e. the distance between the extreme triangulation stations, and then using this as a base from

TABLE 6


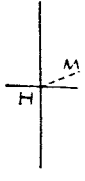

SOLUTION OF TRIANGLES					
Triangle (diagram)	Station (letter & name)	Corrected Angle "	Logarithmic Computation	Length feet/metres	
	K	47 22 8	$L \sin$ 9.866 7181		
	M <i>op.kn.side</i>	62 10 43	$L \csc$ 0.053 3479		
	HK <i>known side</i>	41 734.50	Log 4.620 4952		
	H	70 27 09	$L \sin$ 9.974 2189		
		180 00 00	Log 4.540 5612	HM 34 718.52	
			Log 4.648 0620	KM 44 469.47	
	<i>op.kn.side</i> <i>known side</i>		$L \sin$ $L \csc$ Log $L \sin$		
		180 00 00	Log		
			Log		

TABLE 7

CO-ORDINATES FROM BEARINGS AND DISTANCES					
Formulae: $\Delta E = s \sin \alpha$; $\Delta N = s \cos \alpha$ Distances in feet/metres					
		1	2		Diagrams
	Station Req'd. from Station ..	M H	M K		 for 1
Data	E (known station)	+ 5 476.10	+ 10 998.76		
	N (known station)	+ 73 815.40	+ 115 182.89		
	Co-ord. Bearing' α (known to required)	078° 03' 24"	140° 14' 07"		
	log distance (s) ..	4.540 5612	4.648 0620		
	log sin α	9.990 4953	9.805 9333		 for 2
	log s	4.540 5612	4.648 0620		
	log ΔE sum; ..	4.531 0567	4.453 9953	Accept: M E + 39 443.06 N + 81 000.20	
	ΔE (see diagram) =	+ 33 966.96	+ 28 444.30		
	E (known station)	+ 5 476.10	+ 10 998.76		
	E (required) (sum)	+ 39 443.06	+ 39 443.06		
	log cos α	9.315 8532	9.885 7442		
	log s	4.540 5612	4.648 0620		
	log ΔN sum; ..	3.856 4144	4.533 8062		
	ΔN (see diagram) =	- 7 184.79	- 34 182.68		
	N (known station)	- 73 815.40	+ 115 182.89		
	N (required) (sum)	+ 81 000.19	+ 81 000.21		

which to plot all the other triangulation stations by methods similar to the plotting of a chain survey. Except for very small surveys with six or fewer triangulation stations such a method is not recommended; long lengths are involved, and, since the method necessitates plotting the survey as a whole, a large table is necessary and the whole process tends to be very cumbersome. Usually, therefore, it is more convenient to plot by rectangular co-ordinates whereby small isolated portions of the survey may be plotted at will. The rectangular co-ordinates of the stations will usually be required in any case since they form the most compact and convenient method of recording the triangulation data; also they are virtually a necessity for the ready adjustment of the traverses which will probably be run for surveying the ground detail.

For most engineering work the computation of simple rectangular co-ordinates will suffice, whereby the co-ordinate bearings of the various sides of the triangulation are found by the successive addition or subtraction of the adjusted* angles to the known (or assumed) bearing of the initial side. The co-ordinates of each station are then found successively from the origin of the survey by adding the departure ($= \text{length of side} \times \text{sine co-ordinate bearing}$) and the diff. latitude ($= \text{length of side} \times \text{cosine co-ordinate bearing}$) to the eastings and northings of the preceding station. Once more much repetition work is involved, and the use of forms or a set lay-out is desirable, such as is shown in Table 7. Two points are worthy of attention: (1) With the exception of the second station it will be possible to compute the co-ordinates of every station from two other stations, and this should always be done as a continuous check on the arithmetical work. (2) The headings of most tables of the trigonometrical functions are printed for the first quadrant of the circle only, which necessitates converting (either mentally or on paper) each whole circle co-ordinate bearing into a quadrantal bearing before the sines and cosines can be extracted. Time can be saved over this step if a pull-out slip is pasted into the tables with information on it as shown in Table 8. e.g. Co-ordinate bearing 325° : It can be seen that sine bearing is negative and will be found in column headed Cosine 55° , while cos bearing is positive and found in column headed Sine 55° .

* See Chap. 6.

TABLE 8
CONVERSION OF QUADRANTAL BEARINGS

Angle x°	Argument to be used $= y^\circ$	Functions of y corresponding to			
		Sin x	Cos x	Tan x	Cot x
0° to 90°	$y = x$	$+\sin y$	$+\cos y$	$+\tan y$	$+\cot y$
90° to 180°	$y = x - 90$	$+\cos y$	$-\sin y$	$-\cot y$	$-\tan y$
180° to 270°	$y = x - 180$	$-\sin y$	$-\cos y$	$+\tan y$	$+\cot y$
270° to 360°	$y = x - 270$	$-\cos y$	$+\sin y$	$-\cot y$	$-\tan y$

RECTANGULAR SPHERICAL CO-ORDINATES.* Although it is not proposed to go deeply into the problem, certain aspects of rectangular co-ordinates must be understood if proper use is to be made of the data that may be supplied by the Ordnance Survey or local land survey.

Plane rectangular co-ordinates have already been discussed (in Vol. I), but, as these co-ordinates take no account of the curvature of the earth, their accuracy is insufficient unless the area covered by the survey is very limited. Consequently for any large survey the formulae for plane rectangular co-ordinates are modified by introducing extra terms which make the necessary allowance for the curvature of the earth. The revised formulae will differ for the projection adopted for the survey, but it is logical to consider first the modifications necessary for the Cassini projection which give rise to *rectangular spherical* or *Cassini* co-ordinates.

(Note. For strict accuracy a further modification is necessary to allow for the spheroidal shape of the earth, but the difference between these rectangular spheroidal and the rectangular spherical co-ordinates is so slight that only the latter will be considered.)

In Fig. 16, A and B are two trigonometrical stations and O is the origin of the co-ordinate system.

OY is the meridian through O, therefore OY produced will pass through the north pole, P.

OrtX, aAuX, and bqBX are great circles perpendicular to OY, and therefore meeting at a pole, X, situated on the equator 90° of longitude east of O.

rqA and tBu are small circles parallel to OY.

The rectangular spherical co-ordinates of A and B are:

$$A, \text{ eastings} = x_A = aA; \text{ northings} = y_A = Oa$$

$$B, \quad \quad = x_B = bB; \quad \quad = y_B = Ob$$

* See also pp. 413-418.

It will be seen also that $aA = bq = Or$
and $bB = au = Ot$

The eastings co-ordinate does, therefore, represent the distance of the station east of the y axis, but the northings co-ordinate exaggerates the true distance north of the x axis; and this exaggeration

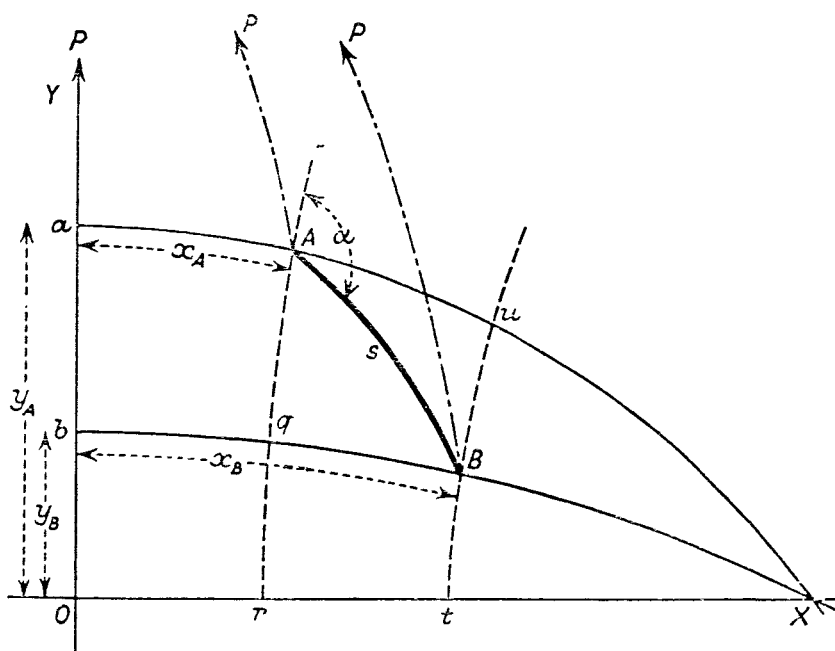


FIG. 16

increases the farther east (or west) the station is from the meridian of origin.

Fig. 17 shows diagrammatically the errors involved in treating the rectangular spherical co-ordinates as if they were plane rectangular co-ordinates.

A' and B' represent the co-ordinated positions plotted on the plane.

S' and β represent the plotted length and direction of $A'B'$.

A, B, S and α represent the true positions, length and direction of AB . It can be seen that there is distortion both of length and direction of the line $A'B'$, and from Fig. 16 it is plain that the amount of distortion will depend on the amount of the easting co-ordinate of

A, x_A ; the direction of AB, α ; and the amount of the difference of easting co-ordinates, Δx or ($x_A \sim x_B$).

The ratio $\frac{S'}{S}$, generally called the 'scale'

$$= 1 + \frac{\cos^2 \alpha}{6r^2} (x_B^2 + x_B x_A + x_A^2)$$

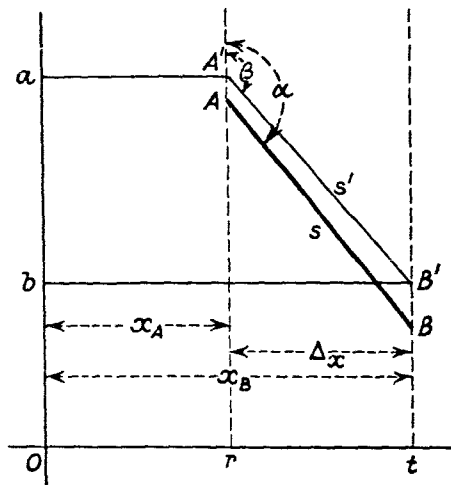


FIG. 17

(where r^2 = radius of curvature along the meridian \times radius of curvature perpendicular to meridian at latitude A), i.e. $r = \sqrt{\rho \nu}$, the quantity $\frac{\cos^2 \alpha}{6r^2} (x_B^2 + x_B x_A + x_A^2)$ being called the 'scale error'.

When A and B are very close together $x_B = x_A$ and

$$\frac{S'}{S} = 1 + \frac{x^2}{2r^2} \cos^2 \alpha$$

which

$$= 1 \text{ when } \alpha = 90^\circ \text{ (or } 270^\circ \text{)}$$

and

$$= 1 + \frac{x^2}{2r^2} \text{ when } \alpha = 0^\circ \text{ (or } 180^\circ \text{)}$$

'David Clark' gives the following table of approximate maximum linear and angular errors at various distances from the central meridian.

	Dist. in miles from central meridian				
	50	100	150	200	250
Maximum error in distance (fraction of length)	$\frac{1}{12510}$	$\frac{1}{3142}$	$\frac{1}{1396}$	$\frac{1}{785}$	$\frac{1}{503}$
Max. error in bearing	<div> <div>angle (nearest ")</div> <div>lateral displacement of length</div> </div>	<div> <div>angle (nearest ")</div> <div>lateral displacement of length</div> </div>	<div> <div>angle (nearest ")</div> <div>lateral displacement of length</div> </div>	<div> <div>angle (nearest ")</div> <div>lateral displacement of length</div> </div>	<div> <div>angle (nearest ")</div> <div>lateral displacement of length</div> </div>
	0' 8"	0' 33"	1' 14"	2' 11"	3' 25"
	$\frac{1}{25154}$	$\frac{1}{6289}$	$\frac{1}{2791}$	$\frac{1}{1571}$	$\frac{1}{1005}$

Computing rectangular spherical co-ordinates from bearing and distance. Formulae which can be used for computing rectangular spherical co-ordinates from bearings and distances, when A is the known station and B the unknown, are

$$\text{easting } x_B = x_A + S \sin \alpha - \frac{S^2 \cos^2 \alpha}{6r^2} (S \sin \alpha + 3x_A)$$

$$\text{northing } y_B = y_A + S \cos \alpha + \frac{S \cos \alpha}{6r^2} (3x_B^2 - S^2 \sin^2 \alpha)$$

$$\text{and the reverse bearing } BA = \alpha' = \alpha \pm 180^\circ - \frac{S \cos \alpha (x_A + x_B)}{2r^2 \sin 1''}$$

Computing bearing and distance from rectangular spherical co-ordinates. This reverse process will more commonly be needed by the engineer basing a survey on two of the local government survey trigonometrical points.

It is first necessary to compute the plane rectangular bearing (β) and distance (S') from:

$$\tan \beta = \frac{x_B - x_A}{y_B - y_A}$$

$$\text{and } S' = (x_B - x_A) \operatorname{cosec} \beta = (y_B - y_A) \sec \beta$$

$$\text{then } S = S' \left(1 - \frac{K \cos^2 \beta}{2r^2} \right)$$

$$\text{and } \alpha = \beta + \frac{\sin \beta \cos \beta}{2r^2 \sin 1''} K + \frac{(y_B - y_A) (2x_A + x_B)}{6r^2 \sin 1''}$$

$$\text{where } K = \left[\left(\frac{x_B + x_A}{2} \right)^2 + \frac{1}{3} \left(\frac{x_B - x_A}{2} \right)^2 \right]$$

Caution. The α in the above formulae is the co-ordinate bearing of B from A and differs from the true bearing by the convergency

between the meridian through A and the meridian through the origin; similarly α' is the co-ordinate bearing of A from B.

TRANSVERSE MERCATOR CO-ORDINATES.* It has been shown above that the use of rectangular spherical co-ordinates produces distortion in the northing co-ordinates but not in the easting, the net result being a distortion both in length and distance of the line joining two co-ordinated points, the amount of both distortions depending upon the direction of the line. It can be seen from Figs. 16 and 17 that, if Δx were distorted in the same proportion as Δy , it would result in an even greater distortion in the length of the line, but *for short lines* it would remove distortion from the direction; in other words the resulting projection would be orthomorphic. This modification of the rectangular spherical co-ordinate system gives the Transverse Mercator or Gauss Conformal projection and co-ordinates which have now been adopted for the maps of Great Britain by the Ordnance Survey and are in use in many other countries.

Ordnance Survey data based on the Transverse Mercator projection. As far as work in this country is concerned, the Ordnance Survey (through H.M. Stationery Office) publish *Projection Tables* and an accompanying explanatory pamphlet *Constants, Formulae and Methods used in Transverse Mercator Projection* which contain all the necessary material for solving any of the problems likely to arise even in the most precise work. The pamphlet, in addition, contains much interesting information about the Ordnance Survey and the Transverse Mercator projection, and for this chapter it is relevant to quote from the final section 'Notes on the Practical Use of the Projection'.

'For minor surveys in small areas (i.e., not exceeding 10 km. in extent), where scale errors not exceeding 1/2000 and directional errors not exceeding 10" may be tolerated, calculations may be done by plane rectangular co-ordinates without any correction terms; and measurements on the ground may be taken as grid distances and vice versa.

Where greater accuracy is required the following figures will be some guide to the errors likely from the use of plane rectangular co-ordinates and direct measurements.

Scale. The scale may be taken as constant from north to south along any easting grid line, with an error nowhere exceeding 1 in

* See also p. 417.

600,000. The scale for a given easting, interpolated from the table of scale factors (printed below), may be accepted for any line whose ends do not depart by more than 10 km. from the given easting. The error will never be greater than 1 part in 10,000.

If greater accuracy is required the scale calculated for the mid point of the line by the formula (given in the pamphlet) will be correct to 1 in 600,000 for any line extending not more than 30 km. in easting.

Care must be taken in accurate calculations to reduce terrestrial measurements to mean sea-level.

Orientation. To convert grid bearings to true azimuths the convergence must always be applied.

The approximate formula

$$\text{convergence}' = \text{diff. long.} \times \sin \text{lat. of place}$$

will give values correct to quarter of a minute of arc. The latitude of place may be taken from a map to the nearest 5 minutes of arc.

TABLE OF LOCAL SCALE FACTOR

National grid easting (km.)		Scale factor F	National grid easting (km.)		Scale factor F
400	400	0.99960	250	550	0.99988
390	410	60	240	560	92
380	420	61	230	570	0.99996
370	430	61	220	580	1.00000
360	440	62	210	590	04
350	450	63	200	600	09
340	460	65	190	610	14
330	470	66	180	620	20
320	480	68	170	630	25
310	490	70	160	640	31
300	500	72	150	650	37
290	510	75	140	660	43
280	520	78	130	670	1.00050
270	530	81			
260	540	0.99984			

Use of Scale Factor

$$S' = S \times F$$

where S' = distance in the projection

S = distance on the spheroid at mean sea level

F = local scale factor from table.

If greater accuracy is required, the (t - T) correction must be considered (calculated by the formulae given in the pamphlet).

At a distance of 250 km. from the central meridian and for a line extending over 10 km. in northing the (t - T) correction is about $6''.5$. For estimating purposes it may be taken that the value varies directly as the distance from the central meridian and as the extent of the line in northing. Hence the approximate value of the correction can be judged for any other conditions; e.g., 100 km. from the central meridian the correction for a line extending over 15 km. in northing would be approximately

$$6''.5 \times \frac{100}{250} \times \frac{15}{10} = 4''$$

It is clear that, over most of the country, (t - T) need only be considered for very long rays or very accurate work.

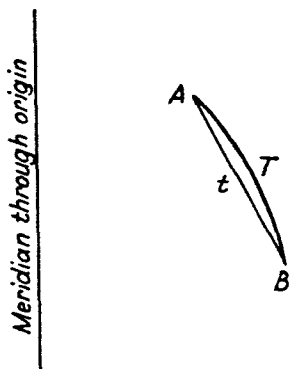


FIG. 18

The (t - T) correction needs a little explanation.

t = straight line direction joining two points on the projection

T = direction on the projection of the projected geodesic or line of sight joining two points.

On any Mercator projection, transverse or otherwise, the shortest distance between two points, i.e. the great circle track for the sphere or the geodesic for the spheroid is represented on the projection as a curve which, in the transverse case,

is always concave to the central meridian. As can be gathered from the last paragraphs of the quotation above, the (t - T) correction is nil between two stations having the same northing co-ordinate and maximum between two stations having the same easting co-ordinate, this maximum being $7''$ of arc at the extreme easterly and westerly limits of the map. Consequently, in the extreme case an angle measured at a point between a station due north and a station due south might differ from the plotted angle by $14''$, but in practice it is very rare that the difference will ever exceed the $10''$ given in the first paragraph of the quotation above.

When no special tables published by the local survey department are available, the following formulae may be used:

Computing Transverse Mercator co-ordinates from bearing and distance.
Easting

$$x_B = x_A + S \sin \alpha - \frac{S^2 \cos^2 \alpha}{6r^2} (S \sin \alpha + 3x_A) + \frac{(x_B^3 - x_A^3)}{6r^2}$$

Northings

$$y_B = y_A + S \cos \alpha + \frac{S \cos \alpha}{6r^2} (3x_B^2 - S^2 \sin^2 \alpha)$$

Reverse bearing, $\alpha_r = \alpha \pm 180^\circ - \frac{S \cos \alpha (x_A + x_B)}{2r^2 \sin 1''}$

in which, as before (p. 70), $r = \sqrt{\rho v}$ (extracted for mid-lat. of line)
and α = Transverse Mercator co-ordinate bearing.

Computing bearing and distance from Transverse Mercator co-ordinates.

$$S = S' - \frac{S'}{2r^2} \left[\left(\frac{x_B + x_A}{2} \right)^2 + \frac{1}{3} \left(\frac{x_B - x_A}{2} \right)^2 \right]$$

Bearing A \rightarrow B $\alpha = \beta + \frac{(y_B - y_A)(2x_A + x_B)}{6r^2 \sin 1''}$

„ B \rightarrow A $\alpha' = \beta \pm 180^\circ + \frac{(y_A - y_B)(2x_B + x_A)}{6r^2 \sin 1''}$

in which $\tan \beta = \frac{x_B - x_A}{y_B - y_A}$

Scale and scale error (short lines only)

$$\frac{S'}{S} = 1 + \frac{x^2}{2r^2} + \frac{x^4}{24r^4}$$

$$\frac{S}{S'} = 1 - \frac{x^2}{2r^2} + \frac{5}{24} \frac{x^4}{r^4}$$

Scale factor. It can be seen that the scale error increases with the square of the distance east or west from the central meridian, and when this distance is 125 miles the scale error is about $\frac{1}{20000}$, and further, that if the scale is true along the central meridian, S' will always be greater than S . In practice, national surveys which use the Transverse Mercator projection usually halve the scale error by making the scale true along (co-ordinate) meridians about two-thirds of the way between the central meridian and the east (and west) limits of the projection. When this is done, the scale along the central meridian will be too small by the same amount as the scale is too large at the eastern and western limits of the projection.

Examination of the table printed on p. 73 will show that, in the case of the Ordnance Survey, the scale is correct, 180 km. east and west of the central meridian, while on the central meridian the ratio $\frac{S'}{S}$ decreases to $\frac{9996}{10000}$; and, at about 255 km. east and west of the central meridian, it increases to $\frac{10004}{10000}$.

The ratio $\frac{9996}{10000}$ is referred to either as *the* scale factor or in full as the scale factor on the central meridian. The Ordnance Survey Projection Tables referred to above allow for this factor, but, if the general formulae given on p. 75 are being used, distances on the earth must be multiplied by the scale factor before being used in the formulae.

COMPUTING GEOGRAPHICAL CO-ORDINATES. The surveyor engaged on minor triangulation work will seldom be required to carry out this computation. Formulae for converting rectangular spherical co-ordinates and Transverse Mercator co-ordinates into geographical co-ordinates and the inverse problem are given in *Plane and Geodetic Surveying for Engineers* by the late David Clark, Vol. II, *Higher Surveying*, and also in the pamphlet published by the Ordnance Survey mentioned on p. 72 as well as in other standard works.

MID-LATITUDE FORMULAE. Geographical co-ordinates, however, must not be dismissed in quite such an airy fashion as the above paragraph suggests, for the surveyor or engineer engaged on minor triangulation based on the local government survey may frequently be supplied with data given in this form, and so will need to calculate the bearing and distance between two datum triangulated points, of which only the latitude and longitude are known. In the absence of any special locally-prepared tables and formulae, the most convenient formulae are Gauss' mid-latitude formulae which, though approximate, give results accurate to one foot for lines not exceeding 25 miles in length, in latitudes less than 60°.

The four formulae, which may also be used for the inverse problem, are given below, in which

ϕ_A and ϕ_B are the latitudes of A and B and $\Delta\phi$ their diff. in latitude
 L_A , L_B and ΔL are their longitudes and diff. in longitude

α_A , α_B are azimuths at A and B measured clockwise from north

$\Delta\alpha$ is the convergency between the meridians through A and B

S is the distance, at sea-level, between A and B

ρ and ν are the radii of the earth's curvature at latitude measured along the meridian (ρ) and along the prime vertical (ν)

(It should be noted that $\frac{1}{\rho \sin 1''}$ and $\frac{1}{\nu \sin 1''}$ are tabulated in several standard works, but the values vary for the different figures of the earth, those of Airy, Bessel, Clarke, Everest, etc., and, obviously, it is important that, for consistency, the figure of the earth adopted by the local survey should be used.)

$$\tan \frac{\Delta\alpha}{2} = \tan \frac{\Delta L}{2} \sin \left(\phi_A + \frac{\Delta\phi}{2} \right) \sec \frac{\Delta\phi}{2} \quad . \quad . \quad (1)$$

$$\Delta\phi = \frac{S}{\rho \sin 1''} \cos \left(\alpha_A + \frac{\Delta\alpha}{2} \right) \quad . \quad . \quad . \quad (2)$$

$$\Delta L = \frac{S}{\nu \sin 1''} \frac{\sin \left(\alpha_A + \frac{\Delta\alpha}{2} \right)}{\cos \left(\phi_A + \frac{\Delta\phi}{2} \right)} \quad . \quad . \quad . \quad (3)$$

$$\tan \left(\alpha_A + \frac{\Delta\alpha}{2} \right) = \frac{\nu \Delta L}{\rho \Delta\phi} \times \cos \left(\phi_A + \frac{\Delta\phi}{2} \right) \quad . \quad . \quad . \quad (4)$$

(Note. Formula (4) = $\frac{\text{formula (3)}}{\text{formula (2)}}$, simplified)

When computing bearing and distance from geographical co-ordinates the steps are

(a) Compute $\Delta\alpha$ from (1)

(b) From (4) compute $\alpha_A + \frac{\Delta\alpha}{2}$ and hence α_A and $\alpha_B (= \alpha_A + \Delta\alpha \pm 180^\circ)$

(c) S can be found from (2) or (3) according to whether $\Delta\phi \cdot \rho \sin 1''$ or $\Delta L \cdot \nu \sin 1''$ is the larger.

Notes. Values of $\frac{1}{\rho}$ and $\frac{1}{\nu}$ are for the mid-latitude, $\phi_A + \frac{\Delta\phi}{2}$

Sign conventions must be followed: If N and E are +, and S and W are —, then for values of

$$\alpha_A + \frac{\Delta\alpha}{2} \begin{cases} 000^\circ - 090^\circ & \Delta\phi \text{ is } + & \Delta L \text{ is } + & \Delta\alpha(\text{N.lat.}) + & (\text{S.lat.}) - \\ 090 - 180 & - & + & + & - \\ 180 - 270 & - & - & - & + \\ 270 - 360 & + & - & - & + \end{cases}$$

Should the inverse problem have to be computed it will be in the form of requiring the latitude and longitude of B and the reverse azimuth $B \rightarrow A$, from the known azimuth and distance from A, a station whose geographical co-ordinates are known. The computation is not quite so simple as the direct problem because approximate values of $\Delta\phi$ and ΔL are required before the computation proper can start. The steps are:

Using five-figure logs:

- (d) Compute the approximate diff. latitude and mid-latitude from:

$$\Delta'\phi = S \cos \alpha_A \frac{1}{\rho \sin 1''}$$

(ρ being extracted for latitude ϕ_A)

and
$$\phi'_M = \phi_A + \frac{1}{2}\Delta'\phi$$

- (e) Compute the approximate difference of longitude from:

$$\Delta'L = S \sin \alpha_A \frac{1}{\nu \sin 1''} \sec \phi'_M$$

(ν being extracted for latitude ϕ'_M)

- (f) Compute the value of $\frac{\Delta\alpha}{2}$ from formula (1)

and thence
$$\left(\alpha_A + \frac{\Delta\alpha}{2} \right)$$

Using seven-figure logs:

- (g) Compute the true difference of latitude from formula (2)
(ρ being extracted for lat. ϕ'_M) and hence the latitude of B

and the true middle latitude from
$$\phi_M = \frac{\phi_A + \phi_B}{2}$$

- (h) Compute the true difference of longitude from formula (3)
(ν being extracted for lat. ϕ_M)

Finally compute the true azimuth $B \rightarrow A$ from

$$\alpha_B = \alpha_A \pm 180^\circ \pm \Delta\alpha$$

Examples of the computations are given in many standard works together with the sign conventions which must be adhered to rigidly.

INTERSECTION

In the minor triangulation that has been discussed so far the positions of triangulated points have been fixed successively by

rays observed both to and from them. It frequently happens, however, that points have to be included in the triangulation from which it is either impossible or inconvenient to observe; such points must be fixed solely by intersecting rays from surrounding stations and are known as 'intersected points'.

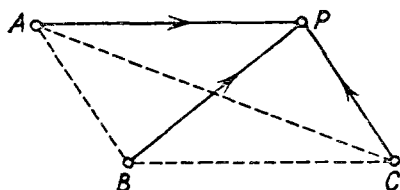


FIG. 19

Fig. 19 shows such a point P intersected from rays observed from A, B and C.

If P is of such importance that it has to be included in the main triangulation scheme, then the angles at P must be computed from

$$\text{APB} = 180^\circ - (\text{PAB} + \text{PBA})$$

and

$$\text{BPC} = 180^\circ - (\text{PBC} + \text{PCB})$$

and then, using these angles, the figure may be adjusted with the rest of the triangulation (see Chap. 6).

More frequently intersected points, although required as part of the 'control', will not form part of the triangulation scheme itself, in which case it is sufficient to compute the co-ordinates of P independently in the three triangles ABP, BCP, and ACP and then accept an arbitrarily weighted mean, the weights being influenced by the relative distance of P from the observing stations and the size of the receiving angle at P.

SEMIGRAPHIC INTERSECTION. Frequently more than three intersecting rays are available, and in such a case the 'weighting' becomes a matter of some difficulty, and it is more satisfactory to plot the rays being received at P on a large scale. The most probable position of P can then be determined by inspection of the plot, on which faulty rays will usually be obvious. The method is known as 'semigraphic intersection'.

In Fig. 20, A, B, C, D and E are Δ s whose co-ordinates have been calculated, and from each of which rays into P have been observed. Proceed as follows:

- (a) Convert these rays into co-ordinate bearings.
- (b) From any conveniently placed pair of Δ s, calculate the approximate co-ordinates of P, P'_E and P'_N .

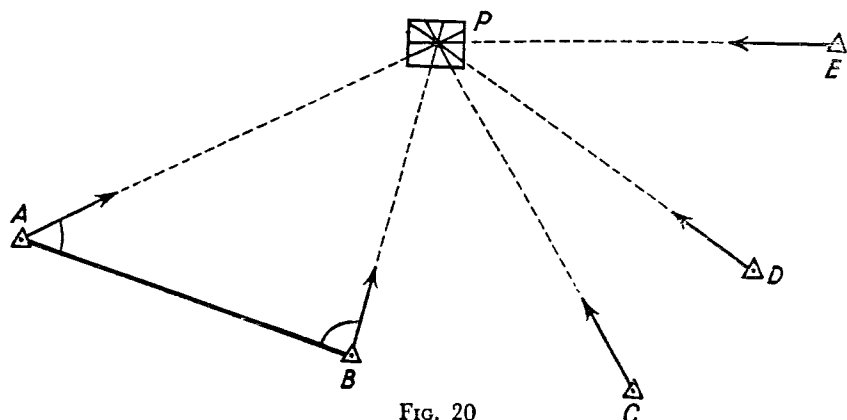


FIG. 20

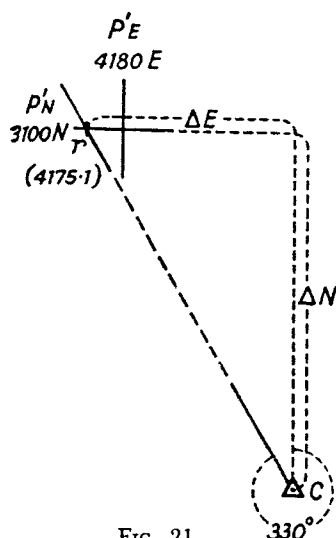


FIG. 21

(c) On graph paper, choosing a convenient scale such as $\frac{1}{2}'' = 1$ foot, draw the co-ordinate axes of P' and label them with the values P'_E and P'_N .

(d) Taking each Δ in turn calculate where the rays from each will cut the appropriate axis; the appropriate axis being the E-W when the co-ordinate bearing of the ray lies between $315^\circ - 045^\circ$ and $135^\circ - 225^\circ$; and the N-S axis when it lies between $045^\circ - 135^\circ$ or $225^\circ - 315^\circ$ (i.e. the axis which makes the larger angle with the intersecting ray).

This is done as follows:

say P'_E is +4,180 ft. and P'_N is +3,100 ft., and the ray in question is that observed from ΔC , whose co-ordinates are (say) +6,600 E and -1,100 N and the bearing of the ray is 330° .

In this case the eastings of the point where this ray cuts the axis 3,100 N is required. Call this point r .

Considering r and C:

$$\begin{aligned}\text{diff. northings} &= 3,100 - (-1,100) = 4,200 \\ \text{diff. eastings} &= \text{diff. northings} \tan \text{bearing} \\ (\text{in this case}) &= 4,200 \tan 330^\circ \\ &= -2424.9\end{aligned}$$

i.e. the eastings of r are $6,600 - 2424.9 = 4,175.1$ E

(e) Tick off this point r and with a protractor draw in a portion of the ray $C \rightarrow P$ ($= C \rightarrow r$) through this point on the bearing 330° .

(f) In a similar way draw the rays from the other Δ s, and the result will be a graph on which the receiving ends of the intersecting rays are drawn to scale.

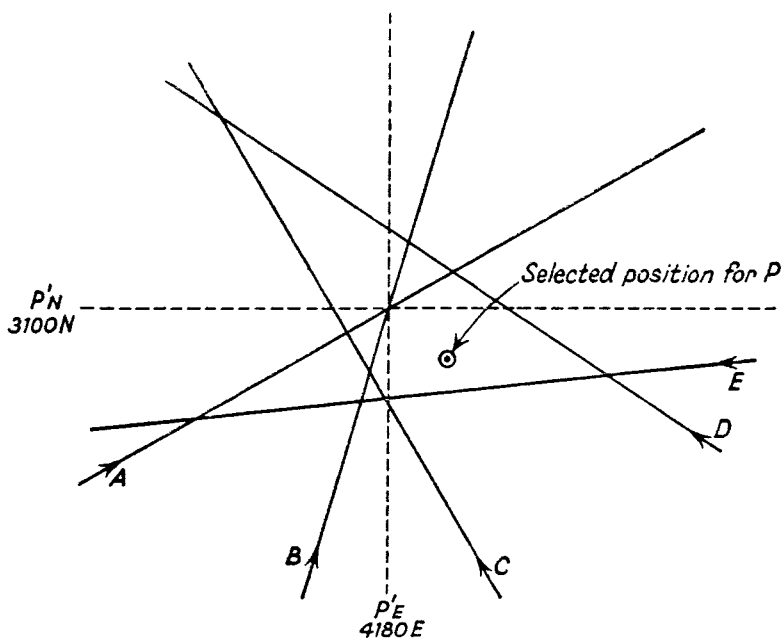


FIG. 22

(g) A position for P can then be selected by inspection and plotted, and its co-ordinates read off the graph. When selecting the position of P , the rule to follow is that the length of the

perpendicular from P to any ray should be proportional to the distance from P to the trig point from which that particular ray was observed.

The computation may be arranged very compactly.

An alternative method is to adopt an enclosing rectangle instead of the axes through P'. In the example a rectangle might be chosen with northing co-ordinates 3,105 N and 3,095 N, and eastings of 4,175 E and 4,185 E. The computation then takes the form of finding the cutting points of each ray on the two appropriate sides of the rectangle (e.g. the cutting points of the ray from C on the northing lines 3,105 N and 3,095 N). No protractor is required, the path of the rays being plotted by joining up the cutting points.

Sometimes, owing to the adjustment of the triangulation, the sum of (say) the observed angles BCP and DCP no longer equal the adjusted angle BCD. In such a case the co-ordinate bearing $C \rightarrow P$ used for the above computation should be a mean one.

Note also that the method should not be used with rectangular spherical co-ordinates if the distance east or west of the meridian of origin is great; but the method is 'safe' for orthomorphic projections such as the Transverse Mercator.

TRIGONOMETRICAL INTERPOLATION OR RE-SECTION. Cases frequently arise in minor triangulation (though less frequently than in reconnaissance work) when it is desirable to fix the position of a new control point by angles observed from that position to previously fixed triangulation stations. Provided the stations to which the observations are being made are suitably placed the resulting fixation should be perfectly reliable.

Three cases arise whereby the position may be calculated from:

(a)	horizontal angles observed from 1 unfixed point to 3 known points
(b)	“ “ “ “ 2 “ “ “ 2 “ “
(c)	“ “ “ “ 1 “ “ “ 2 “ “

plus an azimuth to one of them.

(a) This is the familiar plane-table '*Three-point*' problem.

Let A, B, and C be the three known points (with C as the central one) and P the unknown point at which are observed the angles $APC = \alpha$ and $BPC = \beta$. The three angles to be calculated for the solution of the problem are:

Observing station	A	B	C	D	E
Bearing	<i>Used for computing co-ordinates of P'. Rays from these two stations therefore automatically pass through P'</i>		CA 291° 10' 30"		
Observed angle			ACP 38 49 30		
Sum or diff. = bearing			CP 330 00 00		
Observing station E or N			N - 1 100		
Trial grid E or N			N + 3 100		
* ΔE or ΔN			$\Delta N + 4 200$		
$\log \Delta E$ or ΔN			3.623 2493		
* Bearing L. tan or L. cot			$\tan 9.761 4394$		
Sum (= $\log \Delta N$ or ΔE)			3.384 6887		
ΔN or ΔE			$\Delta E - 2 424.9$		
Observing station N or E			E + 6 600.0		
Intersection on grid E or N			<u>E + 4 175.1</u>		

* Use E and L. cot when bearing is between 045° & 135°, or 225° & 315°
 " N and L. tan " " " " 315° & 045°, or 135° & 225°

$ACB = C$ (always to be measured on the side of AB towards P)

$PAC = x$

$PBC = y$

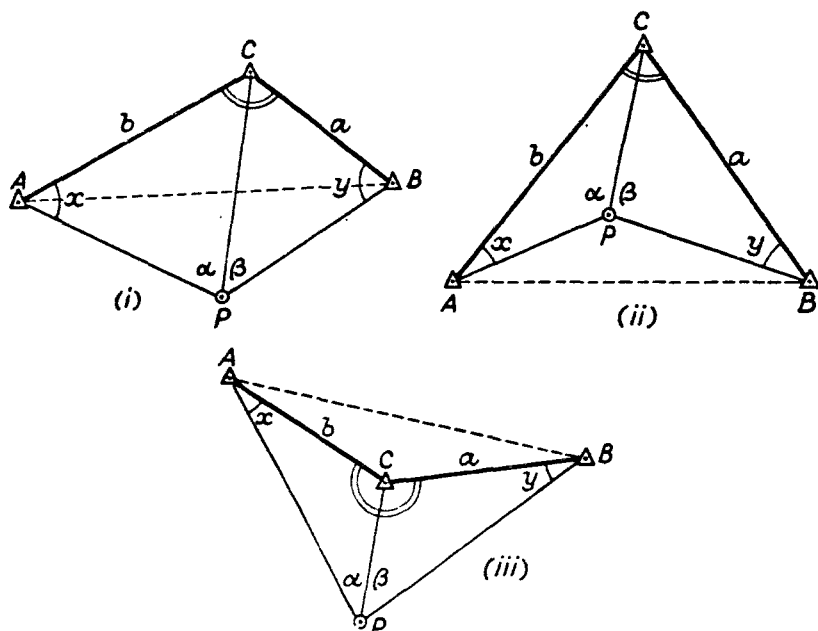


FIG. 23

(There are three different arrangements of A, B, C and P as shown above, but in all the proof is the same. In (i) it can be seen that if A, C, B , and P are on the circumference of the same circle the problem is insoluble; (ii) and (iii) are safe in this respect.)

In all: $x + y = 360^\circ - (\alpha + \beta + C)$ (I)

In $\triangle ACP$, $\frac{PC}{\sin x} = \frac{AC}{\sin \alpha}$ and in $\triangle BCP$, $\frac{PC}{\sin y} = \frac{BC}{\sin \beta}$

so, $\frac{PC}{\sin y} = \frac{BC}{\sin \beta}$ i.e. $\frac{\sin x}{\sin y} = \frac{a \sin \alpha}{b \sin \beta}$

To simplify the subsequent computation let this equal $\tan \phi$

$$\text{i.e.} \quad \tan \phi = \frac{a \sin \alpha}{b \sin \beta} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then it can be deduced that:

$$\tan (\phi - 45^\circ) = \frac{\sin x - \sin y}{\sin x + \sin y}$$

$$\text{and} \quad \tan \frac{x-y}{2} = \tan \frac{x+y}{2} \times \frac{\sin x - \sin y}{\sin x + \sin y}$$

$$\text{i.e.} \quad \tan \frac{x-y}{2} = \tan \frac{x+y}{2} \times \tan (\phi - 45^\circ) \quad . \quad . \quad (3)$$

From these three formulae the angles x and y may be calculated and the triangles ACP and BCP solved as follows:

(i) Calculate the angle C (usually by the difference between the co-ordinate bearings of CA and CB)

(ii) Calculate $\frac{x+y}{2}$ from (1)

(iii) Calculate ϕ from (2)

(iv) Find $(\phi - 45^\circ)$ and note that it will frequently be a minus quantity

(v) Calculate $\frac{x-y}{2}$ from (3)

(vi) Find x and y from the sum and difference of (ii) and (v); and note x is frequently smaller than y (see below)

(vii) Calculate CP from $b \operatorname{cosec} \alpha \sin x$ and $a \operatorname{cosec} \beta \sin y$, thus providing a check on the computation

(viii) Find the angles

$$\text{ACP} = 180 - (\alpha + x) \quad \text{and} \quad \text{BCP} = 180 - (\beta + y)$$

and so obtain the co-ordinate bearing of C to P

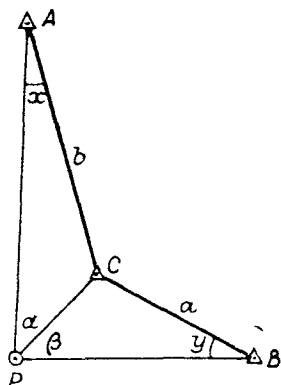
(ix) Obtain difference of co-ordinates between C and P by CP sin bearing and CP cos bearing.

Note. When $(\phi - 45^\circ)$ is negative $\frac{x-y}{2}$ as calculated from (3) will

also be negative; but if any doubt still exists as to whether x or y should be the larger it may be dispelled by the following rules:

(i) when $x + y < 180^\circ \dots$ If $\phi > 45^\circ$ x is $> y$, and vice versa.

(ii) when $x + y > 180^\circ \dots$ If $\phi < 45^\circ$ x is $> y$, and vice versa.



Data: A 15240 E 40350 N
 B 34400 E 4360 N
 C 19500 E 14450 N
 α $39^{\circ} 30' 25''$
 β $47^{\circ} 10' 55''$

FIG. 24

A	15240	40350		B	34400	4360
C	19500	14450		C	19500	14450
ΔE	4260		$\log 3.629\ 4096$		14900	$\log 4.173\ 1863$
ΔN	25900		$\log 4.413\ 2998$		10090	$\log 4.003\ 8912$
$C \rightarrow A$	$350^{\circ} 39' 34.9''$		$\frac{L.\tan 9.216\ 1098}{L.\cos 9.994\ 2036} -$	$C \rightarrow B$	$124^{\circ} 06' 18.4''$	$\frac{L.\tan 0.169\ 2951}{L.\sin 9.918\ 0357}$
ΔN			$\log 4.413\ 2998 +$	ΔE		$\log 4.173\ 1863$
b (= CA)			$\log 4.419\ 0962$	a (= CB)		$\log 4.255\ 1506$
β			$L.\sin 9.865\ 4094$	α		$L.\sin 9.803\ 5744$
$b \sin \beta$			$\log 4.284\ 5056$	$a \sin \alpha$		$\log 4.058\ 7250$
				$b \sin \beta$		$\log 4.284\ 5056$
				ϕ	$30^{\circ} 44' 07.4''$	$L.\tan 9.774\ 2194$

From above: $C = 226^{\circ} 33' 16.5''$

$\alpha = 39^{\circ} 30' 25''$

$\beta = 47^{\circ} 10' 55''$

$x + y$	$\text{Sum } 313\ 14\ 36.5$	$\phi - 45^{\circ} = -14\ 15\ 52.6$	$L.\tan 9.405\ 2424$
$(= 360^{\circ} - \text{Sum})$	$46\ 45\ 23.5$	$\frac{1}{2}(x + y)$	$\frac{23\ 22\ 41.75}{L.\tan 9.635\ 7735}$
		$\frac{1}{2}(x - y)$	$\frac{-6\ 16\ 18.8}{L.\tan 9.041\ 0159}$
		$x'(\text{sum})$	$17\ 06\ 23$ (ACP = $123\ 23\ 12$)
		$y'(\text{diff})$	$29\ 39\ 00.5$ (BCP = $103\ 10\ 04.5$)
		\therefore bearing $C \rightarrow P$	$= 227^{\circ} 16' 22.9''$

b	$\log 4.419\ 0962$	a	$\log 4.255\ 1506$
α	$L.\text{cosec } 0.196\ 4256$	β	$L.\text{cosec } 0.134\ 5906$
	$L.\sin 9.468\ 5643$	y	$L.\sin 9.694\ 3442$
CP	$\log 4.084\ 0861$	CP	$\log 4.084\ 0854$
bng.	$L.\sin 9.866\ 0481$	bng.	$L.\cos 9.831\ 5534$
ΔE	$\log 9.950\ 1342$	ΔN	$\log 3.915\ 6388$

$\Delta E = 8915.3$

$\Delta N = 8234.5$

$C_E + 19500.0$

$C_N + 14450.0$

$P + 10584.7\ E$

$+ 6215.5\ N$

It will be realized that there is no check on the observations unless a fourth triangulated point is observed. If such a fourth point has been observed, the re-section may be re-computed using a different combination of three observed points, but see 'Semigraphic Re-section', p. 91, for a better way of utilizing any extra rays which may have been observed.

The computation may be arranged compactly as shown on opposite page.

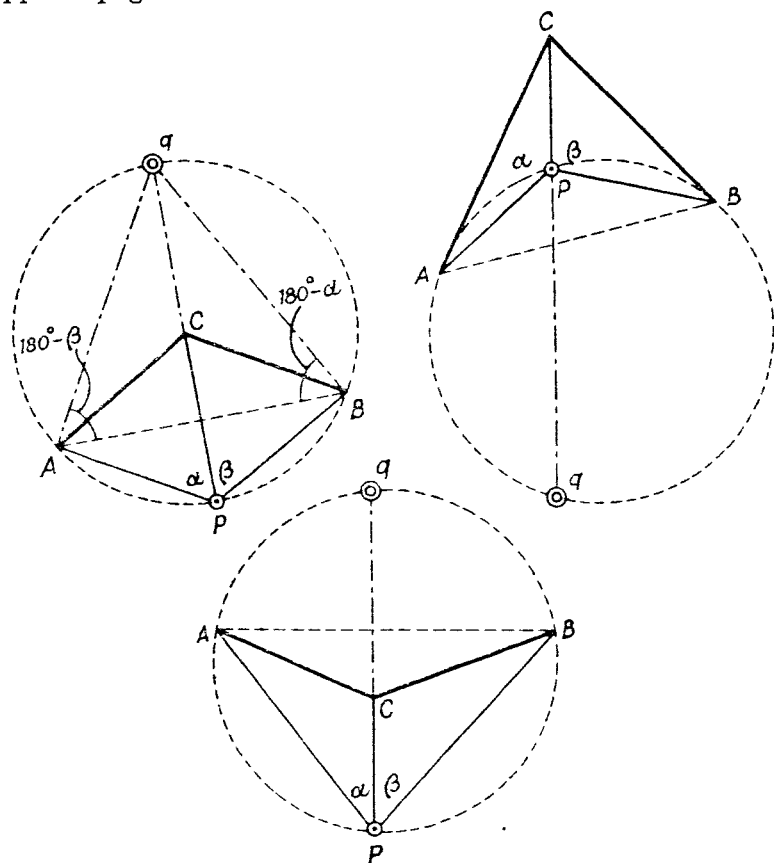


FIG. 25

Collins point re-section. The three-point problem may also be solved by introducing an auxiliary point (*the Collins point, q*) situated at the intersection of the circumference of the circle drawn through A, B and P and the line CP produced (see Fig. 25 in which the three

cases of Fig. 23 are reproduced). In each case $\angle ABq = 180^\circ - \alpha$ and $\angle BAq = 180^\circ - \beta$, and, since AB is known, (i) the triangle ABq may be solved, from which (ii) the position of q may be calculated and hence, by difference of co-ordinates, (iii) the bearing of the line q to C which is the same as C to P. Since the length and bearing Aq are known from (i), the angle AqP can be found and, as APq ($= \alpha$) is already known, the triangle APq may be solved (iv) to give AP and hence the position of P. Solving the triangle BPq will supply the necessary check to the computation. Examples of the construction and computation are given in many text books, but the method appears to have no advantages over the one previously described.

(b) *Fixing the position of two points by angles taken from them to two known points.* This is frequently known as the 'Inaccessible Base' problem, since the observations and calculations enable the direction and dis-

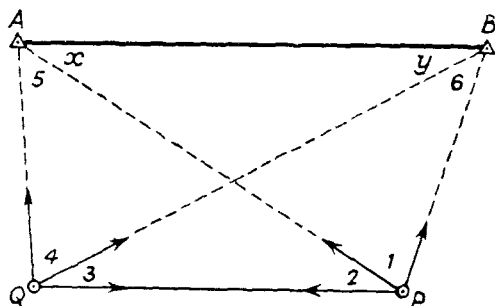


FIG. 26

tance between two points outside the survey area to be transferred to a new base within it.

The positions of A and B are known. It is required to fix the positions of P and Q from the angles 1, 2, 3, and 4 observed at them.

For the computation it is necessary to obtain the values of the angles 5 and 6 and x and y (see Fig. 26):

$$5 = 180^\circ - (2 + 3 + 4) \text{ and } 6 = 180^\circ - (1 + 2 + 3) \quad (1)$$

$$x + y = 2 + 3 \quad (2)$$

From the ordinary trigonometrical relations between angles in a

$$\text{rectangle} \quad \frac{\sin x}{\sin y} = \frac{\sin 1 \times \sin 3 \times \sin 5}{\sin 2 \times \sin 4 \times \sin 6}$$

Let this equal R.

Since $\tan \frac{x-y}{2} = \tan \frac{x+y}{2} \times \frac{\sin x - \sin y}{\sin x + \sin y}$ it can be seen that

$$\tan \frac{x-y}{2} = \tan \frac{2+3}{2} \times \frac{R-1}{R+1} \quad (3)$$

When x and y have been found the positions of P and Q may be calculated by solving the triangles ABP and ABQ. The computation is as follows:

(i) Calculate co-ordinate bearing and distance of AB (if not already known)

(ii) Find angles 5 and 6 from (i)

(iii) Calculate $R \left(= \frac{\sin 1 \times \sin 3 \times \sin 5}{\sin 2 \times \sin 4 \times \sin 6} \right)$

(iv) Calculate $\frac{x-y}{2}$ from (3)

(v) Find x and y from the sum and difference of

$$\frac{x-y}{2} \text{ and } \frac{2+3}{2}$$

(vi) Solve the triangles ABP and ABQ to obtain the co-ordinates of P and Q

(vii) Check the computation by finding the co-ordinates of Q in the triangle APQ using AP as the known side.

A worked example is given on the next page.

(c) *Fixing the position of a point by an angle observed at that point between two known points and an azimuth to one of them.*

A and B are two points whose geographical co-ordinates are

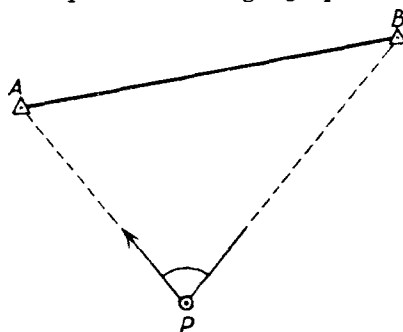


FIG. 27

known. The angle APB and the azimuth $P \rightarrow A$ are observed (and so the azimuth of $P \rightarrow B$ is also known).

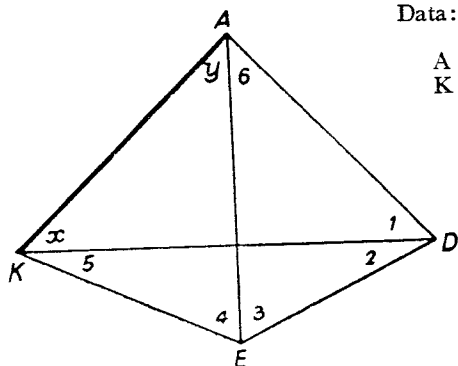


FIG. 28

Data:

$$\begin{array}{r} \text{E} \\ \text{A} + 10 \ 000 \cdot 0 \\ \text{K} + 1 \ 925 \cdot 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{N} \\ + 14 \ 000 \cdot 0 \\ + 7 \ 977 \cdot 0 \\ \hline 6 \ 023 \cdot 0 \end{array}$$

$$8 \ 074 \cdot 6 \log \ 3 \cdot 907 \ 1210$$

$$3 \cdot 907 \ 1210$$

$$6 \ 023 \cdot 0 \log \ 3 \cdot 779 \ 8129$$

$$\text{bng. AK} \quad \text{L.tan} \ 0 \cdot 127 \ 3081$$

$$\text{L.sin} \ 9 \cdot 903 \ 9400$$

$$\text{Log AK} \ 4 \cdot 003 \ 1810$$

$$\text{AK} \ 233^\circ \ 16' \ 48''$$

$$10 \ 073 \cdot 5$$

Observed angles	1	50°	14'	24"
	2	19	59	13
	3	51	43	31
	4	66	45	39
	2 + 3	71	42	44

$$121 \ 57 \ 08 \therefore 6 = 58^\circ \ 02' \ 52''$$

$$138 \ 28 \ 23 \therefore 5 = 41^\circ \ 31' \ 37''$$

1	50	14	24	L.sin	9.885	7740
3	51	43	31	L.sin	9.894	8972
5	41	31	37	L.sin	9.821	4953
2	19	59	13	L.cosec	0.466	2203
4	66	45	39	L.cosec	0.036	7479
6	58	02	52	L.cosec	0.071	3534
				log R	0.176	4881
				R =	1.50137	

$$R - 1 = 0.50137 + \log \ 1.700 \ 1583$$

$$R + 1 = 2.50137 \text{ co-log } 1.601 \ 8221$$

$$\frac{1}{2}(2 + 3) \ 35 \ 51 \ 22 \text{ L.tan } 9.858 \ 9656$$

$$\text{L.tan } 9.160 \ 9460$$

$$\frac{1}{2}(x - y) + 8 \ 14 \ 33$$

$$x(\text{sum}) \ 44 \ 05 \ 55$$

$$y(\text{diff.}) \ 27 \ 36 \ 49$$

$$\begin{array}{r} \text{bng. AK } 233^\circ \ 16' \ 48'' \\ \Delta \text{ AKD} \quad (y + 6) \ 85 \ 39 \ 41 \\ \text{bng. AD } 147 \ 37 \ 07 \end{array}$$

$$\text{AK log } 4.003 \ 1810$$

$$1 \text{ L.cosec } 0.114 \ 2260$$

$$x \text{ L.sin } 9.842 \ 5440$$

$$\text{AD log } 3.959 \ 9510$$

$$\text{bng. sin } 9.728 \ 8021 \cos \ 9.926 \ 6006$$

$$3.688 \ 7531 \quad 3.886 \ 5516$$

$$\Delta E + 4 \ 883 \cdot 7 \quad \Delta N - 7 \ 701 \cdot 1$$

$$\text{A} + 10 \ 000 \cdot 0 \quad + 14 \ 000 \cdot 0$$

$$\text{D} - 14 \ 883 \cdot 7 \text{ E} \quad + 6 \ 298 \cdot 9 \text{ N}$$

$$\begin{array}{r} \text{bng. KA } 053^\circ \ 16' \ 48'' \\ \Delta \text{ AKE} \quad (x + 5) \ 85 \ 37 \ 32 \\ \text{bng. KE } 138 \ 54 \ 20 \end{array}$$

$$\text{AK log } 4.003 \ 1810$$

$$4 \text{ L.cosec } 0.036 \ 7479$$

$$y \text{ L.sin } 9.666 \ 0559$$

$$\text{KE log } 3.705 \ 9848$$

$$\text{bng. sin } 9.817 \ 7651 \cos \ 9.877 \ 1565$$

$$3.523 \ 7499 \quad 3.583 \ 1413$$

$$\Delta E + 3 \ 340 \cdot 0 \quad \Delta N - 3 \ 829 \cdot 5$$

$$\text{K} + 1 \ 925 \cdot 4 \quad + 7 \ 977 \cdot 0$$

$$\text{E} + 5 \ 265 \cdot 4 \text{ E} \quad - 4 \ 147 \cdot 5 \text{ N}$$

Check:

$$\begin{array}{r} \Delta \text{ ADE} \quad \text{DA} \quad 327^\circ \ 37' \ 07'' \\ (1 + 2) \ 70 \ 13 \ 37 \\ \therefore \text{DE} \ 257 \ 23 \ 30 \end{array}$$

$$\text{AD log } 3.959 \ 9510$$

$$3 \text{ L.cosec } 0.105 \ 1028$$

$$6 \text{ L.sin } 9.928 \ 6466$$

$$\text{DE log } 3.993 \ 7004$$

$$\text{bng. sin } 9.989 \ 3987 \cos \ 9.339 \ 0243$$

$$\Delta E \log \ 3.983 \ 0991 \quad \Delta N \log \ 3.332 \ 7247$$

$$\Delta E - 9 \ 618 \cdot 3 \quad \Delta N - 2 \ 151 \cdot 4$$

$$\text{DE} - 14 \ 883 \cdot 7 \quad \text{DN} + 6 \ 298 \cdot 9$$

$$\text{E} - 5 \ 265 \cdot 4 \text{ E} \quad + 4 \ 147 \cdot 5 \text{ N}$$

If the approximate geographical co-ordinates of P are calculated, or obtained by direct plotting, they may be used to calculate the reverse azimuths $A \rightarrow P$ from the formula

$$\text{Az. } A \rightarrow P = 180^\circ + \text{Az. } P \rightarrow A \pm \Delta\alpha$$

in which $\Delta\alpha = \text{diff. long. AP} \times \sin \frac{\text{lat. A} + \text{lat. P}}{2}$

(The reverse azimuth $B \rightarrow P$ may be calculated in a similar way.)
The steps in the complete computation are:

- (i) Calculate AB, the azimuth $A \rightarrow B$ and the reverse azimuth $B \rightarrow A$ (by the 'mid-lat' formula, see p. 77).
- (ii) Calculate the azimuth $A \rightarrow P$ (and $B \rightarrow P$, as a check) by the formula given above.
- (iii) Find the angle BAP from the difference between the azimuths $A \rightarrow B$ and $A \rightarrow P$ (and the angle PBA as a check).
- (iv) Solve the triangle ABP by the sine formula to obtain AP and BP, and thence the co-ordinates, either rectangular or geographical, of P.

SEMIGRAPHIC RE-SECTION. In practice, in most cases of re-section under case (a) (p. 82) the observations made at the position being re-sected will include rays to four or more of the surrounding triangulated points.

A satisfactory method of selecting the most probable position of the re-section is by inspection of all the observed rays plotted by the method of semigraphic re-section (similar to semigraphic intersection described on p. 79).

The procedure adopted in this method is:

- (i) Obtain approximate co-ordinates of P by computing the re-section from the three stations most suitably positioned using the method described on pp. 84-86.

This will not only establish the co-ordinates of the trial grid axis but also the co-ordinate bearings from the inter-section of these axes to the three stations selected.

- (ii) Using these first approximation bearings calculate bearings to all other observed stations by adding or subtracting the angles observed at P.

- (iii) Using the method described for semigraphic intersection calculate the cutting points of all the observed rays on the first approximation grid axis.

(iv) Draw in the rays on squared paper on a suitable scale.

(v) Select the position of P by the rules used for a plane-table re-section; viz. all the rays must be swung the same way and through the same angle (i.e. the amount of shift of any ray must be proportional to the distance of P from the trig point to which the ray was observed).

Should the first approximation grid axes be badly in error a second approximation should be made and the whole process repeated.

It is possible to obtain the first approximation grid axes by graphic plotting on a plane-table; in such a case it is advisable to compute at first only for three well-placed stations. Calculate the co-ordinate bearing from this first approximation P to the most distant of the three selected stations and by addition and subtraction of observed angles find the bearings to the other two.

Plot the three rays, and against them mark their distances (in convenient units, say 1000's of feet) from P.

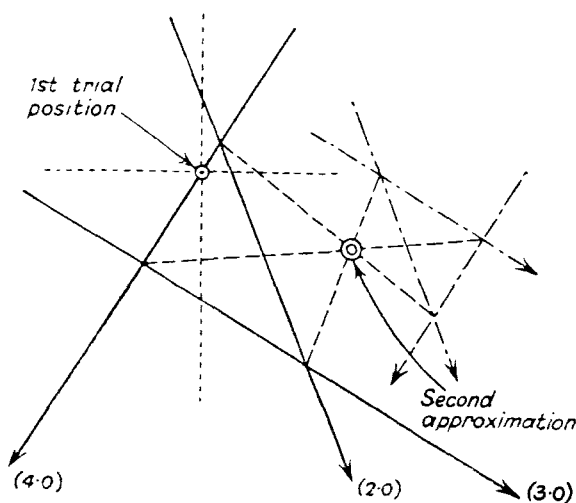


FIG. 29

Having decided which way the rays should be moved, rule in lines parallel to them at a distance proportionate to the distances marked on the rays, thus obtaining a 'cocked hat' similar in shape to the first. Join corresponding intersections, and these joining lines will cut in a point which may be used for the second approximation grid axis.

CHAPTER 4

TRIGONOMETRICAL AND PRECISE LEVELLING

THE VERTICAL control of a primary survey is, when the general topography permits, established by 'Precise Levelling', similar in principle to the engineering levelling described in Vol. I but carried out with greater refinement of instruments and technique. 'Trigonometrical Levelling', another method of establishing vertical control, is employed to supplement and/or replace the precise levelling in mountainous regions or other unsuitable terrain.

TRIGONOMETRICAL LEVELLING

It might have been inferred from the subject matter of Chapter 3 that the 'control' established by minor triangulation was horizontal control only; but in many surveys the vertical control also, especially in the initial stages, may have to be determined by angular measurement. For economy's sake the vertical angles will have been observed concurrently with the horizontal angles, and for that reason it is convenient to deal here with the determination of height by observation of vertical angles.

ANGLE MEASUREMENT. The method and best times of observation given on pp. 57 and 60 may be amplified slightly. It must be realized that, unless special instruments are provided for the purpose, the standard of accuracy of the observed vertical angles cannot attain that of the horizontal angles, for the following reasons: Firstly, with most theodolites the diameter of the vertical circle is less than that of the horizontal. Secondly, the construction of the vertical circle of normal theodolites does not allow a change of zero; consequently, however many repetitions are made of an angle, all are made on the same portion of the circle and errors of graduation cannot be eliminated. Thirdly, the R.O. for all vertical angles is the horizontal line through the instrument as determined by the altitude spirit level, which in most modern instruments is secured to the vertical frame; so that ultimately the accuracy of an

individual reading depends largely on the sensitivity of this level which, in some instruments, may be only 20–30 secs. per 2-mm. division.

When vertical angles are being observed for determining differences of height, the measurement of the heights of the theodolite axis above the observing station and of the observed point of the target above its station are essential parts of the observation.

CURVATURE AND REFRACTION

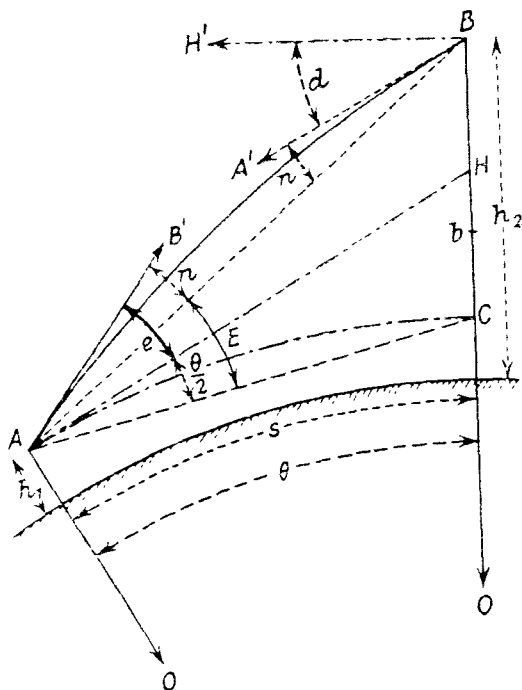


FIG. 30

A and B are two points elevated h_1 and h_2 above MSL. Join A and B to centre of earth and on OB mark C so that $OC = OA$.

Let the angle subtended at the centre of the earth by $AC = \theta$. The arc AC is parallel to MSL and the tangent AH represents the horizontal line through A.

The arc AB (which may be considered to be the arc of a circle) represents the ray of light between A and B curved by refraction; from A, therefore, B appears to be in the direction B' .

The angle $B'AH = e$ represents the angle of elevation observed to B from A. It can be seen that it exceeds the angle BAH by n , the angle of refraction.

By definition $\frac{n}{\theta} = \text{the coefficient of refraction} = \mu$ and $n = \mu \theta$

The angle $BAC = E$

$$= e + \frac{\theta}{2} \text{ (the angular correction due to curvature)} \\ - n \text{ (the angle of refraction); and}$$

$$\text{the angle } ABC = 180^\circ - \left[E + \left(90^\circ + \frac{\theta}{2} \right) \right] = 90^\circ - \left(E + \frac{\theta}{2} \right)$$

Then by the sine formula:

$$h_2 - h_1 = BC = AC \sin E \sec \left(E + \frac{\theta}{2} \right) \\ = S \frac{(R + h_1)}{R} \sin \left(e + \frac{\theta}{2} - n \right) \sec (e + \theta - n)$$

$$\text{or, very nearly} \quad = S \tan E \left(1 + \frac{h_2 + h_1}{2R} \right)$$

The term $\left(\frac{h_2 + h_1}{2R} \right)$ is so small that in most minor triangulation work it can be ignored and the formula written

$$h_2 - h_1 = S \tan E \\ = S \tan \left(e + \frac{\theta}{2} - n \right)$$

since $n = \mu \theta$ the formula becomes

$$h_2 - h_1 = S \tan \left[e + \theta \left(\frac{1}{2} - \mu \right) \right]$$

R represents the mean radius of curvature of the earth in the direction AC so $\theta = \frac{S}{R \sin 1''}$. R is a variable quantity depending on the latitude but a mean value may be taken as 20,890,000 feet; μ is also a variable quantity but its mean value has been found to be 0.07. If these values are used it will be found that $\theta \left(\frac{1}{2} - \mu \right) = S$ in feet $\times 0.00425$ sec.

SIMULTANEOUS RECIPROCAL OBSERVATIONS. As stated above, the coefficient of refraction of the atmosphere is a variable quantity,

the variations depending chiefly on changes of the temperature and density of the layers of atmosphere through which the ray of light passes.

Referring again to Fig. 30, let the horizontal through B be BH' and let BA' be drawn tangential to the arc BA. It can be seen then that H'BA' represents the angle of depression d observed from B to A and that the angle of refraction A'BA once more $= n$.

Since $AO = CO$, the angle $CAO = ACO = ABC + BAC$
 It can be seen that $BAO = 90^\circ + e - n$
 and $ABO (= ABC) = 90^\circ - d - n$

E, the angle required for finding $h_2 - h_1$ (see above)
 $= BAC$
 $= BAO - CAO$
 $= BAO - (ABC + BAC)$
 so $E = 90^\circ + e - n - (90^\circ - d - n) - E$
 i.e. $2E = e + d$ or $E = \frac{e + d}{2}$

so $h_2 - h_1 = S \tan \frac{e + d}{2}$

It can be seen from Fig. 30 that if, as frequently happens, B is between H and C, say in the position b where the angle $HAb > n$, then both the reciprocal angles will be angles of depression and the angle e becomes a minus quantity; d and e may then be written D_B and D_A and the formula becomes

$$h_2 - h_1 = S \tan \frac{D_B - D_A}{2}$$

When *simultaneous* reciprocal angles have been observed, it is convenient to use the formulae just given, but if the reciprocal angles are not simultaneous it is perhaps safer to work out each observation separately from the formulae

$h_2 - h_1 = S \tan (K + e) = S \tan (K - d)$
 where $K = \theta (\frac{1}{2} - \mu)$ [$=$ on the average, S (ft.) \times .00425 sec.]

ERRORS IN HEIGHT DUE TO USING INCORRECT COEFFICIENT OF REFRACTION. In general, it may be stated that mean values of the coefficient of refraction range from extremes of .060 for rays over the land in tropical countries to .081 for rays over the sea in

temperate countries. For rays 10 miles long the linear error in height caused by using a mean value of $\cdot 07$ instead of the extreme value amounts to no more than 1.4 feet.

DETERMINATION OF COEFFICIENT OF REFRACTION. The coefficient of refraction may be determined by simultaneous reciprocal observations as follows:

In Fig. 30 n = angle of refraction
 θ = angle subtended at centre of earth by AB

and $\mu = \frac{n}{\theta}$

It can be seen that $n = 90^\circ + e - \text{BAO}$
 and also $= 90^\circ - d - \text{ABO}$
 $\therefore 2n = 180^\circ - (d - e) - (\text{BAO} + \text{ABO})$
 and since $(\text{BAO} + \text{ABO}) = 180^\circ - \theta$
 $2n = \theta - (d - e)$ or $n = \frac{1}{2} [\theta - (d - e)]$

Dividing by θ $\mu = \frac{1}{2} \left[1 - \frac{(d - e)}{\theta} \right]$

or when both observed angles are depressions

$$\mu = \frac{1}{2} \left[1 - \frac{(D_A + D_B)}{\theta} \right]$$

EYE AND OBJECT CORRECTION

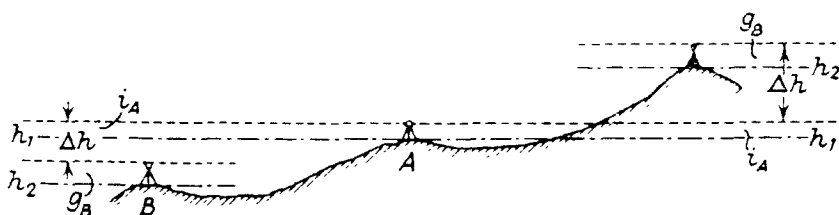


FIG. 31

Normally the $h_2 - h_1$ required is the difference between ground heights whereas the Δh observed is the difference in height between the theodolite trunnion and the observing point on the signal. In Fig. 31 these heights above their respective ground levels are represented by i_A and g_B .

Let y be the linear correction due to an angle of refraction n .
 n is a small angle and since AC approximates to S,

$$y = Sn \sin 1'' \text{ nearly}$$

$$\text{As was seen on p. 95 } n = \mu\theta = \mu \frac{S}{R \sin 1''}$$

$$\text{so } y = S \frac{\mu S \sin 1''}{R \sin 1''} = \frac{\mu S^2}{R}$$

Then Q , the combined correction due to curvature and refraction,

$$= x - y = \frac{S^2}{2R} - \frac{\mu S^2}{R} = \frac{S^2}{R} \left(\frac{1}{2} - \mu \right)$$

Using mean values $R = 20,890,000$ ft. and $\mu = .07$

$$Q = .2058 S^2 \times 10^{-7}$$

or if S is given in miles $= .574 S^2$ or $\frac{4}{7} S^2$ nearly

(A table giving Q for different values of S is given on p. 48 and will be found useful for plane table-work.)

PRECISE LEVELLING

The term *precise levelling* is grossly overworked, its meaning ranging from that of the land surveyor who uses it to mean geodetic levelling for the primary height control of a country to that of the student, graduating from a hand level, who regards any work done with a tilting level fitted with a fine levelling screw as precise levelling. Here it is taken to mean levelling carried out with that refinement of accuracy in instruments and methods which is required for geodetic levelling, though it must be understood that geodetic levelling itself is the concern of the National Survey department and is carried out as a major operation and seldom falls to the lot of the engineer or private practitioner.

INSTRUMENTS. In principle the instruments used do not differ from those used for ordinary engineering, but special attention is paid by the manufacturers to the following constructional details which add to their precision.

Stability. Provided by a broad levelling base and keeping the part of the instrument above the base as low as possible.

Level tube. The curvature of the tube must be of large radius and very uniform, the value of a 2-mm. division usually being from 1.2 to 3 secs.

Telescope. Must have superior definition. The aperture should be at least $1\frac{1}{2}$ inches and the focal length about 15 to 20 inches. Magnification ranges from about 30 to over 50 diameters. The focusing motion, usually internal, must be truly axial. Stadia lines should be fitted. Many telescopes can be 'reversed', i.e. rotated in their collars through 180° .

Reading. The bubble must be capable of being read at the eye end of the telescope, and in some models of precise levels it is read in the main telescope itself. In many types both ends of the bubble are seen together; in others a constant length of bubble is maintained by the provision of an air chamber.

Levelling arrangements. The tilting level device is used whereby the final adjustment for level in the direction of the line of sight is made by a fine levelling screw usually fitted under the eyepiece.

Parallel plate micrometer. This clever device (described in Vol. I), which enables an ordinary staff to be read directly to $\cdot 001$ feet and by estimation to $\cdot 0001$ feet, is usually fitted.

Levelling staff. Even graduation and constant length are the desiderata. Most precise staffs are made in one length of 10–14 feet, though that adopted by the Ordnance Survey is a folding staff hinged in the middle. Usually the graduations are cut on a strip of invar or nillex steel which is free to slide in the woodwork, being rigidly secured only at the base of the staff. Graduations are either to $\cdot 02$ or $\cdot 01$ feet. (Metric staff graduations range from 1 mm. to 1 cm.) Provision is frequently made for reading the temperature of the staff. Staffs are fitted with steadying handles and rods, spirit levels and plumb bobs to enable them to be held vertically and steady. They are invariably held on a foot plate or a pin driven nearly flush with the ground, the latter being the most satisfactory for precise work.

FIELD WORK. It is important to organize the routine of the field work to make the maximum use of the instrumental refinements listed above. The most important consideration is that the length of the back- and fore-sight should be equal, and in order that this may be checked readings of the stadia lines should be made at the same time as the centre line. To minimize errors caused through any alteration in height of the instrument between observations, as little time as possible should elapse between fore- and back-sights; to reduce this time it is customary to use two staffs, and, since the

index error of one staff may differ from that of the other, it is important that the booking should indicate which staff was observed for any one reading.

Temperatures should be taken at intervals so that the graduation length of the staffs may be corrected. In this connection it should be noted that change of temperature has other adverse effects on accuracy—in altering the value of the coefficient of refraction, in causing shimmer and in causing unequal expansion and contraction of the instrument. Of these effects the first probably cancels out between fore- and back-sights; the second is minimized as far as possible by never observing on the bottom foot of the staff; and the third must be guarded against by keeping the instrument shaded from hot sun during both observation and transport.

To minimize errors of reading, sights should be kept short; 120 to 150 ft. should be regarded as a maximum, though W. W. Williams states that the ideal length of sight is only 70 ft.*

The routine adopted in the geodetic levelling of Ceylon is given below as suitable for any form of precise levelling.

A chaining party went ahead of the levelling party, drove pickets for the staffs and marked positions for the instrument. Sights were limited to 125 feet with a maximum discrepancy of 2 feet between the length of the fore- and back-sights. Two staffs were used, the same staff always being read first (i.e. fore- and back-sight first alternately). Stadia lines were read each time with the plate micrometer in addition to the centre line reading. The order of reading was:

- | | | |
|-----------------------|---|---------------------|
| 1 Top stadia line | } | Odd numbered staff |
| 2 Bottom „ „ | | |
| 3 Centre line | | |
| 4 Centre line | } | Even numbered staff |
| 5 Top stadia line | | |
| 6 Bottom „ „ | | |
| Throw off bubble and | | |
| 7 re-read centre line | } | Odd numbered staff |
| 8 Centre line | | |

Booked readings were checked before moving on by calculating the sums and differences of

$$\begin{array}{ll}
 1 \text{ and } 2, \text{ sum} = A_1 \text{ diff.} = C_1 \\
 3 \text{ and } 8 \text{ „} = B_1 \text{ „} = D_1 \\
 4 \text{ and } 7 \text{ „} = B_2 \text{ „} = D_2 \\
 5 \text{ and } 6 \text{ „} = A_2 \text{ „} = C_2
 \end{array}$$

* E.S.R., Vol. I, No. 5, 1932.

Tolerances allowed were

$C_1 \sim C_2$	(distance)	2 feet
D_1 and D_2	(centre line readings)	·0015 feet
$A_1 \sim B_1$	(sums of centre lines and	·0030 feet
$A_2 \sim B_2$	stadia lines)	

No readings were permitted on bottom foot of the staff.

Computing the levels. Precise staffs should be calibrated in a laboratory, and allowances must be made for errors of graduation and also for temperature. Other errors such as collimation and curvature tend to cancel out if fore-sights and back-sights are equalized; and, generally speaking, levels may be reduced in the normal way, though opinions differ as to whether the readings of the stadia lines should be included in the computation, the majority opinion being that only centre line readings should be used.

RECIPROCAL LEVELLING. Sometimes it is impossible to avoid really long sights when crossing ravines or wide rivers, and, since it is usually impossible to equalize fore- and back-sights (as well as undesirable, because of uncertainty of refraction), other means must be used to cancel, or minimize, errors caused by curvature and refraction.

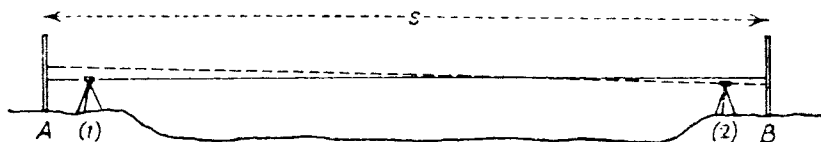


FIG. 33

In Fig. 33 let us suppose the levelling has reached position A and established the reduced level of the ground at that point. If staffs are set up at A and B, and readings taken on them by a level set up at (1), about 20 feet from A, the difference between the readings will give a value for the difference of height between A and B which will be in error because of the combined effect of the curvature of the earth and refraction; but if the level is now set up at (2), about 20 feet from B, and the readings and calculations repeated, the error will be repeated but with the opposite sign; so the mean of the differences of height obtained will be rid of any error caused by the curvature of the earth. Error due to refraction, however, can

The staff positions should be carefully prepared so that any chance of their height being altered when the staffs change position is reduced to a minimum. Instruments are set up in positions (1) and (2), and on a given signal each observer makes an agreed number of readings (6 or 8 are usual) on the staff on the opposite side of the ravine, after which he observes his own staff. Four to six (or more) of such groups of observations are made, each group being started by signal, after which observers, levels and staffs change position to the opposite side and repeat the same number of groups from the exchanged positions. The mean difference of height thus obtained should be rid of most error, but some degree of uncertainty, due to uneven refraction, will always remain, particularly when one end of the line of sight is over water and the other over dry sand.

Readings are made with the centre wire on the top target = t
when the bubble is central = c
with the centre wire on the bottom target = b
(Readings to the targets ignore the bubble.)

If H is the height of the bottom target above the zero of the staff and D is the distance apart of the targets, then the required staff reading is given by $H + \frac{(b-c)}{(b-t)}D$

ORTHOMETRIC AND DYNAMIC HEIGHTS. Although of small concern in minor triangulation, the difference between orthometric elevations and dynamic heights should be understood. The force of gravity varies with latitude, being less at the equator than at the poles. Consequently less work is required at the equator to raise unit mass to a given height above mean sea-level than at the pole; in other words the free surface of a large lake elevated at (say) 1000 feet above M.S.L. and stretching from equator to pole would not

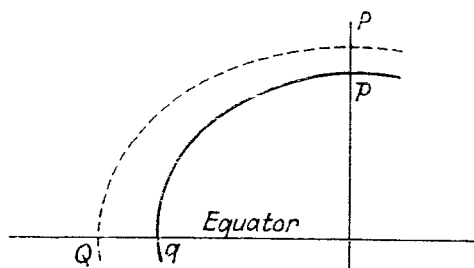


FIG. 34

lie parallel to M.S.L. but would converge from equator to pole as shown in exaggerated form in Fig. 34. The spirit levels of any instrument will lie in the equipotential surface, and consequently lines of levels run from Q to P would converge on those run from q to p . Q has the same dynamic height as P although the linear distance Qq is greater than Pp . Qq and Pp are the orthometric elevations of Q and P , and these are the heights that are shown on the Ordnance Map; it can be seen, however, that hydraulic engineers are chiefly concerned with dynamic heights. The equipotential surfaces under discussion take no account of local deviations of the plumb-bob, and consequently a line of constant dynamic height will lie parallel to mean sea-level if run along a parallel of latitude. It is thus possible to establish a relationship between orthometric elevations and dynamic heights in terms of latitude as follows: In any country a standard latitude is selected (in this country it is the

latitude of 53° N.); if H = orthometric elevation of a point in latitude ϕp and h = its dynamic height and ϕo = the latitude of the standard parallel, then:

$$h - H = H \left[\frac{1 + 0.00532 \sin^2 \phi p}{1 + 0.00532 \sin^2 \phi o} - 1 \right]$$

(*Note.* For the summit of Ben Nevis, 4406 feet in height in latitude $56^{\circ} 48' \text{ N.}$, $h - H = 1.46 \text{ ft.}$)

CHAPTER 5

PRECISE TRAVERSING

'PRECISE TRAVERSING' is usually understood to mean traversing carried out for establishing 'control' in countries unsuited for triangulation, e.g. in very flat country, especially if heavily forested, or in country where climatic conditions normally prevent the range of visibility necessary for economic triangulation. Surveys of comparatively small extent made for engineering purposes may frequently be 'controlled' more efficiently by precise traverse than by triangulation.

Generalizing, one may say that, except for the more precise methods adopted for making the measurements, both angular and linear, there is no difference between precise traversing and ordinary engineering traversing as described in Vol. I. Traverse legs are kept as long as possible, deviations, as illustrated in Fig. 39, being used if it is impossible to make the linear measurement of any leg in one straight line; the angles are measured with greater precision, and the measured sides are measured in catenary or along graded surfaces, usually by invar tapes; in addition frequent observations for azimuth are made. It is important, therefore, to decide the standard of accuracy desired and the precision necessary to achieve this standard before work is started.

STANDARDS OF ACCURACY. Precise traverses run for the control of national surveys regularly achieve closing errors of the order of 1 : 70,000 to 1 : 100,000, but for the traversing being considered here a reasonable closing error to strive for is about 1 : 30,000. An angle of 6.9 seconds will displace the end of a line by $\frac{1}{30000}$ of its length; consequently the standards of accuracy to be achieved must be a *p.e.* of $\frac{1}{30000}$ for linear measurements and a *p.e.* of 7 seconds for angular measurements. Both these probable errors will depend on the total length of the traverse and on the number of angle stations, but the routine of measurement given below is likely to achieve the desired standard of accuracy.

LINEAR MEASUREMENTS

These may be made with a 100-ft. steel tape $\frac{1}{8}$ inch wide supported in catenary or, with greater expedition, by a 300-ft. steel tape in catenary with a tension of 30 lb. With the longer tape it is usual to support it between the ends either with one support at 150 feet or two supports at 100 and 200 feet. These supports, which must be lined in carefully, both horizontally and vertically, between the end marks, may be of several kinds; most simple is a polished nail driven in perpendicularly to a stake, the necessary height adjustment being obtained by the angle at which the stake is supported on the ground by a labourer. More convenient perhaps

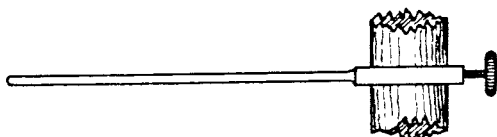


FIG. 35

is the rod (Fig. 35) similar to that on a retort stand, which may be slid up and down a pole and clamped in position; this may be improved if a bicycle-wheel hub is mounted with its ball bearings on the rod. The tension may be applied by weights as described in Chapter 2 or by spring balance; in either case an anchoring trestle is frequently used at one end and the tension applied at the other end only. The trestles may be similar to those described in Chapter 2 or may simply be poles with a hook on a strap or loose sleeve, which can be slid up and down the pole for height adjustment, to which the tape is secured; skill is required by the holders of such poles to keep them rigid and steady, but it is soon acquired.

The end marks for each tape length may be an ordinary measuring tripod (see p. 24) or a mark on a nail head driven into a stout wooden post about 3 feet high. Different methods of reading the tape against these marks are used, the method being dictated frequently by the type of graduation on the tape. Both ends of the tape may be read as described in Chapter 2, or the zero on the tape may be held in coincidence with the rear mark and the graduations on the tape read against the fore mark. In this case the signal to read must be given from the rear end. With tapes graduated to feet only, the difference between the fore-end mark and the

nearest foot graduation must be read by an auxiliary scale graduated to tenths and hundredths (or five-hundredths). Yet another method is to dispense with every other end mark and to set up the theodolite instead. The reading is then taken to the trunnion of the theodolite, which of course must be sighted on the end mark before



FIG. 36

the reading is taken. Fig. 36 illustrates the method (which is described in detail in Vol. I) and which, as can be seen, affects a considerable saving in the amount of equipment required. Slopes, for the slope correction, of each leg are obtained by vertical angle on the theodolite, while the intermediate supports are being lined in. In the other methods described the slopes can be measured as described in Chapter 2. Temperatures are observed for every set-up of the tape by a cased thermometer suspended horizontally near the tape.

A method much used in America is to substitute a small table 4 to 5 inches square for the end mark, these tables being mounted with a ball and socket joint on a tripod. The tables frequently carry a strip of copper on which the fore-end mark can be scratched with a knife against the end graduation of the tape; the rear-end zero mark of the succeeding tape length is then held against this scratch. Alternatively the tables carry a short graduated scale, placed with the zero towards the direction of measurement so that the reading of the fore-end graduation mark on the scale is always a positive correction.

ODD-LENGTH BAYS. For the sake of both convenience and accuracy traverse legs should, when possible, be made an exact number of tape lengths. Odd lengths which cannot be avoided should be measured directly against the graduations on the tape itself (if the tape is suitably graduated); otherwise a graduated tape must be kept specially for this purpose. Special grips for holding the tape at a position away from its end can be supplied by most instrument-makers.

CHECK TAPING. For the avoidance of gross errors it is essential that a check taping should be carried out. This taping can be conveniently measured on the flat during the setting-out process (see below). A standard tension is applied and slopes of more than 1 : 20 are corrected, but smaller slopes and temperature corrections are ignored.

SURFACE TAPING. In built-up areas and others where there are graded surfaces the taping may be done on the flat, and in America it is common practice to make the measurements along a railway-line using a glass-cutter to scratch the position of the forward end of each tape. The same care in applying a constant tension and in measuring temperature and slope is employed as for the catenary measurement. It should be noted in this connection that it is not necessary that the linear measurement should follow the line of the traverse leg, and in the case of measurements along a railway-line it is seldom that they do; the measurement is transferred to the traverse leg by means of offset angles and distances as shown in Fig. 37.

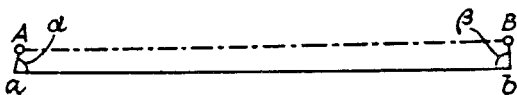


FIG. 37

AB = traverse leg

ab = measured length

also measured Aa , Bb , and α and β .

STANDARDIZATION OF FIELD TAPES. Field tapes should be standardized against a regularly standardized 'standard' tape at intervals of about 20 miles. In these standardization comparisons it is important that, when measuring the test bay, the standard tape should be used in the same way as it was when standardized (i.e. probably with weights suspended at each end), and similarly when the field tape is in the test bay the measuring procedure should follow the field practice, e.g. one end anchored and tension by spring balance at the other.

CORRECTIONS TO BE APPLIED TO LINEAR MEASUREMENTS. These are similar to those applied when measuring a base and may be summarized:

Standardization.

Temperature.

Tension. (Only necessary when tensions differ from those at standardization.)

Catenary. (Not necessary except for odd lengths if standardizations are made in catenary. Necessary in surface taping only when the tape has to cross gullies, etc., unsupported.) Note, a 300-ft. tape with intermediate supports at 100 and 200 feet carefully aligned may be treated as three 100-ft. lengths for this correction.

Slope. (Note, the correction for deformation of the catenary should be applied if the slope exceeds $1/10$, the approximate formula for the deformed catenary of $C \times \cos^2\theta$, where C = the ordinary catenary correction and θ = the slope, being sufficiently accurate.)

Horizontal alignment. (Not usually necessary.)

Height above sea-level. (The approximate formula for the correction $0.000048 \times H$ (ft. above M.S.L.) per 1000 feet of measured length, with H taken to the nearest 100 feet, is sufficiently accurate.)

Additional corrections which may have to be applied are: correction for index error of spring balance and correction for change of gravity. These corrections, if necessary, are required only for correcting the length of the standard tape before determining the length of the test bay to be used for comparison of the field tapes.

MEASURING ACROSS OBSTRUCTIONS. Rivers, gorges, high cliffs, etc., which obstruct the traverse must be bridged by triangulation, the bases for such triangulation being either legs of the main traverse or special subsidiary traverses run for that specific purpose.

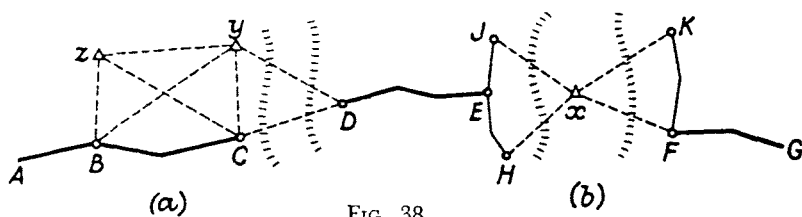


FIG. 38

Fig. 38 (a) and (b) show typical examples in which ABCDEFG is the main traverse; EH, EJ, and FK are subsidiary traverses and

z , y and x are triangulation stations. It is desirable, though not essential, that B and C, H and J, F and K should be intervisible.

ANGULAR MEASUREMENT

Angles should be measured with a theodolite of equivalent accuracy to the double-reading *Tavistock*, Watts *Microptic No. 2*, Wild T_2 or a 5-inch micrometer (see. p. 55). With the double-reading type a suitable observing routine for the main traverse angles is 4 or 6 readings, each on a different zero, with a change of face and swing between each zero, 6" to 8" being the maximum allowable discrepancy between a single reading and the mean reading. (When a maximum discrepancy of less than 6" is being achieved regularly 4 readings are sufficient.)

Observations may be made in daylight, but the signals used must be small and centred with great care. For lines of moderate length the targets used in the three-tripod system are ideal; for short lines a plumb-bob cord* or the nail driven into a stake which marks the traverse change point is suitable. For longer lines (it is assumed these will not exceed 3000 yards) a ranging pole may be used, but this must be plumbed and centred accurately.

Generally the type of traverse under consideration will be run from one triangulation station to another, and the size of the misclosure at the terminal station will indicate the accuracy of the work. It should be noted that a correspondingly small misclosure of the traverse of a closed figure which ends on the starting point is not a satisfactory test, since errors of standardization will not be disclosed.

RECONNAISSANCE AND SETTING OUT. A careful reconnaissance must be made before the main work is started, chiefly to choose the best line for the taping. Legs of the traverse should be as long as possible (though this is not as important as it is in primary traverse); in general, it is unlikely that legs will exceed about 3000 yards, and they should not be less than about $\frac{1}{4}$ mile; it is a convenience if the legs can be an exact number of tape lengths. It is desirable that the main angle stations should be on slightly elevated ground for the avoidance of grazing rays. When the route has been selected, lines must be cleared to ensure inter-visibility between angle stations

* Blind cord with the top six inches, next to the supporting tripod, painted alternately black and white, is suitable, the bob itself being steadied in a jam-jar of water.

and to enable the taping to be carried out. Sometimes it will be found that it is inconvenient or impossible for the taping to follow the direct line between angle stations and a deviation must be made; such deviations, though frequent in primary traverse, are usually unnecessary in the traversing being considered where shorter legs are permissible. Fig. 39 illustrates a deviation where

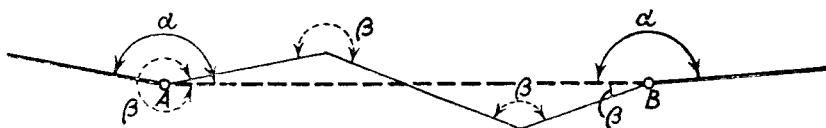


FIG. 39

the angles marked β are used solely for computing the length of the side AB and so may be observed with slightly less precision than the main traverse angles marked α . The legs are then set out for taping. A small theodolite is set up at the initial station, and, by means of it and a steel tape, pegs are driven in, in the correct alignment, at the appropriate field-tape-length intervals. If the measurement is to be made with a 300-foot tape in catenary, the appropriate interval will be about 0.9 in. less than 300 feet. The pegs will vary according to the measuring apparatus available. If measuring tripods are being used, short pegs suffice, but otherwise it is more convenient to use stout poles which will stand about 3 feet above the ground, on the head of which may be driven a nail as a terminal mark or a strip of copper may be fixed (p. 107). If the measurement is to be made to the trunnion of a theodolite, odd numbered pegs from the rear station should be long and even numbered ones (over which the theodolite will be set up) can be short. In spite of the extra time involved, it is recommended that the setting-out taping, although done as surface taping, should be carried out with sufficient precision to serve as the check taping necessary for the avoidance of gross errors (p. 109). The check is of much greater value if the length of the setting-out tape differs from that of the field tape, e.g. field tape, 300 feet, setting-out tape, 100 m. The angle stations or turning points should be marked in some semi-permanent way. Concrete pillars similar to the observing pillars used by the Ordnance Survey are undoubtedly the best form, but perhaps are over-elaborate for the type of work under consideration. During the setting-out it will sometimes be found

that by moving an angle station a few feet an exact number of tape lengths for the preceding leg may be obtained; such a movement is advisable, provided the setting-out is being carried out in correct sequence.

CONNECTION WITH TRIANGULATION. As a general rule the triangulation stations between which the traverse is to be run will be situated on high hills and ill-placed for direct linear measurements to be made from or to them. In such cases the connection between the triangulation station and the terminal traverse station must be made by triangulation by methods similar to those shown in Fig. 38. In Fig. 38 (*a*), D would be the triangulation station and C the terminal of the linear measurements; in Fig. 38 (*b*), x would be the triangulation station and either E or F the terminal of the linear measurements.

ROUTINE OF MEASUREMENTS. There will seldom be sufficient staff available for the various operations which have been described to be carried on simultaneously, and local conditions must decide whether linear or angular measurements are to be made first and also how far the one shall be carried before going back to do the other. At whatever stage the horizontal angular measurements are made, the measurement of the vertical angles and the measurement of azimuths (see below) should be made at the same time. Vertical angles are observed for the determination of difference of height between adjacent angle stations, the determination of height above mean sea-level for the correction of the linear measurements, and the determination of slope of each tape length for the same purpose. Only the first of these requires any great precision (pp. 60-61) and all may be dispensed with if a line of levels is being run along the traverse.

AZIMUTHS. An important way of controlling the direction of the traverse is by observing azimuths of heavenly bodies at regular intervals. Primary traverse demands azimuth observations to a high order of precision at intervals of not more than 50 miles; in this 'minor' precise traversing, the precision of the observations need not be so high, but as a general rule the azimuth stations should not be more than 10 miles apart. In appropriate latitudes the observations may be made to Polaris, in which case a suitable

observing routine is *face left* and *face right* on four different zeros, the local time being determined by radio or star (see Chapter 7). Elsewhere the observations should be made to stars at or near elongation. Stars should be observed in pairs, and both declination and altitude of the two in a pair should be in close agreement. Four to six pairs should suffice, and it is desirable that their azimuths should be well spread round the horizon. Time azimuths should be used if radio time signals are available; otherwise altitude azimuths are recommended. Treating the azimuth stations as the ends of sections, American practice allows a maximum discrepancy of $2n''$ between the observed and computed azimuth at the end of a section, where n is the number of main angle stations in the section.

LAY-OUT OF CONTROL TRAVERSES

When the main survey control is being established by precise traversing, it is convenient to lay out the work in the form of a grid, with a series of roughly parallel traverses being cut perpendicularly by a similar series of parallel traverses. Obviously the topography, both natural and artificial, will be the deciding factor in the final siting of the various traverses, and a route which offered easy and accurate traversing, such as one along a main road, would never be abandoned simply for the sake of conforming to an idealized geometrical pattern. (See also p. 111.) The grid type of lay-out simplifies the subsequent breaking down of the control by secondary traverse, and also lends itself to the simplified traverse adjustment described in Chapter 6 (p. 149).

As an alternative to the grid pattern the work may be laid out as a series of triangles. Since all the sides will have been measured, the angles may be calculated, and from these calculated angles the 'trilateration' may be adjusted as a triangulation as detailed in Chapter 6.

COMPUTATION OF MINOR PRECISE TRAVERSE

For many purposes the methods of computing an ordinary engineering traverse by plane rectangular co-ordinates will suffice, but if the purpose of the traverse is for establishing control over a comparatively large area, and if it is being adjusted between triangulation stations, extra computation will be required to give the corrected co-ordinates of the change points. Primary traverse

is frequently computed in terms of geographical co-ordinates, which can then be converted at will to the co-ordinates of the particular projection in local use, but with minor precise traverse it is more convenient to use plane rectangular co-ordinates and then convert them to the local system by approximate formulae. Approximate formulae for Transverse Mercator co-ordinates are given in Chapter 6; it should be noted that the method described demands that the traverse should be divided into fairly short sections, say 10 to 12 miles, with an azimuth observed at the end of each section.

CHAPTER 6

ADJUSTMENT OF MINOR TRIANGULATION AND PRECISE TRAVERSES

IN ALL accurate surveys of some extent, the linear measurements should be examined to see if a reduction to mean sea-level would appreciably affect the results. Assuming that the earth is spherical, then the horizontal length of a line is the distance between the verticals at its two ends, and it is clear from Fig. 40 that this

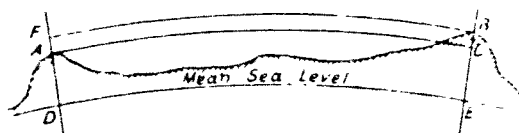


FIG. 40

differs with the level at which that distance is measured. Thus FB is longer than AC.

REDUCTION TO SEA-LEVEL. Hence if different sides of a traverse or triangulation were measured independently at different levels, then the measurements would not agree. It is therefore desirable to have a common datum; hence all surveys are reduced to mean sea-level. Thus if DE in the figure is at mean sea-level, then either FB or AC would be represented by the length DE.

To determine the corrections to be applied to reduce a measurement to M.S.L., let $AD = h$ and $r =$ radius of the sphere at mean sea-level. Then suppose AC is measured:

$$\frac{DE}{AC} = \frac{r}{r+h} = \left(1 - \frac{h}{r}\right)^{-1}$$

or $DE = AC = AC \cdot \frac{h}{r}$ very nearly, as $\frac{h}{r}$ is always a small fraction.

Hence the correction to be subtracted is $AC \times \frac{h}{r}$

Now as $r = 20,900,000$ feet (approx.), if h is, say, 1000 feet,

$$\frac{h}{r} = \frac{1}{20900}$$

which is greater than the probable error of the best measurements.

ADJUSTMENT OF MINOR TRIANGULATION

In order to arrive at a simultaneous correction of the whole of a triangulation, the most probable values of all the angles should be worked out together, using the method of least squares. However, to do this even for a limited number of stations would involve such lengthy computations that in general the triangulation is divided into sections which are adjusted independently. The complete theoretical result is therefore forfeited in order to contain the calculations within reasonable limits.

The first consideration is to see that the observations taken at each station balance within themselves irrespective of the results obtained for other stations. This in general consists of seeing that the horizon closes, i.e. that all the angles comprising the complete rotation about the station sum to 360° , and that all other geometrical conditions are fulfilled such as certain angles being the summation of other angles.

The adjustment of the angles at one station can be carried out very simply. If the horizon is closed by measuring each angle of which it is composed, separately, then, if the weights of the observations are equal, equal corrections can be applied to each reading. If the weights of the observations are unequal, then corrections can be applied in inverse proportion to the weights. In the same way if two or more angles are observed at a station and also their summation, and a discrepancy exists, then correction can be made in inverse proportion to the respective weights of the observations and with the sign to the total angle of opposite value of that to the part angles.

SPHERICAL EXCESS. The second consideration is to see that the angles in each triangle have the proper summation, and also to see how the interlacing of triangles affects the individual angles. When the triangles of the system are large, as mentioned in Chapter 3, spherical excess has to be taken into account. Spherical excess is

the amount by which the angles of a spherical triangle exceed 180° . If A is the area of the triangle and r the mean radius of the earth's curvature at the place, then the spherical excess

$$E = \frac{A}{r^2 \sin 1''} \text{ secs.}$$

For great precision the earth could even be taken as a spheroid instead of a sphere to calculate the excess, in which case

$$E = \frac{A}{\rho \nu \sin 1''} \text{ secs. where}$$

ρ = the radius of curvature of a meridian section at the latitude concerned, and

ν = the radius of curvature of the normal surface curve made by the perpendicular to the meridian at the latitude concerned.

It is convenient when spherical excess has to be taken into account to use Legendre's theorem, which is discussed in Chapter 3.

SIMPLE POLYGON NET. The way in which the triangles are interlaced introduces equations of condition depending on the particular arrangement involved. It is only possible to deal in general terms with the simpler systems of triangulation.

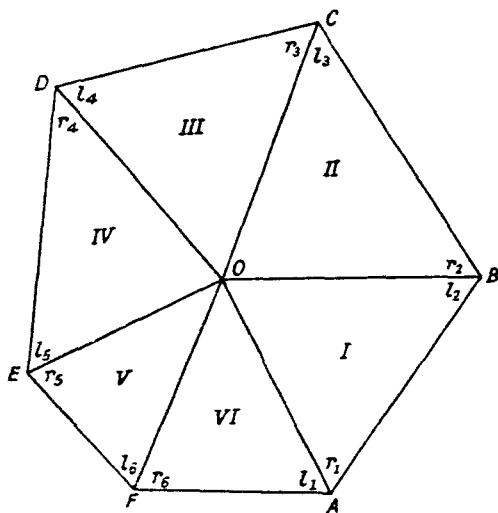


FIG. 41

Consider a system of several triangles fitting together to form a closed polygon ABCDEF (Fig. 41) with O as some station point

within the polygon. Then the polygon may be considered as made up of the triangles AOB, BOC, COD, DOE, EOF and FOA. Then considering the angles reduced for spherical excess, the angles of each component triangle must sum to 180° .

Now consider the local condition at station O. In this case the sum of the angles must be 360° . If the angles are observed in a continuous round with a theodolite checking back to zero on the first station, then the angles being obtained by successive subtraction must sum correctly. Nevertheless, each angle is as liable to error as if observed independently, when even the sum in all probability would not be correct.

Suppose now that the angles have been corrected to satisfy the above two conditions and the polygon is drawn out starting at the triangle AOB and continuing the next triangle BOC from the side

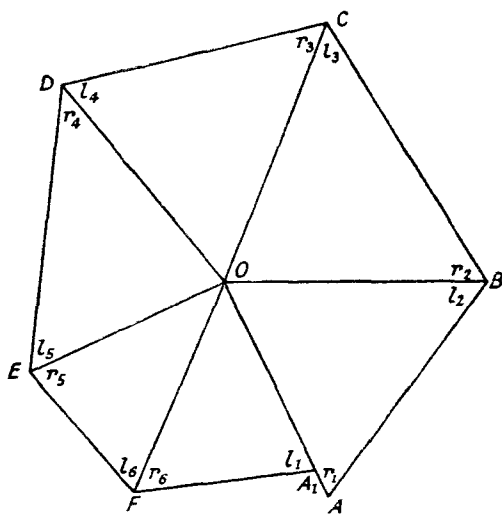


FIG 42

OB. The triangles will fit together as in Fig. 42. The angles at the centre station O adding correctly to 360° and the angles in each triangle adding correctly to 180° . Even so, there is no condition to ensure that the side OA_1 in the final triangle FOA_1 shall be the same in length as OA in the first triangle drawn. It is obvious therefore that it is necessary to introduce another condition. Let the left- and right-hand angles of the triangles be numbered as in Figs. 41 and 42.

Now by plane trigonometry

$$\begin{array}{ll} \frac{OB}{OA} = \frac{\sin r_1}{\sin l_2} & \frac{OE}{OD} = \frac{\sin r_4}{\sin l_5} \\ \frac{OC}{OB} = \frac{\sin r_2}{\sin l_3} & \frac{OF}{OE} = \frac{\sin r_5}{\sin l_6} \\ \frac{OD}{OC} = \frac{\sin r_3}{\sin l_4} & \frac{OA_1}{OF} = \frac{\sin r_6}{\sin l_1} \end{array}$$

Multiply these equations together

$$\begin{aligned} \frac{OB \times OC \times OD \times OE \times OF \times OA_1}{OA \times OB \times OC \times OD \times OE \times OF} \\ = \frac{\sin r_1 \times \sin r_2 \times \sin r_3 \times \sin r_4 \times \sin r_5 \times \sin r_6}{\sin l_1 \times \sin l_2 \times \sin l_3 \times \sin l_4 \times \sin l_5 \times \sin l_6} \end{aligned}$$

and for OA_1 to equal OA the products on the left of the equation cancel to give 1.

That is, in any closed polygon, composed of triangles, the continued product of the sines of the right-hand angles is equal to the continued product of the sines of the left-hand angles. The equation which results from this condition is called a side equation, and it is usual to take the more convenient form of the log sines in lieu of the natural sines.

The three conditions of a perfect polygon are therefore:

1. The three angles of each triangle must sum two right angles after correction for spherical excess.
2. The angles of the central point must sum four right angles.
3. The sum of the log sines of the right-hand angles must equal the sum of the log sines of the left-hand angles.

Very similar conditions exist where the station O falls outside the polygon or in other systems of interlacing.

The method used for carrying out the adjustment of the observed values depends upon the time available for making computations and the degree of accuracy required. The most accurate method is to choose that distribution of corrections which makes the sum of the squares of the corrections a minimum according to the principle of least squares. To apply this method rigidly would involve a simultaneous adjustment of all the angles in the system to comply with the geometrical and log sine conditions and the condition that the sums of the squares of the errors should be a minimum. This involves a large amount of arithmetical labour, however, as will be

clear from an example given later, and usually much of the work is reduced by judiciously dividing the whole triangulation into separate figures.

METHOD OF TRIAL AND ERROR. Many surveyors prefer to ignore the exact mathematical methods of procedure and proceed to an adjustment by trial and error. In such cases the only definite principle usually involved is a limitation of the maximum size of an error allowed. For example, the maximum correction to any one observed angle might be set at $30''$.

A sketch of the triangulation is first prepared approximately to scale, the angles laid off by means of a protractor and the triangles numbered as in Fig. 41. A computation table is then drawn up as shown in Table 9 and the observed angles entered into the appropriate places as indicated, $O_1, O_2, \dots, l_1, l_2, \dots, r_1, r_2, \dots$. The centre angles are summed and compared to 360° , and the angles of each triangle centre, left and right, summed and entered in the appropriate column. The log sines of the left- and right-hand angles are also entered from seven-figure logarithmic tables and also the log difference for $1''$ in each case. This is taken from the seven-figure tables at the same time as the log sines, and means the increase in the log sine due to an increase of $1''$ in the angle, and this, when multiplied by the correction to the angle, gives the corresponding change in log sine, for which there is a column in the table headed 'Corrected difference'. If the angle is greater than 90° , the log difference for $1''$ must be entered minus, because an increase in the angle means a decrease in the sine. The two columns of log sines can now be added. By a comparison of the sums of the individual triangles to 180° , the errors are found and entered in the last column. The adjustment now consists of judiciously shifting the corrections in order to satisfy simultaneously the log sine condition, the triangle conditions and the central angle condition. Various methods are adopted as a first trial, one being simply to divide the total error of each triangle equally between the three angles. The shifting is in general, however, simply carried out according to the judgment of the computer. In passing from polygon to polygon one finds certain triangles common to more than one, and all corrections applied to these triangles in the polygon treated first must, of course, be carried down to the later polygons, and must not again be altered, unless the first polygon is re-corrected.

T A B L E 9
CORRECTION OF ANGLE OF POLYGON (CENTRE O)

No. of tri-angles	Centre			Left				Right				Sum of observed angles ° / "	Corr.
	Observed angles ° / "	Corr. "		Observed angles ° / "	Log sines	Log diff. for 1"	Corr. "	Observed angles ° / "	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	
I	O ₁			l ₃				r ₁					
II	O ₂			l ₃				r ₂					
III	O ₃			l ₄				r ₃					
IV	O ₄			l ₅				r ₄					
V	O ₅			l ₆				r ₅					
VI	O ₆			l ₁				r ₆					
	sum				sum				sum			sum	

METHOD OF EQUAL SHIFTS. The disadvantage of this haphazard reliance in the dexterity of the computer is that it is difficult for one computer to take over from another, as it is entirely unsystematic. The method of equal shifts was therefore introduced, which, although it is as devoid of mathematical justification as the method

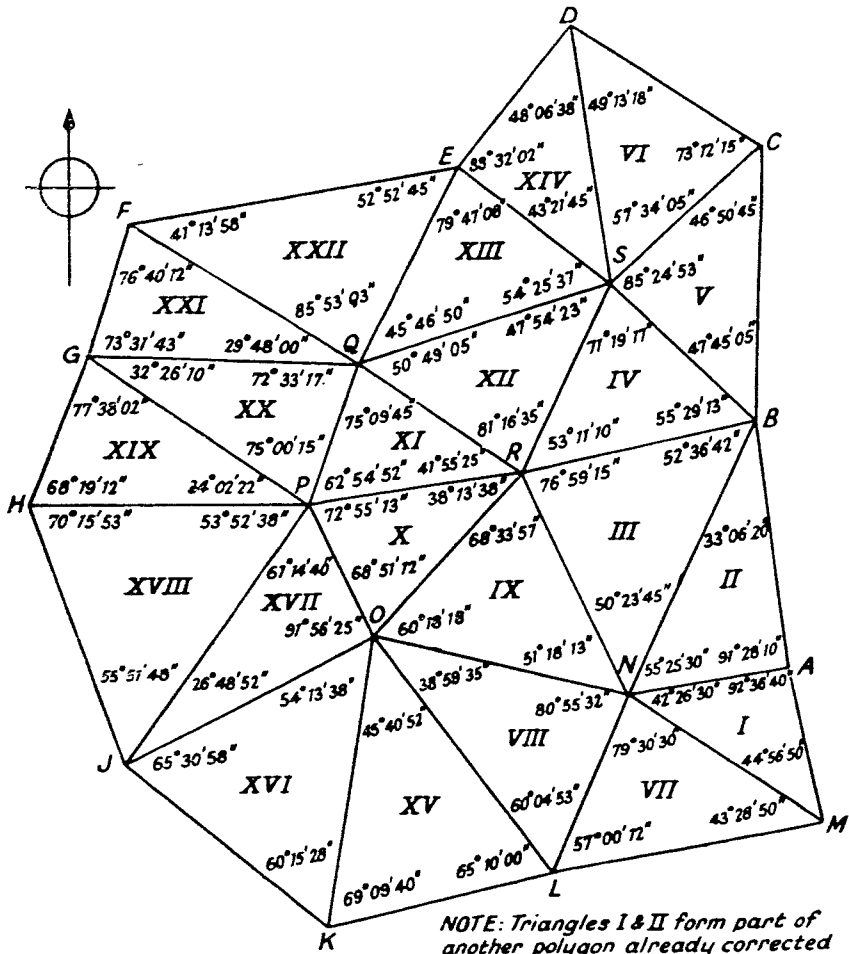


FIG. 43

of trial and error, in that it only gives one of an indefinite number of solutions, does give a systematic treatment with a definite procedure. The underlying principle of the method is that, if any shift

is necessary to satisfy the local equation of a station, then it should be the same for each angle completing the station, and that this should also be true of any further shift necessary to satisfy the side equations.

The method is best illustrated with a numerical example. Refer to Fig. 43 which is a portion of the *Malta Triangulation*, each observed angle being entered in its proper place on this diagram. Triangles I and II form part of another polygon already corrected, and therefore need not be altered. The correction is carried out as follows:

First complete a table similar to Table 9 for the first polygon to be considered with centre N. This is done in Table 10. Then correct each centre angle of a polygon by one third of the total error in the triangle. Sum these corrections and see how the result affects the sum of the centre angles. It can then easily be found how much each correction must be increased or decreased, the shift being equal in all.

Take the first polygon with centre N. Triangles I and II are already corrected. Consider triangles III, IX, VIII and VII. The corrections for these triangles are respectively $+18''$, $-28''$, zero and $+28''$ from Table 10. Divide these corrections by three, and the trial corrections to the centre angles are $+6.0$, -9.3 , zero and $+9.3$, of which the sum is $+6.0$.

Add the local angles at the centre N in Table 10 to give $360^{\circ} 00' 00''$. So the correction to the centre angle should be zero, not $+6.0$.

Since triangles I and II are already corrected, there are only four angles to which an adjustment can be made; hence each must be decreased by one quarter of 6.0 , or 1.5 . Hence the corrections to the centre angles $+4.5$, -10.8 , -1.5 , $+7.8$.

Now subtract each of these from the total error of the corresponding triangle, and one half of the results will give the trial corrections to the left-hand and right-hand angles. Thus for triangle III

$$\begin{aligned} \text{the summation correction} &= +18'' \\ \text{applied to centre angle} &= +4.5'' \\ \text{remainder} &= +13.5'' \end{aligned}$$

Trial correction to left- and right-hand angles is $6.8''$ (working to first decimal place). The log differences for $1''$ belonging to the left- and right-hand angles of the triangle III are 4.9 and 16.1 respectively (from Table 10). These are multiplied by the trial corrections to find the corrected differences.

T A B L E 10
CORRECTION OF ANGLES OF POLYGON (CENTRE N)

No. of tri-angles	Centre			Left				Right				Sum of observed angles ° ' "	Corr.
	Observed angles ° ' "	Corr. "	Observed angles ° ' "	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	Observed angles ° ' "	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	
I	42 26 30	—	92 36 40	9.9995483	—	—	—	44 56 50	9.849846	—	—	180 00 00	—
II	55 25 30	—	33 06 20	9.7373383	—	—	—	91 28 10	9.9998571	—	—	180 00 00	—
III	50 23 45		76 59 15	9.9887020	4.9			52 36 42	9.9001148	16.1		179 59 42	+18
IX	51 18 13		60 18 18	9.9388572	12.0			68 23 57	9.9683761	8.3		180 00 28	-28
VIII	80 55 32		60 04 53	9.9378862	12.1			38 59 35	9.7988068	26.0		180 00 00	00
VII	79 30 30		43 28 50	9.8376568	22.2			57 00 12	9.9236078	13.7		179 59 32	+28
	360 00 00			1.4399893					1.4398472				

When this procedure is followed out for each triangle in the polygon, the results can be tabulated.

TABLE 11
TRIAL CORRECTED DIFFERENCES

No. o triangles	Left		Right	
	Correction	Corrected diff.	Correction	Corrected diff.
III	+ 6.8	+ 33	+ 6.8	+ 110
IX	- 8.6	- 103	- 8.6	- 71
VIII	+ 0.8	+ 10	+ 0.8	+ 21
VII	+ 10.1	+ 224	+ 10.1	+ 138
	Sum =	+ 164		+ 198

Hence the trial corrections have increased the log sines of the left-hand angle by 164, and also increased those of the right-hand by 198, giving a resultant increase on the right (as compared with the left) of $198 - 164 = 34$. Now the original sums of the log sines for this polygon are (Table 10)

Left . . . $\bar{1} \cdot 4399893$

Right . . . $\bar{1} \cdot 4398472$

Right too small by 1421

Hence the increase of 34 on the right is in the proper direction, but is much too small. A relative increase of $1421 - 34 = 1387$ is still required. Now add up the log differences for 1" on both sides for those triangles which are being corrected (from Table 10)

<i>left</i>	<i>right</i>
4.9	16.1
12.0	8.3
12.1	26.0
<u>22.2</u>	<u>13.7</u>
51.2	64.1

This shows that a shift in each triangle from left to right, or vice versa, will affect the summation of the log sines by $51.2 + 64.1 = 115.3$. Hence to produce a correction of 1387, still required as above, the number of seconds to be shifted is $\frac{1387}{115.3} = 12$ seconds, very nearly. As the right-hand side is to be relatively increased, add

12" to each right-hand correction, and deduct 12" from each left-hand correction.

The results are again multiplied by the differences for 1", to find the corrected differences as here shown.

TABLE 12
FINAL CORRECTED DIFFERENCES

No. of triangle	Left			Right		
	Trial correction	Final correction	Corrected difference	Trial correction	Final correction	Corrected difference
III	+ 6.8	- 5.2	- 25	+ 6.8	+ 18.8	+ 303
IX	- 8.6	- 20.6	- 247	- 8.6	+ 3.4	+ 28
VIII	+ 0.8	- 11.2	- 136	+ 0.8	+ 12.8	+ 333
VI	+ 10.1	- 1.9	- 42	+ 10.1	+ 22.1	+ 303
		Sum =	- 450			+ 967

The results show a decrease on the left of 450 and an increase on the right of 967, or a relative increase to the right of $967 + 450 = 1417$ instead of the 1421 required to balance exactly. The discrepancy of a few units like this is negligible. The whole net can now be proceeded with, remembering that corrections applied to any triangle in one polygon must be carried forward to the next, and must not be altered unless it is desired to re-correct the first polygon.

For example, if the next polygon taken is centre R, the triangles III and IX must be left unchanged as I and II were in the polygon centre N. If the polygons R, S, O, Q and P are then taken in turn, in this case the maximum single correction is found to be 37", and the sum of the squares of all the corrections is about 12,000. If one follows a hard and fast system like this and carries on from polygon to polygon, it is inevitable, of course, that certain polygons will be adversely affected by the corrections brought down to them, and the magnitude of the corrections will gradually increase. This, of course, is the advantage of the judicious shifting of errors without a definite method, when the corrections can be limited and no large corrections carried on.

The results of the equal shift method could possibly be improved by taking the polygons in a different order, beginning with the worst one. It is much better, however (and not very much more

laborious), to take five or six polygons at a time, and this can be illustrated by correcting the whole net shown in Fig. 43 at once.

In order to carry this out, it is necessary to draw up tables similar to Table 10 for all the polygons involved. The tables for all the polygons involved in the example are drawn up in Table 13. The various errors in summation are shown in this table, but no corrections are filled in until all the calculations are completed. For easy reference, use the following notation: B_3 = angle at B, in triangle No. III, and so on; b_3 = correction to angle B_3 , and so on; ΣN = total summation error at N; Σ_3 = total summation error in triangle No. III; ΣR = sum of log sines of right-hand angles; ΣL = sum of log sines of left-hand angles. Then, taking the whole net, first find the corrections at each central station to satisfy the local equations as before.

Where a triangle is common to two polygons, two of its angles will be at centre stations, and trial values of the corrections to these two will be settled in this way. The third trial correction is then also settled, as the sum of the three must be equal to the summation error for the triangle.

If a triangle belongs to three polygons, each of its trial values would be settled by the local equations, and, as they would seldom sum to the proper amount, make the trial values in such triangles equal to one third of the whole error. Thus, in this case, triangles Nos. IX, X, XI and XII, all belonging to three polygons, can be dealt with first. The corrections are drawn up in Table 14, with the trial corrections found by dividing the total error in each triangle equally between the three angles. This accounts for angles N_9 , O_9 , R_9 , Q_{10} , P_{10} , R_{10} , P_{11} , Q_{11} , R_{11} , Q_{12} , R_{12} and S_{12} .

Triangle No. III has two angles N and R at central stations.

Taking N first, apply one third the error in each triangle of the polygon to the centre angle, and then correct these to satisfy the local equation at N as before, except that the trial correction to N_9 has already been settled, and must not be altered.

n_1 and n_2 must be zero as I and II are fixed. For first trials put

$$n_3 = \frac{+18}{3} = +6; n_8 = 0; n_7 = \frac{+28}{3} = +9.3$$

and

$$n_9 \text{ already fixed at } -9.4$$

The sum of these trial corrections is $+6 + 0 + 9.3 - 9.4 = +5.9$, whence the total shift required is -5.9 .

T A B L E 13
EXAMPLE OF CORRECTIONS TO POLYGON ANGLES

Centre			Left				Right				Sum of angles	Corr.	
No. of tri-angles	Observed angles ° ' "	Corr. "	Observed angles ° ' "	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	° ' "	"
POLYGON POINT N CENTRE													
I	42 26 30		92 36 40	9.9995483	—			9.8490846	—			180 00 00	—
II	55 25 30		33 06 20	9.7373383	—			9.9998571	—			180 00 00	—
III	50 23 45		76 59 15	9.8887020	4.9			9.9001148	16.1			179 59 42	+18
IX	51 18 13		60 18 18	9.9388572	12.0			9.9683761	8.3			180 00 28	-28
VIII	80 55 32		60 04 53	9.9378862	12.1			9.7988068	26.0			180 00 00	00
VII	79 30 30		43 28 50	9.8376568	22.2			9.9236078	13.7			179 59 32	+28
	360 00 00			1.4399893				1.4398472					
POLYGON POINT R CENTRE													
III	76 59 15		52 36 42	9.9001148	16.10			9.8867540	17.40			179 59 42	+18
IV	53 11 10		71 19 17	9.9765012	7.10			9.9159257	14.47			179 59 40	+20
XII	81 16 35		50 49 05	9.8893822	17.15			9.8704335	19.00			180 00 03	-03
XI	41 55 25		62 54 52	9.9495499	10.80			9.9852719	5.60			180 00 02	-02
X	38 13 38		68 51 12	9.9697234	8.13			9.9804111	6.47			180 00 03	-03
IX	68 23 57		51 18 13	9.8923561	16.90			9.9388572	12.00			180 00 28	-28
	360 00 00			1.5776276				1.5776534					

T A B L E 13 (Contd.)

Centre			Left				Right				Sum of observed angles °	Corr. "
No. of tri- angles	Observed angles °	Corr. "	Observed angles °	Log sines	Log diff. for 1"	Corr. "	Corr. diff.	Observed angles °	Log sines	Log diff. for 1"		
			POLYGON POINT Q CENTRE									
XXII	50 49 05		47 54 23	9.8704335	19.00			81 16 35	9.9949465	3.23	180 00 03	-03
XXIII	45 46 50		79 47 08	9.9930617	3.80			54 25 37	9.9102905	15.00	179 59 35	+25
XXXII	85 53 03		41 13 58	9.8189644	24.00			52 52 45	9.9016569	15.93	179 59 46	+14
XXXI	29 48 00		73 31 43	9.9818011	6.20			76 40 12	9.9881389	4.98	179 59 55	+05
XXX	72 33 17		75 00 15	9.9849522	5.63			32 26 10	9.7294554	33.13	179 59 42	+18
XXI	75 09 45		41 55 25	9.8248669	23.45			62 54 52	9.9495499	10.80	180 00 02	-02
	360 00 00			1.4740798					1.4740381			
			POLYGON POINT P CENTRE									
XXI	62 54 52		75 09 45	9.9852719	5.60			41 55 25	9.8248669	23.45	180 00 02	-02
XXX	75 00 15		32 26 10	9.7294554	33.13			72 33 17	9.9795500	6.62	179 59 42	+18
XXIX	34 02 22		68 19 12	9.9681379	8.37			77 38 02	9.9898052	4.62	179 59 36	+24
XXVIII	53 52 38		55 51 48	9.9178737	14.28			70 15 53	9.9737109	7.55	180 00 19	-19
XXVII	61 14 40		91 56 25	9.9997509	0.72			26 48 52	9.6542753	41.67	179 59 57	+03
X	72 55 13		38 13 38	9.7915375	26.73			68 51 12	9.9697234	8.13	180 00 03	-03
	360 00 00			1.3920273					1.3919317			

TABLE 14
SHIFT CORRECTIONS

Angle	Trial correction	Shift	Final value
N ₉	— 9·4	$\frac{1}{2}(x_R - x_O)$	— 18·2
O ₉	— 9·3	$\frac{1}{2}(x_N - x_R)$	— 15·3
R ₉	— 9·3	$\frac{1}{2}(x_O - x_N)$	+ 5·5
O ₁₀	— 1·0	$\frac{1}{2}(x_R - x_P)$	+ 2·8
P ₁₀	— 1·0	$\frac{1}{2}(x_O - x_R)$	+ 7·8
R ₁₀	— 1·0	$\frac{1}{2}(x_P - x_O)$	— 13·6
P ₁₁	— 0·7	$\frac{1}{2}(x_R - x_Q)$	0
Q ₁₁	— 0·6	$\frac{1}{2}(x_P - x_R)$	— 4·4
R ₁₁	— 0·7	$\frac{1}{2}(x_Q - x_P)$	+ 2·4
Q ₁₂	— 1·0	$\frac{1}{2}(x_R - x_S)$	+ 7·9
R ₁₂	— 1·0	$\frac{1}{2}(x_S - x_Q)$	— 9·2
S ₁₂	— 1·0	$\frac{1}{2}(x_Q - x_R)$	— 1·7
N ₃	+ 4·0	$-\frac{1}{2} x_R$	+ 5·0
R ₃	+ 5·6	$+\frac{1}{2} x_N$	— 1·4
B ₃	+ 8·4	$\frac{1}{2}(x_R - x_N)$	+ 14·4
N ₈	— 2·0	$+\frac{1}{2} x_O$	+ 5·8
O ₈	+ 5·3	$-\frac{1}{2} x_N$	+ 12·3
L ₈	— 3·3	$\frac{1}{2}(x_N - x_O)$	— 18·1
N ₇	+ 7·4	—	+ 7·4
L ₇	+ 10·3	$-x_N$	+ 24·3
M ₇	+ 10·3	$+x_N$	— 3·7
R ₄	+ 6·4	$-\frac{1}{2} x_S$	+ 16·3
B ₄	+ 6·6	$\frac{1}{2}(x_S - x_R)$	— 2·3
S ₄	+ 7·0	$+\frac{1}{2} x_R$	+ 6·0
S ₅	— 14·0	—	— 14·0
B ₅	— 14·5	$-x_S$	+ 5·3
C ₅	— 14·5	$+x_S$	— 34·3
S ₆	+ 7·6	—	+ 7·6
C ₆	+ 7·2	$-x_S$	+ 27·0
D ₆	+ 7·2	$+x_S$	— 12·6
S ₁₄	— 8·1	—	— 8·1
D ₁₄	— 8·5	$-x_S$	+ 11·3
E ₁₄	— 8·4	$+x_S$	— 28·2
S ₁₃	+ 8·5	$-\frac{1}{2} x_Q$	+ 10·2
E ₁₃	— 13·0	$\frac{1}{2}(x_Q - x_S)$	— 21·2
Q ₁₃	+ 3·5	$+\frac{1}{2} x_S$	— 6·4

TABLE 14 (Contd.)

Angle	Trial correction	Shift	Final value
Q_{22}	0	—	0
E_{22}	+ 7.0	- x_Q	+ 10.3
F_{22}	+ 7.0	+ x_Q	+ 3.7
Q_{21}	- 3.1	—	- 3.1
F_{21}	+ 4.0	- x_Q	+ 7.3
G_{21}	+ 4.1	+ x_Q	+ 0.8
Q_{20}	+ 1.2	- $\frac{1}{2} x_P$	+ 6.0
G_{20}	+ 12.6	$\frac{1}{2}(x_P - x_Q)$	+ 9.4
P_{20}	+ 4.2	+ $\frac{1}{2} x_Q$	+ 2.6
P_{19}	+ 6.2	—	+ 6.2
G_{19}	+ 8.9	- x_P	+ 18.5
H_{19}	+ 8.9	+ x_P	- 0.7
P_{18}	- 8.1	—	- 8.1
H_{18}	- 5.5	- x_P	+ 4.1
J_{18}	- 5.4	+ x_P	- 15.0
P_{17}	- 0.7	- $\frac{1}{2} x_O$	- 8.5
J_{17}	- 2.7	$\frac{1}{2}(x_O - x_P)$	+ 9.9
O_{17}	+ 6.4	+ $\frac{1}{2} x_P$	+ 1.6
O_{16}	+ 4.0	—	+ 4.0
J_{16}	- 4.0	- x_O	- 19.6
K_{16}	- 4.0	+ x_O	+ 11.6
O_{15}	- 5.4	—	- 5.4
K_{15}	- 13.3	- x_O	- 28.9
L_{15}	- 13.3	+ x_O	+ 2.3

Since there are only three angles to which this shift can be applied, a shift of $\frac{-5.9}{3}$, or nearly two seconds, must be applied to each correction n_3, n_8, n_7 .

Giving their trial values, $n_3 = +4.0$; $n_8 = -2.0$; $n_7 = +7.4$. Put these trial corrections in Table 14.

Similarly for the station R, for a first trial put $r_3 = +6.0$; $r_4 = +6.7$, each being one third of the error in the corresponding triangle. From previous deliberation

$$r_9 = -9.3; r_{10} = -1.0; r_{11} = -0.7; r_{12} = -1.0$$

The sum of all the corrections is

$$-9.3 - 1.0 - 0.7 - 1.0 + 6.0 + 6.7 = +0.7$$

H—K

when the shift on r_3 and r_4 to satisfy the local equation at R is $\frac{-0.7}{2}$

or say -0.4 on r_3 and -0.3 on r_4 , giving the values shown in Table 14. The trial values of $n_3 = +4.0$ and $r_3 = +5.6$ have now been decided. The sum of these is $+9.6$, and the correction required for the whole of triangle III is $+18.0$. Hence the trial value of the remaining correction b_3 is $18.0 - 9.6$ or $+8.4$. The triangle VII has only one angle at a central station. Its trial correction has been settled at $n_7 = +7.4$. The total correction required $+28.0$. Hence the trial values for the remaining corrections l_7 and m_7 are $\frac{28.0 - 7.4}{2} = +10.3$ each.

Consider station O.

Trial values are $o_8 = 0.0$; $o_{15} = -10.7$; $o_{16} = -1.3$; $o_{17} = +1.0$. Hence, since o_9 and o_{10} have been fixed at -9.3 and -1.0 respectively, the sum of the corrections is now

$$-10.7 - 1.3 + 1.0 - 9.3 - 1.0 = -21.3$$

Hence the trial values must have a shift of $+\frac{21.3}{4}$ making

$$o_8 = +5.3; o_{15} = -5.4; o_{16} = +4.0 \text{ and } o_{17} = +6.4$$

In triangle VIII n_8 and o_8 have been fixed at -2.0 and $+5.3$ respectively. The total correction required is zero, so that

$$l_8 = -5.3 + 2.0 = -3.3$$

Triangles XV, XVI and VII have only one central station each. The central angle has been corrected in each case so that

$$k_{15} = l_{15} = \frac{-3.2 + 5.4}{2} = -1.3$$

and
$$j_{16} = k_{16} = \frac{-4.0 - 4.0}{2} = -4.0$$

and
$$l_7 = m_7 = \frac{+28 - 7.4}{2} = +10.3$$

Consider the central station at S. Take the following trial corrections at one third the triangle corrections :

$S_4 = +6.7$; $S_5 = -14.3$; $S_6 = +7.3$; $S_{14} = -8.3$; $S_{13} = +8.3$ and S_{12} is already fixed at -1.0

The summation of the corrections is

$$+6.7 - 14.3 + 7.3 - 8.3 + 8.3 - 1.0 = -1.3$$

So the shift to be applied to the five trial corrections is $\frac{+1.3}{5}$ making

$$S_4 = +7.0; S_5 = -14.0; S_6 = +7.6; S_{14} = -8.1; S_{13} = +8.5$$

Since the total correction for triangle IV is $+20.0$ and $r_4 = +6.4$ and $s_4 = +7.0$ then $b_4 = 20.0 - 6.4 - 7.0 = 6.6$.

Triangles V, VI and XIV have only one central angle at S so that

$$b_5 = c_5 = \frac{-43 + 14.0}{2} = -14.5$$

$$c_6 = d_6 = \frac{+22 - 7.6}{2} = +7.2$$

$$d_{14} = e_{14} = \frac{-25 + 8.1}{2} = -8.5 \text{ and } -8.4 \text{ respectively}$$

In this way all the trial corrections shown in Table 14 can be computed and filled in.

It is now necessary to balance the side equations for the whole net taken simultaneously. The principle is that all shifts for any polygon are to be equal as before; that, when any angle belongs to two polygons and requires a shift in each, the actual shift should be the mean of those belonging to the separate polygons, and that all shifts must be arranged so as not to alter the local equation of any point, or the triangle equations. It is convenient to use the following notation:

Let x_N be the shift required in a polygon whose centre is at N, x_R in that which has its centre at R, and so on, and mark them positive on left-hand angles and negative on right-hand angles throughout. Then, for example, in triangle No. IX, the angle N_9 is a left-hand angle for the polygon with centre at R, and hence it should have a shift of $+x_R$ in that polygon; but, as a right-hand angle of the polygon with O as centre, it would require a shift of $-x_O$. Hence the final shift on the angle N_9 would be $\frac{x_R - x_O}{2}$

Similarly the shift R_9 is $\frac{x_O - x_N}{2}$ and that on O_9 is $\frac{x_N - x_R}{2}$. The sum of these three shifts is zero, and hence does not affect the total on the triangle IX. The same principle applies to all triangles common to three polygons.

For triangle No. III, B_3 is a left-hand angle in the polygon with R as centre, and a right-hand angle in that with N as centre. The shift on B_3 is therefore $\frac{x_R - x_N}{2}$. The angle R_3 is a left-hand angle in the polygon with N as centre, and should therefore be given a shift x_N . But it is a centre angle in the polygon with R as centre, and hence should have no shift. Therefore, taking the mean, make the shift on R_3 equal to $+\frac{x_N}{2}$ only.

Similarly that on $N_3 = -\frac{x_R}{2}$.

The same principle applies to all triangles common to two polygons.

In triangle No. VII the shift on M_7 is $+x_N$ and that on L_7 is $-x_N$, while that on N_7 is zero. The same principle applies to all triangles belonging to only one polygon. All the shifts are filled in in the Table 14 working on the same lines as above, and once the idea is grasped the preparation of this table occupies about half an hour for the six polygons.

The calculations have now reached a stage where the final values of the corrections must be found, and they equal the trial corrections adjusted by the shifts. So in order to find the final corrections it is necessary to find the values of the six unknowns, x_N , x_R , x_S , x_Q , x_P and x_O , which balance the side equations for each polygon.

Take any polygon and multiply each correction, as expressed by the trial correction adjusted by the shift in the Table 14, by the log difference for 1" of the corresponding angle in the Table 13. Call all the products plus for the left-hand angles and minus for the right-hand angles. Then equate the algebraic sum of the products to $\Sigma R - \Sigma L$ (the difference must be taken in the order shown and proper attention paid to signs), and an equation will be arrived at which will ensure the satisfaction of the side equation of the polygon. Thus for the polygon with N as centre, the left-hand angles to be corrected are R_3 , O_9 , L_8 and M_7 and the log differences from Table 13 are 4.9; 12.0; 12.1 and 22.2, and the formulae for the corrections from Table 14 are

$$+5.6 + \frac{1}{2}x_N; -9.3 + \frac{1}{2}(x_N - x_R) \\ -3.3 + \frac{1}{2}(x_N - x_O) \text{ and } +10.3 + x_N \text{ respectively}$$

The right-hand angles to be corrected are B_3 , R_9 , O_8 and L_7 and the log differences from Table 13 are 16.1; 8.3; 26.0 and 13.7 and the formulae for the corrections from Table 14 are

$$+ 8.4 + \frac{1}{2}(x_R - x_N); - 9.3 + \frac{1}{2}(x_O - x_N) \\ + 5.3 - \frac{1}{2}x_N \text{ and } + 10.3 - x_N \text{ respectively}$$

$$\begin{aligned} \text{For this polygon} \quad \Sigma_R &= \bar{1}.4398472 \\ \Sigma_L &= \bar{1}.4399893 \\ \Sigma_R - \Sigma_L &= - \quad 1421 \end{aligned}$$

Hence the equation is

$$\begin{aligned} 4.9[5.6 + \frac{1}{2}x_N] + 12.0[- 9.3 + \frac{1}{2}(x_N - x_R)] \\ + 12.1[- 3.3 + \frac{1}{2}(x_N - x_O)] + 22.2[10.3 + x_N] \\ - 16.1[8.4 + \frac{1}{2}(x_R - x_N)] - 8.3[- 9.3 + \frac{1}{2}(x_O - x_N)] \\ - 26.0[5.3 - \frac{1}{2}x_N] - 13.7[10.3 - x_N] = - 1421 \end{aligned}$$

Form the equations for the other five polygons in the same way and simplify all to the six following simultaneous equations:

$$\begin{aligned} 75.6x_N - 14.0x_R - 10.2x_O &= - 1189 \\ - 14.0x_N + 75.6x_R - 11.7x_O - 15.9x_S - 14.9x_Q - 6.8x_P &= + 295 \\ - 15.9x_R + 120.2x_S - 3.5x_Q &= - 2336 \\ - 10.2x_N - 11.7x_R + 102.9x_O - 34.2x_P &= + 2102 \\ - 14.9x_R - 3.5x_S + 108.1x_Q - 28.3x_P &= + 15 \\ - 6.8x_R - 34.2x_O - 28.3x_Q + 107.1x_P &= - 1452 \end{aligned}$$

Solving these simultaneous equations

$$\begin{aligned} x_Q &= - 3.3; x_P = - 9.6; x_R = - 1.9; x_S = - 19.8; \\ x_O &= + 15.6; x_N = - 14.0 \end{aligned}$$

Substitute these values in the Table 14 to obtain the final values of the corrections as there tabulated. Finally insert these values in the correction Table 13 and check the balance of the local triangle and side equations. It may be necessary in this final stage to add or subtract 0.1 seconds here or there as required, but this will not appreciably affect the result. This final check is not shown but the student may work it for himself.

The whole correction of six such polygons means about one day's work for a tolerably capable computer. The maximum correction in this example is $- 34.3''$ and the sum of the squares of the corrections about 9000, which gives a favourable result compared with the previous method of correcting each polygon independently. To correct the same net by the method of least squares would involve the solution of 28 simultaneous equations and many times the amount of arithmetical work here required.

METHOD OF LEAST SQUARES. So far the most accurate method of adjustment has not been dealt with, that is, the method of least squares. To apply this method, in addition to the geometrical and side conditions the sums of the squares of the errors must be a minimum. The calculations involved, even in a simple network, are exceedingly laborious, and, as already mentioned, the triangulation is usually separated in smaller figures in order to reduce the size and number of the equations. The method has been used for many important triangulations, and is best explained with a numerical example. The example to be solved consists of a quadrilateral with diagonals, but the methods involved can be applied to more complicated networks. Refer to Fig. 44, each observed

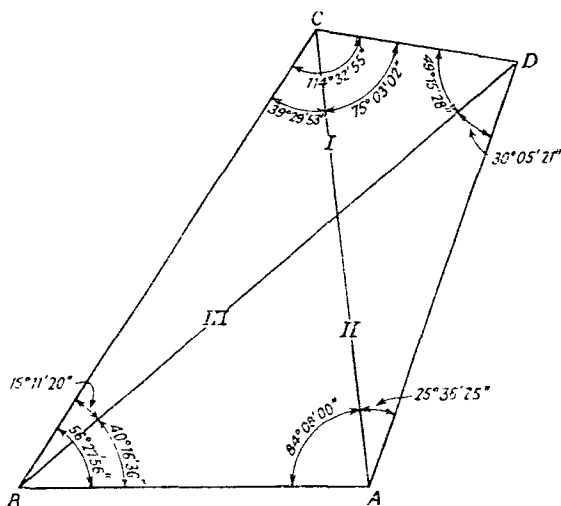


FIG. 44

angle being entered in its proper place on this diagram. The triangles are numbered as follows:

BCD—triangle No. I; BDA—triangle No. II

ABC—triangle No. III.

Use the same notation for numbering the angles and corrections to angles as previously. Then taking B as the centre angle draw up Table 15 in a similar manner to the table drawn up for the method of equal shifts. (It will be noticed that the log sine difference for 1" is negative when the angle is greater than 90°.)

Write down the equations of condition:

$$b_1 + b_2 - b_3 = 0 \text{ (local equation at B)}$$

$$b_1 + c_1 + d_1 = + 17 \text{ (to balance triangle I)}$$

$$b_2 + d_2 + a_2 = - 22 \text{ (to balance triangle II)}$$

$$b_3 + c_3 + a_3 = + 11 \text{ (to balance triangle III)}$$

To obtain the side equation take the difference of the long sines for the left- and right-hand angles.

$$\begin{array}{r} \bar{1}.6567148 \\ \bar{1}.6566606 \\ \hline \end{array}$$

542

Then multiply each log difference by its respective correction and equate the difference between the right- and left-hand products to the correction 542 needed.

$$18.1 d_1 - 7.6 a_2 + 25.6 c_3 + 9.6 c_1 - 36.4 d_2 - 2.1 a_3 = 542$$

There are thus five equations of condition, and there are nine variables, three for each triangle. Hence choose any four of these as independent variables and express all the rest in terms of them by means of the five equations of condition.

Taking a_2 , b_3 , c_1 , and d_2 as the four chosen variables, the nine observation equations become:

$$(1) a_2 = a_2$$

$$(2) b_3 = b_3$$

$$(3) c_1 = c_1$$

$$(4) d_2 = d_2$$

$$(5) b_2 = - 22 - a_2 - d_2$$

$$(6) b_1 = b_3 + 22 + a_2 + d_2 \text{ (since } b_1 = b_3 - b_2)$$

$$(7) d_1 = - 5 - b_3 - a_2 - d_2 - c_1 \text{ (since } d_1 = 17 - c_1 - b_1)$$

To obtain the other two equations substitute in terms of the independent variables in the side equation:

$$\begin{aligned} - 18.1 (5 + b_3 + a_2 + d_2 + c_1) - 7.6 a_2 + 25.6 (11 - b_3 - a_3) \\ + 9.6 c_1 - 36.4 d_2 - 2.1 a_3 = 542 \end{aligned}$$

that is

$$- 27.7 a_3 = 350.9 + 25.7 a_2 + 43.7 b_3 + 8.5 c_1 + 54.5 d_2$$

$$\text{or } (8) a_3 = - 12.7 - 0.93 a_2 - 1.58 b_3 - 0.31 c_1 - 1.97 d_2$$

$$\text{and } (9) c_3 = 23.7 + 0.93 a_2 + 0.58 b_3 + 0.31 c_1 + 1.97 d_2$$

$$\text{(since } c_3 = 11 - b_3 - a_3)$$

The normal equations can now be obtained by the methods used in Chapter 1, and, if any of the angles measured have any

T A B L E 15
EXAMPLE OF ANGULAR CORRECTIONS

Centre			Left				Right				Sum of observed angles ° ' "	Corr.	
No. of tri- angles	Observed angles ° ' "	Corr.	Observed angles ° ' "	Log sines	Log diff. for 1"	Corr.	Corr. diff.	Observed angles ° ' "	Log sines	Log diff. for 1"	Corr.	Corr. diff.	
I	16 11 20		114 32 55	9.9588548	-9.6			49 15 28	9.8794707	18.1			179 59 43 +17
II	40 10 36		30 05 21	9.7001385	36.4			109 44 25	9.9736973	-7.6			180 00 22 -22
III	56 21 56		84 08 00	9.9977215	2.1			39 29 53	9.8034926	25.6			179 59 49 +11
				1.6567148					1.6566606				

weight other than unity, then that weight can be applied to the observation equation concerned.

a_2	b_3	c_1	d_2	N
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
-1	0	0	-1	-22
+1	+1	0	+1	+22
-1	-1	-1	-1	-5
-0.93	-1.58	-0.31	-1.97	-12.7
+0.93	+0.58	+0.31	+1.97	+23.7

The normal equations which result are:

$$5.72 a_2 + 4.01 b_3 + 1.58 c_1 + 6.66 d_2 + 82.85 = 0$$

$$4.01 a_2 + 5.84 b_3 + 1.67 c_1 + 6.26 d_2 + 60.82 = 0$$

$$1.58 a_2 + 1.67 b_3 + 2.20 c_1 + 2.22 d_2 + 16.28 = 0$$

$$6.66 a_2 + 6.26 b_3 + 2.22 c_1 + 11.76 d_2 + 120.71 = 0$$

And from these simultaneous equations

$$a_2 = -7.9; b_3 = +1.9; c_1 = +4.5; d_2 = -7.7$$

The remaining corrections can be found from the nine observation equations, whence

$$b_2 = -6.4; b_1 = +8.3; d_1 = +4.2; a_3 = +5.2; c_3 = +3.9$$

The student should solve this last example by the method of equal shifts and compare the results obtained. One of the polygons in Fig. 43 can also be solved by the least square method as an exercise and the results compared with the equal shift results.

COMPUTATION AND ADJUSTMENT OF PRECISE TRAVERSES

A traverse may be computed either with rectangular co-ordinates or else directly in terms of geographical co-ordinates. In the latter case they can then be converted at will to the co-ordinates of the particular projection in local use, but with minor precise traverses it is more convenient to use plane rectangular co-ordinates and then convert them to the local system by approximate formulae. The approximate formulae for Transverse Mercator co-ordinates only will be described later.

RECTANGULAR CO-ORDINATES. The conditions to be fulfilled are a closure in latitude and departure and an agreement in azimuth observations. The main difficulty in arriving at an adjustment of the traverse is in the assessment of the relative weights to be given to angular and linear measurements. This difficulty is further complicated when networks of traverses are involved, also when the traverse closes to points fixed by triangulation.

The only adjustment possible to an open traverse is to the angles by finding the azimuths of lines in the traverse with astronomical observations, correcting for the convergence of the meridians and comparing with the angles of the traverse.

To overcome this difficulty, Bowditch assumed that the errors in linear measurements are proportional to \sqrt{S} where S is the length of the line and that errors in angular bearings are proportional to $1/\sqrt{S}$. The latter assumption in particular is to be suspected, however, assuming it to be correct. If AB is any side of a traverse such that $AB = S$, then according to Bowditch's assumption, the corrected line AB would be some such line as AC , and if l is the error in latitude (or the latitude of BC) and d the error in departure (or the departure of BC) then $BC = \sqrt{l^2 + d^2}$ and is proportional to the \sqrt{S} .

If L is the total error in latitude and D is the total error in departure then by the method of least squares the sum of the weighted squares of the errors must be a minimum, or

$$\Sigma \left(\frac{l^2 + d^2}{S} \right) = \text{a minimum and } \Sigma(l) = L \text{ and } \Sigma(d) = D$$

Differentiating

$$\Sigma \left(\frac{l \delta l}{S} + \frac{d \delta d}{S} \right) = 0; \Sigma(\delta l) = 0; \Sigma(\delta d) = 0$$

Multiply the last two equations by $-\lambda_1$ and $-\lambda_2$ respectively, add all three equations together and equate the coefficients of each δl and δd to zero to give

$$\frac{l_1}{S_1} = \lambda_1; \frac{l_2}{S_2} = \lambda_1 \text{ etc. and } \frac{d_1}{S_1} = \lambda_2 = \frac{d_2}{S_2} = \lambda_2 \text{ etc.}$$

Substitute these values in the original equations

$$\lambda_1 \Sigma(S) = L \text{ and } \lambda_2 \Sigma(S) = D$$

or

$$\lambda_1 = \frac{L}{\Sigma(S)}; \lambda_2 = \frac{D}{\Sigma(S)}$$

so the corrections of latitude are

$$l_1 = \frac{S_1 L}{\Sigma(S)}; l_2 = \frac{S_2 L}{\Sigma(S)} \text{ etc.}$$

and the corrections of departure are

$$d_1 = \frac{S_1 D}{\Sigma(S)}; d_2 = \frac{S_2 D}{\Sigma(S)} \text{ etc.}$$

If the lengths of the sides have weights w_1, w_2, w_3 , etc., then taking

$\Sigma \left[\frac{w}{S} (l^2 + d^2) \right]$ = a minimum it can be shown in a similar manner

that
$$l_1 = \frac{S_1}{w_1} \frac{L}{\Sigma \left(\frac{S}{w} \right)} \text{ and } d_1 = \frac{S_1}{w_1} \frac{D}{\Sigma \left(\frac{S}{w} \right)}$$

Thus each co-ordinate can readily be adjusted.

This method was originally associated with the magnetic compass, and many surveyors prefer to use the *transit rule*, claiming that the angles are less affected by this method of correction. The method is purely empirical and takes the latitude (or departure) correction as proportional to the latitude (or departure) of the line concerned instead of the length of the line.

Then if A_1, A_2 , etc., and Δ_1, Δ_2 , etc., are the latitudes and departures of the sides

$$l_1 = \frac{A_1 L}{\Sigma(A)} \text{ etc. and } d_1 = \frac{\Delta_1 D}{\Sigma(\Delta)}$$

CRANDALL'S METHOD. Since in general angular measurement is more precise than linear measurement, many surveyors prefer to adjust any small angular errors as a preliminary operation and then use an orthomorphic method such as Crandall's Method, where the bearings remain unaltered.

Let the correction of a side of length S be equal to xS . Then if A and Δ are respectively the latitude and departure of the side, the correction of latitude is xA and of departure is $x\Delta$.

Then if the linear measurement follows the square root law, the weight will be proportional to $1/S$ and therefore by the method of least squares

$$\Sigma \left(\frac{x^2 S^2}{S} \right) = \Sigma (x^2 S) = \text{a minimum}; \Sigma xA = L; \Sigma x\Delta = D$$

therefore $\Sigma(xS\delta x) = 0$; $\Sigma(A\delta x) = 0$; $\Sigma(\Delta\delta x) = 0$

Multiply the last two by $-\lambda_1$ and $-\lambda_2$ respectively, add the three equations together and equate the coefficients to zero as before to give

$$x_1 = \frac{\lambda_1 A_1 + \lambda_2 \Delta_1}{S_1}; x_2 = \frac{\lambda_1 \Delta_2 + \lambda_2 \Delta_2}{S_2} \text{ etc.}$$

Substitute in the former equations to give

$$\lambda_1 \Sigma\left(\frac{A^2}{S}\right) + \lambda_2 \Sigma\left(\frac{A\Delta}{S}\right) = L$$

and

$$\lambda_1 \Sigma\left(\frac{A\Delta}{S}\right) + \lambda_2 \Sigma\left(\frac{\Delta^2}{S}\right) = D$$

from which λ_1 and λ_2 can be calculated.

The corrections of latitude are then equal to

$$l_1 = x_1 A_1 = \frac{\lambda_1 A_1^2 + \lambda_2 A_1 \Delta_1}{S_1} \text{ etc.}$$

and of departure $d_1 = x_1 \Delta_1 = \frac{\lambda_1 A_1 \Delta_1 + \lambda_2 \Delta_1^2}{S_1} \text{ etc.}$

ORMSBY'S METHOD. Ormsby's simple conventional method gives results very similar to Crandall's method but is much easier to apply. In this method two correction factors x and y are used. All sides whose direction lies in the N.E. and S.W. quadrants have a correction xS , and all sides whose direction lies in the N.W. and S.E. quadrants have a correction yS . If L , the total correction in latitude, is greater than D , the total correction in departure, make the sign of all the latitude corrections the same as the sign of L . Then make the signs of all the departure corrections consistent with the latitude correction already chosen for the particular side. For example, if a positive correction is applied to a positive latitude, then the length of the side is increased (as if a negative correction is applied to a negative latitude), and so the sign of the correction applied to the departure must have the same sign as the departure.

If, however, L is less than D , all the departure corrections are given the same sign as D , and the signs of the latitude corrections made consistent with this choice as before. Then sum the latitude corrections and the departure corrections so obtained, and equate

to L and D respectively. This will give two simultaneous equations in x and y , which in turn will give the corrections to be applied.

Ormsby's method is, as stated, similar to Crandall's method in that the bearings remain unaltered, all the corrections being made in the lengths. It does not however satisfy the least squares condition, but it will be found that in many cases the sum of the squares of the errors by Ormsby's method is not much greater than the minimum sum obtained by the more exact method.

However, even if the angular measurements satisfy their geometrical conditions, it is no indication that the bearings should remain unaltered for a rational adjustment of the traverse. A logical method of adjustment would allow for the relative weights of the linear and angular measurements not in an arbitrary manner as in Bowditch's method but according to a theoretically justified method. Such methods have been suggested by Higgins, Rappleye and others, but the calculations involved are extensive and laborious and would only be used where the utmost accuracy of adjustment was required.

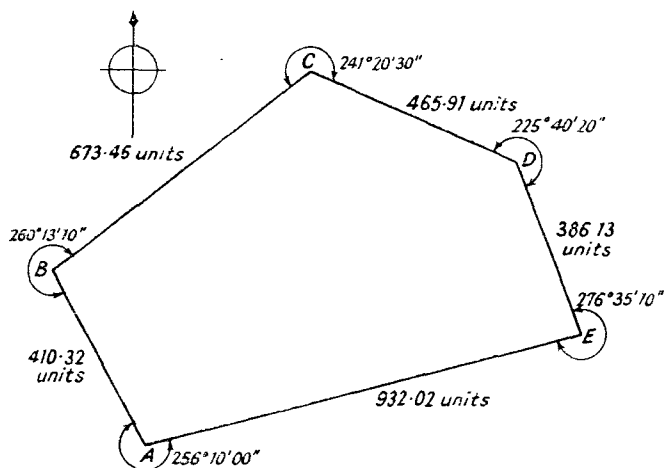


FIG. 45

Example

Adjust the traverse shown in Fig. 45 by the methods of Crandall and Ormsby, taking the bearing of AB as north $28^{\circ}20'00''$ west. The exterior angles are measured in each case and sum to $1259^{\circ}59'10''$ which amounts to $50''$ short of the correct total. To adjust the angles add $10''$ to each of the five exterior angles. The computation by Crandall's method is shown in Table 16. First calculate the reduced bearings, and then the latitude and departure for each line. The next

T A B L E 16
ADJUSTMENT OF PRECISE TRAVERSE

	Reduced bearing	Length	Latitude	Departure	$(\text{lat.})^2$ side	$\frac{(\text{lat.} \times \text{dep.})}{\text{side}}$	$\frac{(\text{dep.})^2}{\text{side}}$	Correction to lat.	Correction to dep.
A	N 28° 20' 00" W	410.32	+ 361.16	— 194.74	318.0	— 171.4	92.4	+ 0.19	— 0.10
B	N 51° 53' 20" E	673.46	+ 415.65	+ 529.89	256.6	+ 327.1	426.9	+ 0.17	+ 0.21
C	S 66° 46' 00" E	465.91	— 183.79	+ 428.13	72.5	— 168.9	393.4	+ 0.04	— 0.10
D	S 21° 5' 30" E	386.13	— 360.26	+ 138.95	336.2	— 129.7	50.0	+ 0.21	— 0.08
E	S 75° 29' 50" W	932.02	— 233.41	— 902.32	58.5	+ 226.0	873.5	+ 0.04	+ 0.16
A			— 0.65	— 0.09	1041.8	+ 83.1	1826.2	+ 0.65	+ 0.09

three columns to calculate are respectively $\frac{(\text{latitude})^2}{\text{side}}$; $\frac{(\text{latitude} \times \text{departure})}{\text{side}}$; and $\frac{(\text{departure})^2}{\text{side}}$ for each individual side of the traverse.

Sum these three columns to give respectively 1041·8; + 83·1; 1826·2. The errors in latitude and departure respectively are - 0·65 and - 0·09.

$$\begin{aligned} \text{Hence} \quad & 1041\cdot8 \lambda_1 + 83\cdot1 \lambda_2 = 0\cdot65 \\ & 83\cdot1 \lambda_1 + 1826\cdot2 \lambda_2 = 0\cdot09 \\ \text{or} \quad & \lambda_1 = 0\cdot000621 \\ & \lambda_2 = 0\cdot000021 \end{aligned}$$

The corrections to latitude can now be calculated and equal

$$\lambda_1 \frac{(\text{latitude})^2}{\text{side}} + \lambda_2 \frac{(\text{latitude} \times \text{departure})}{\text{side}}$$

and corrections to departure equal

$$\lambda_1 \frac{(\text{latitude} \times \text{departure})}{\text{side}} + \lambda_2 \frac{(\text{departure})^2}{\text{side}}$$

Enter these results in the table and sum the corrections to check with the total errors.

To carry out the correction by Ormsby's method, the information is tabulated in Table 17. The factors x and y are allotted to the lines according to the convention already given. The last two columns can only be calculated after x and y have been found. In

TABLE 17
CORRECTION BY ORMSBY'S METHOD

Station	Reduced bearing	Length	Latitude	Departure	Factor	Correction of lat. (Factor \times lat.)	dep. (Factor \times dep.)
A							
B	N 28° 20' 00" W	410·32	+ 361·16	- 194·74	y	+ 0·17	- 0·09
C	N 51° 53' 20" E	673·46	+ 415·65	+ 529·89	x	+ 0·13	+ 0·17
D	S 66° 46' 00" E	465·91	- 183·79	+ 428·13	y	+ 0·09	- 0·21
E	S 21° 05' 30" E	386·13	- 360·26	+ 138·95	y	+ 0·18	- 0·07
A	S 75° 29' 50" W	932·02	- 233·41	- 902·32	x	+ 0·08	+ 0·29
			- 0·65	- 0·09		+ 0·65	+ 0·09

this case, since L (0.65) is greater than D (0.09), make the signs of all the latitude corrections positive and the signs of the departure corrections consistent with this.

Equate the corrections to their respective totals

$$\begin{aligned} 361y + 416x + 184y + 360y + 233x &= + 0.65 \\ - 195y + 530x - 428y - 139y + 902x &= + 0.09 \\ \text{giving } x &= 0.000322 \\ y &= 0.000488 \end{aligned}$$

Hence the corrections in the last two columns.

The sums of the squares of the errors by the two methods are respectively 0.21 and 0.26.

TRANSVERSE MERCATOR CO-ORDINATES. To enable approximations to be made with safety the traverse should be divided into fairly short sections of 10–20 miles, the sections between the azimuth stations being suitable. Corrected co-ordinates are then computed for the terminal stations of each section. Convergency at the starting point will be known, or can be computed from the tables prepared for the particular projection of the local survey, or from the approximate formula:

$$\text{Convergency} = (\text{long. place} - \text{long. origin}) \times \text{sine lat. of place}$$

The convergency at the terminal point of each section must be computed in a similar way. (The same formula must be used throughout, always using local tables if such are available.) To obtain the data for computing these convergencies it may be necessary to compute preliminary co-ordinates of the terminal stations, but usually the data can be taken with sufficient accuracy from a large-scale plot or map. Call these convergencies C_0, C_1, C_2, C_3 , etc. The true bearing of the first leg will be known either by applying the convergency to the known grid bearing or by the initial azimuth observations, and this true bearing will be carried through by the normal traverse method of addition of successive observed angles, but at each terminal station it will be corrected by the difference between the convergency at that station and at the preceding one.

At the end of the traverse the true bearing of the last leg may be compared with the Transverse Mercator bearing + convergency, if the traverse closes on a triangulation station, otherwise with its

observed azimuth. The closing error in azimuth, e , may then be distributed through the terminal stations according to the total number of angle stations in the traverse.

The Transverse Mercator co-ordinates of the section terminal stations may now be calculated successively from the initial station as follows:

Let x_A and y_A be the eastings and northings co-ordinates of the initial station

and let Δ and Λ be the sum of the departures and latitudes of the section A to B

Then the approximate co-ordinates of B are $x_A + \Delta$ and $y_A + \Lambda$ and the approximate mean easting of the section is $x_A + \Delta/2 = x_M$. The corrected co-ordinates of B will be:

$$\text{eastings } x_B = x_A + \Delta(1 + x_M^2/2r^2)$$

$$\text{northings } y_B = y_A + \Lambda(1 + x_M^2/2r^2)$$

Co-ordinates of C will be found from the corrected co-ordinates of B, *et seq.* (r = mean radius of earth, i.e. $r^2 = \epsilon v$).

If the traverse has closed on a triangulation station and the misclosure is within the 1:30,000 tolerance (see p. 106), it is usually sufficient to allot the misclosure amongst the terminal stations by the conventional Bowditch rule.

The linear misclosure due to angular errors of a straight traverse between fixed points is probably double that of a traverse closing on its starting point, an important fact to be remembered when assessing the accuracy of the work.

ADJUSTMENT OF A NETWORK OF TRAVERSES. It may happen that the traverse being adjusted instead of being run from fixed point to fixed point forms part of a net and runs from a fixed point to a junction with other traverses. Primary traverses run as a substitute for triangulation are of this nature, and their adjustment is a most laborious process. For the type of work being considered it is usually sufficient to tabulate the co-ordinates of the junction points as calculated from each traverse and then to calculate a weighted mean, the weights allotted being inversely proportional to the total length of the traverse, as shown in the example below.

Strictly, the weights should be inversely proportional to the squares of the lengths, i.e. $W = 1/s^2$, but the mean thus obtained will seldom differ significantly from that obtained above, where $W = 1/s$.

TABLE 18
CO-ORDINATES OF TRAVERSE JUNCTION POINT

From station	Traverse length/10,000 S	$1/s$	Co-ordinates and corrections			
			Eastings		Northings	
				$\times 1/s$		$\times 1/s$
A	3.60	0.278	115 025.8	1.61	87 579.4	2.61
B	8.05	0.124	24.0	0.50	76.0	0.74
C	7.54	0.133	27.2	0.96	75.4	0.72
D	7.0	0.143	27.4	1.06	80.8	1.54
		0.678	\div 678	4.13	\div 678	5.61
				6.1		8.3

Accept co-ordinates of point 115 026.1 E, 87 578.3 N

CHAPTER 7

FIELD ASTRONOMY

IN SURVEYING operations it may become necessary to determine the latitude and longitude of a survey station by observations on the sun or stars, as also the direction from it of a true north and south line, called the true *meridian* of the station. That branch of surveying which treats of these matters is called field astronomy. It comprises both the observational work and the computations, based on the observations, by which the desired results are obtained. The same underlying principles are used in navigation, but the methods may differ to some extent.

The field astronomer is concerned with the angular directions of sun or stars relative to his horizon, viz. their angular elevation (called *altitude*) above it, and their horizontal direction compared with that of a fixed terrestrial mark or line. As these quantities are in general changing with time, on account of the earth's rotation, the time of the observation is also usually required. Further, the determination of local time may be partly or wholly the object of the observation.

The directions referred to can both be measured with a theodolite, which is accordingly the instrument in most general use in field astronomy. Other instruments are in use but have a more limited application.

Before one can understand and apply with confidence the methods of field astronomy, it is necessary to have some knowledge of the apparent motions of the celestial bodies. Without this knowledge, not difficult to acquire, reliable or intelligent work in field astronomy cannot be done: the essentials will be explained in the succeeding paragraphs.

TERRESTRIAL AND CELESTIAL SPHERES. The earth is a nearly spherical body which rotates from west to east on its polar axis in a day. The *terrestrial poles*, P and P', north and south respectively, are the points where the axis meets the surface, Fig. 46. As a first approximation, it is usual and convenient to regard the earth

as spherical. On this assumption, any plane containing the axis PP' cuts the surface in a great circle. Half of such a great circle, e.g. $PAMP'$ or PBP' , is called a *terrestrial meridian*, and its direction

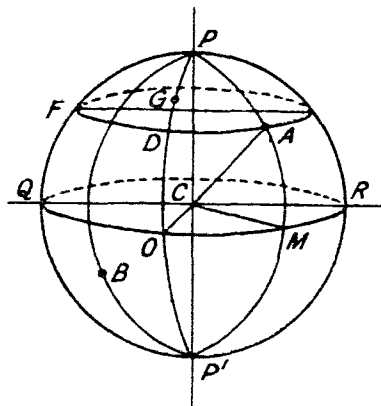


FIG. 46

is at all points true north and south. A plane through C the centre of the sphere and perpendicular to PP' cuts the surface in a great circle, the *equator* QMR . The actual figure of the earth is very nearly an oblate spheroid, or ellipsoid of revolution generated by the rotation of an ellipse round its minor axis, viz. the polar axis. On this figure the terrestrial meridians are ellipses.

Treating the earth as a sphere, the meridian of a point A on its

surface is $PAMP'$ cutting the equator at M . The *latitude* of A is the angle ACM , i.e. the angular distance of A from the equator; it is measured in degrees, minutes and seconds from 0° at the equator to $+90^\circ$ or 90° N at the north pole and to -90° or 90° S at the south pole.

CA and CM being radii of the sphere are also normals to its surface. The latitude of A may, in fact, be defined as the angle between the normal at A and the normal at M , or as the angle between the normal at A and the plane of the equator. The definition of latitude in this way has the advantage that it is applicable to the spheroid, the accepted figure of the earth, as well as to the sphere.

A small circle through A parallel to the equator is the locus of all points which have the same latitude as A . Such a small circle is a *parallel of latitude* ADF . To define the position of A on the earth, it is further necessary to specify the position of the meridian $PAMP'$.

To do this, a *prime meridian* is agreed on. The meridian through a particular instrument in Greenwich is by general consent adopted, the meridian of Greenwich G ; and the angle OCM between the meridian plane PGP' of Greenwich and the meridian plane of A is the *longitude* of A , and of course, of all points on the meridian of A . The latitude of A being also given, its position is perfectly

definite. Longitude is thus the angle between two planes intersecting along the earth's axis. Longitudes are reckoned east or west of Greenwich, from 0° at Greenwich to 180° E or W. As OCM is equal to the spherical angle GPA and to the equatorial arc OM, these also represent the longitude. The latitude and longitude give the *geographical position* of a place.

It is convenient to imagine the stars, which are at vast distances, attached to the surface of a sphere of large or infinite radius, the *celestial sphere*. As the earth rotates and carries the observer and his horizon round with it, different regions of the celestial sphere are progressively presented to view above his horizon, as if the celestial sphere itself were rotating. This apparent rotation is necessarily about an axis parallel to the earth's axis, and is in the opposite direction to the earth's real rotation, and at the same rate; that is, the celestial sphere appears to rotate from east to west in the period of a day.

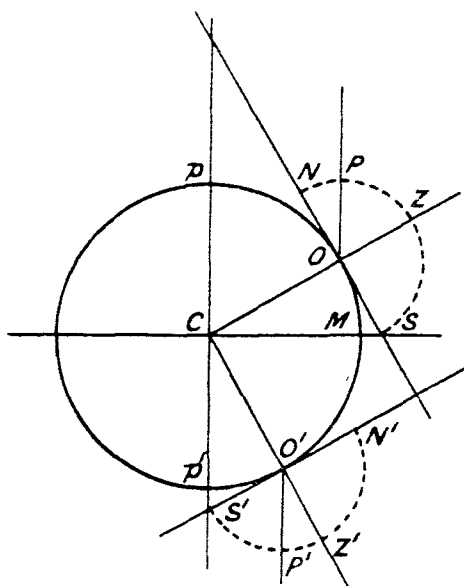


FIG. 47

So far as the stars, properly so called, are concerned, their distances are so vast that the linear displacement of A along the circumference of the small circle which it describes about PP' makes no difference whatever to the apparent relative positions of the

stars. To the observer at A their apparent motion across the sky is exactly as if the celestial sphere were rotating about an axis through A parallel to the earth's axis. The sun, moon and planets, being at distances commensurate with the earth's radius, do suffer a slight displacement when viewed from different points of the earth's surface. This is duly allowed for in observations made on them.

A surveyor on arrival at a station O, Fig. 47, may know neither its geographical position nor the direction of its meridian.

A plumb line will give him the direction of the vertical line OZ, as will also the vertical axis of a well set-up and levelled theodolite, and with this instrument he can sweep out a horizontal circle, i.e. a plane NOS perpendicular to the plumb line. This plane NOS is his *horizon* and the point Z vertically overhead is his *zenith*.

The horizon is a tangent plane to the earth's surface at O; the dotted semicircle indicates the portion of the celestial sphere visible there and OP parallel to pp' is the axis about which it appears to rotate. P is the *north celestial pole* and P', below the horizon of O, is the *south celestial pole*, which is above the horizon of a station such as O' in the southern hemisphere.

As the apparent rotation of the celestial sphere is about the axis POP', the points P and P' keep their places in the heavens, unaffected by the daily rotation. At O, the north celestial pole P is elevated at an angle PON above the horizon, and its horizontal direction is ON, which is in the plane of the meridian of O, i.e. it is due north.

As $\angle PON = \angle OCM$, the latitude of O, the latitude of a station is equal to the altitude of the celestial pole above the horizon of that station, *altitude* being angular elevation above the horizon. This conception of latitude is completely in agreement with the definition already given and, like it, is applicable to the spheroid as well as to the sphere.

More specifically, latitude, as thus defined, is called *astronomical latitude*, to distinguish it from several other kinds of latitude which will be referred to later. Similarly, at O' the altitude of P' above the south point S' of the horizon of O' is the south latitude.

Fig. 48 represents the half of the celestial sphere visible to the observer at O. Any plane, such as OZM, containing the vertical line OZ, is a *vertical plane* and is perpendicular to the horizon NESW. The vertical plane containing OZ and OP cuts the horizon in the north and south points N and S, and coincides with the

terrestrial meridian of O. It is the *celestial meridian* of O, i.e. the celestial meridian is the plane containing the observer, his zenith and the celestial pole. The line NOS is the *meridian line*.

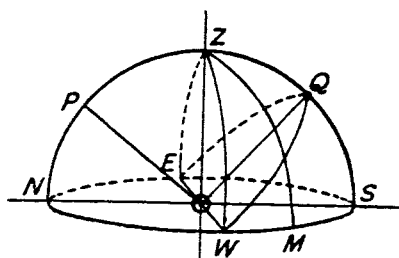


FIG. 48

The plane through O and perpendicular to OP cuts the celestial sphere in the great circle EQW, called the *celestial equator*. The celestial equator meets the horizon plane in the straight line EOW, which is seen from elementary considerations to be at right angles to NOS; E and W are respectively the east and west points of the horizon. The

celestial equator is clearly the 'trace' of the earth's equatorial plane on the celestial sphere. The vertical plane EZW is perpendicular to the meridian. It is called the *prime vertical*.

The position of a star with respect to the observer's horizon is specified by its altitude and its *azimuth*. *Azimuth* is defined as the angle between the celestial meridian plane and the vertical circle through the star.

In Fig. 49 the position of the star X is given by:

- its altitude, i.e. the angle XOM, or the arc XM
- and its azimuth, i.e. the angle NOM, the arc NM,
- or the angle PZX of the spherical triangle PZX.

Altitude and azimuth are thus co-ordinates referred to the horizon; the theodolite is adapted for working with these co-ordinates; such instruments are called *altazimuth instruments*.

Azimuths are by convention usually reckoned clockwise from N, nowadays from 0° to 360° ; the azimuth of the east point of the horizon is 90° and so on. There is, however, no absolute

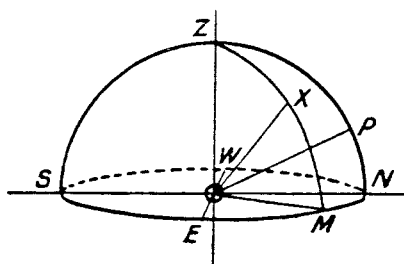


FIG. 49

uniformity; azimuths are still reckoned clockwise from south, occasionally. The angle ZOX, the complement of the altitude h , is the star's *zenith distance*, z or $z.d.$, so that $z = 90^\circ - h$.

The direction of a survey line is given by its azimuth, often called its bearing. Reckoned clockwise from true north, it is called *true bearing* by the Admiralty, to distinguish it from *magnetic bearing*, which is reckoned clockwise from magnetic north.

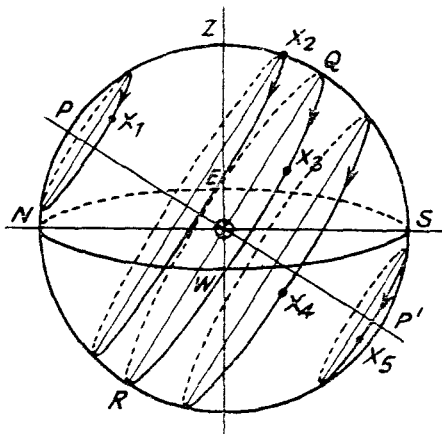


FIG. 50

Fig. 50 represents the celestial sphere for an observer in latitude $\angle PON$. As the celestial sphere rotates with uniform angular velocity about the axis OP parallel to the earth's axis, each star describes a small circle, called a *diurnal circle*, parallel to the equator $EQWR$. The star X_1 describes a small circle lying entirely above his horizon, and the star is always visible unless obscured by cloud or daylight. The star X_2 has the greater part of its path above the horizon, and is therefore above the horizon for more than half of the 24 hours. The star X_3 on the equator (as the sun is at the vernal and autumnal equinoxes) rises in the east point, is above the horizon for 12 hours and sets due west, whatever be the latitude. The star X_4 is only above the horizon for a brief time and X_5 never at all. The effect of a change of latitude can be seen by tilting the horizon NOS in the appropriate direction, keeping OP fixed.

In one complete rotation of the celestial sphere each star must cross the meridian twice, once from east to west and once from west to east. Crossing the meridian is called *transit* or *meridian passage*. The transit of a star across the half-meridian PZP' , which contains the zenith Z , is its *upper transit*; it is the *E to W transit*.

The other transit is its *lower transit*, W to E. It is clear that any star attains its maximum altitude at its upper transit, and its minimum altitude at its lower transit.

Culmination is the attainment of maximum altitude; it is in general identical with transit or meridian passage, except that a celestial body, the angular distance of which from the celestial pole is variable, e.g. the sun, may reach its maximum altitude a little before or after its meridian passage. Such a star as X_1 transits twice daily above the horizon; for X_2 , X_3 and X_4 only the upper transits are above the horizon of the place to which Fig. 50 applies.

The azimuth of a star is in general also continually changing while it describes its diurnal circle, being either 0° or 180° at the star's transit. A star such as X_1 is called a *circumpolar star*; the name is, however, usually restricted to stars within a few degrees of the pole, and such that both upper and lower transits are on the same side of the zenith.

RIGHT ASCENSION AND DECLINATION. In order to record or catalogue the positions of stars or other objects on the celestial sphere, coordinates must be chosen which are independent of the situation of the observer's horizon relative to the celestial sphere. The coordinates adopted are analogous to latitude and longitude on the earth. They are:

Declination. This is the star's angular distance from the celestial equator; it is analogous to latitude on the earth, and is reckoned from the celestial equator along the great circle passing through the celestial poles and the star, from 0° at the equator to $+90^\circ$ or 90° N at the north celestial pole and to -90° or 90° S at the south pole. The great circle is called the star's *declination circle* or *hour circle*. The symbol in general use for declination is δ . The angular distances of a celestial object from the north and south celestial poles are its north-polar distance and its south-polar distance respectively. As declinations south of the equator are reckoned negative, the north-polar distance $+$ the declination is always 90° . The symbol p is commonly used for polar distance.

Right ascension. The *right ascension* (R.A.) of a star is the angle between its declination circle and the declination circle of a certain point on the equator, called the *First Point of Aries*, or simply *Aries*. This point, for which the symbol γ is used, is where the sun crosses the equator from south to north on or about 21st March, when day

and night are of equal length; φ is therefore also called the *vernal equinox*.

In Fig. 51 representing the celestial sphere, P and P' are the N and S celestial poles, the First Point of Aries is φ and the great circle P φ P' is taken as the zero from which right ascensions are reckoned. PXP' is the declination circle of the star X, meeting the equator in M.

The declination of X is the angle XOM and its R.A. is the angle φ OM, or the arc φ M, or the spherical angle φ PX. Right ascensions are reckoned eastwards from φ , and they are usually expressed in hours, minutes and seconds of time at the rate of 360° to 24 hours.

In the Nautical Almanac (N.A.) and in the Star Almanac, the places of stars are tabulated in right ascension and declination for observational work. It is often convenient, for the *reduction*, i.e. computation of observations, to work with the polar distance, $90^\circ - \delta$, instead of the declination δ .

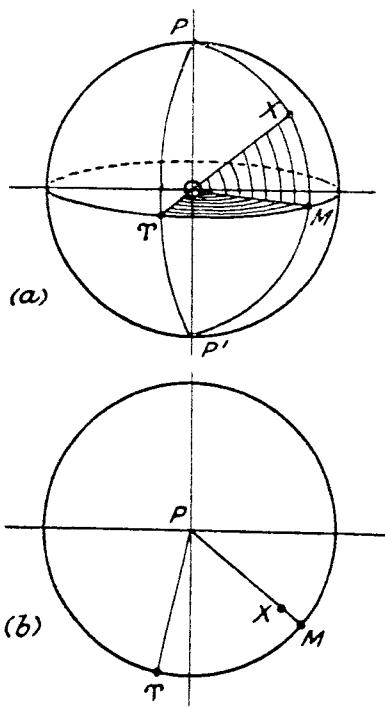


FIG. 51

MEASUREMENT OF TIME. The period of the apparent rotation of the celestial sphere as a whole about its axis is called a *sidereal day*. It is not quite the same as the calendar or civil day, being very nearly four minutes shorter. The difference arises from the fact that the sun is not stationary among the stars, but has a motion among them in a direction contrary to the apparent diurnal rotation; this causes the interval between successive transits of the sun to be longer than the corresponding interval for the stars. The *sidereal day* of astronomers is the interval between successive transits of the First Point of Aries; it is divided into 24 hours as is the civil or mean time day, and it begins for any place when φ transits over the meridian of

that place, irrespective of the position of the sun. Consequently its relation to mean time, which is derived from the sun, is not at first sight obvious.

Fig. 52 represents the celestial sphere and shows the equator EQW, the First Point of Aries γ and a star X, shortly after its

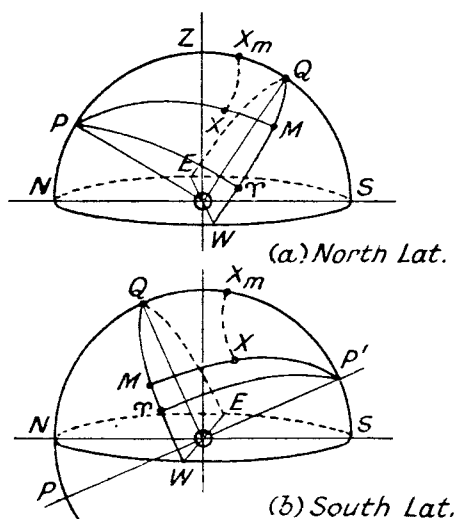


FIG. 52

transit at X_m . The declination circle PXM of the star rotates with uniform angular velocity about P as centre; the angle X_mPX or ZPX which it has turned through since transit at X_m is called the star's *hour angle*, H.A. It is 0° at the star's transit and increases uniformly to 360° at its next transit. Similarly the H.A. of γ increases uniformly from 0° to 360° . *Sidereal time* is measured by the hour angle of Aries, viz. $ZP\gamma$, and it may be expressed in hours, minutes, and seconds of time or in degrees, minutes, and seconds of angle, at the rate of 15° per hour, as may the hour angle of any celestial object.

The hour angle of a celestial object at any place at an instant is then the angle between the celestial meridian of the place and the declination circle of the celestial object, measured westwards from the meridian. The hour angle of a star, being the angle from the celestial meridian reckoned westwards to the star's declination

circle, is necessarily the same at any instant for all places having the same celestial meridian, i.e. for all places on the same terrestrial meridian, i.e. of the same longitude. But a change of longitude, by changing the celestial meridian, alters the hour angle of a star by the amount of the change of longitude.

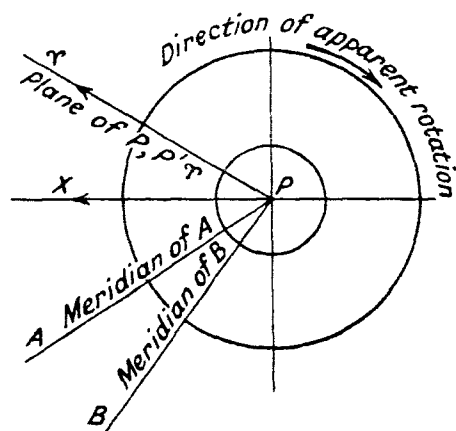


FIG. 53

In Fig. 53 the two concentric circles represent the earth and the celestial sphere, with P the north pole of each, and PX the declination circle of a star X. AP is the meridian plane of a place A, and BP that of a place B, east of A. APB is the difference of longitude. XPA is the star's H.A. at A, and XPB its H.A. at B.

As $XPB = XPA + APB$,

the star's H.A. at B = star's H.A. at A + difference of longitude.

Similarly

H.A. of γ at B = H.A. of γ at A + difference of longitude,

i.e.

the sidereal time at B = sidereal time at A + difference of longitude.

The time at any particular place is therefore referred to as the *local time*, and if necessary the longitude must be specified; the time at any other place of known longitude can then be derived at once as above. This applies not only to sidereal time but also to mean time and to apparent time, which will both be explained later.

If, for example, B is 30° E of A, the local sidereal time (L.S.T.) of B is 2 hrs. ahead of the L.S.T. of A; i.e. if the L.S.T. at A is 13^h the L.S.T. at B is 15^h . This might almost be called self-evident, as 2 sidereal hours are required for the earth to rotate 30° relative to γ . The difference of longitude if expressed in angle is to be converted into time at the rate of 15° per hour.

In Fig. 52 the R.A. of X is $\angle XP\gamma$
 and the H.A. of X is $\angle XPZ$
 adding these gives R.A. of X + H.A. of X = $\angle ZP\gamma$ = L.S.T.
 Therefore the L.S.T. = R.A. + H.A.

This applies to any celestial object. When the object is on the meridian, its H.A. = 0° and the L.S.T. = R.A. of object; hence celestial bodies cross the meridian in the order of their right ascensions, and the L.S.T., when a celestial object transits, is equal to the R.A. of the object. In the observatory, the accurate determination of local time is made by star transits across the meridian. Sidereal clocks are used in observatories and often in field work also. When neither fast nor slow on L.S.T., they indicate at any instant the sidereal time elapsed since the last transit of γ ; in other words they show the R.A. of the meridian at the instant.

Any system of reckoning time for general use must obviously refer directly to the sun. From the sun's hour angle is derived what is called *local apparent time*, L.A.T., or local apparent solar time, which is reckoned as 12^h + the sun's hour angle expressed in time. At *local apparent noon* the sun is on the local meridian, and the L.A.T. is 12^h . When his hour angle is -15° , i.e. 15° east, the L.A.T. is 11^h , and when his hour angle is 30° the L.A.T. is 2^h p.m., or 14^h , and so on. A simple sundial indicates apparent time. But apparent time has the drawback that the lengths of the days measured, say, from one apparent noon to the next are not all equal. In fact, the sun's H.A. does not increase quite uniformly; i.e. apparent time is not a 'uniformly flowing quantity'. For a celestial object to satisfy the condition that its hour angle increases uniformly, it is not necessary that it should keep a fixed position on the celestial sphere; it may have motion, provided only that its motion in R.A. is uniform. Then the angular velocity of its declination circle round the pole, i.e., the rate of increase of its hour angle, will be constant. The true sun does not fulfil this condition; accordingly a fictitious sun, called

the *mean sun*, associated with the true sun as will be explained, serves to provide, by its hour angle, what is called *mean time*.

The reasons for this variability in the 'flow' of apparent solar time will be understood when the motion of the earth in its orbit round the sun and the position of the earth's axis with respect to the plane of that orbit are considered.

The orbit of the earth, in its annual revolution round the sun, is an ellipse of which the sun occupies one focus. As the sun lies in the plane of the orbit, the apparent path of the sun as seen from the earth is a great circle of the celestial sphere, and it is described in a year. This great circle is called the *ecliptic*. It may be regarded as a fixed plane; any change in its position relative to the stars is very slow and small and negligible for all ordinary purposes, and in any

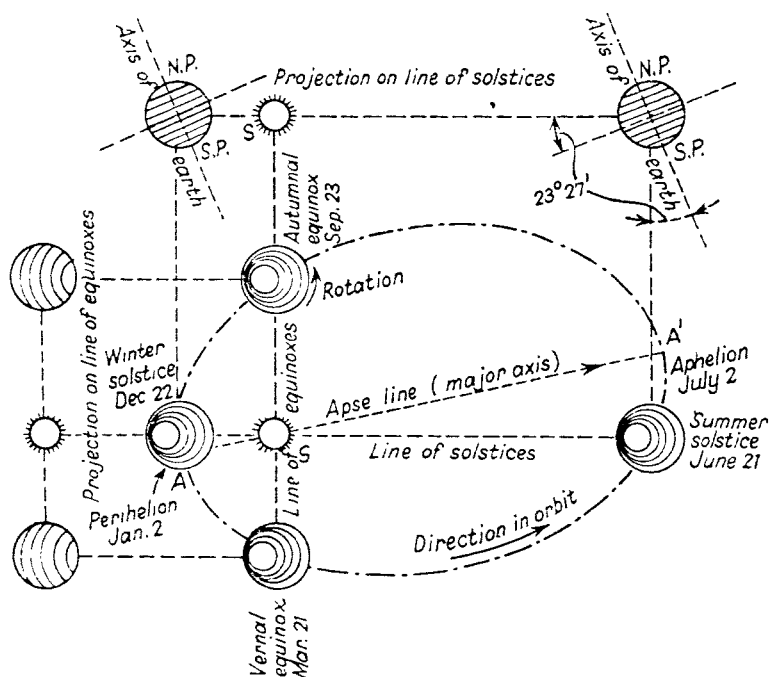


FIG. 54

case does not affect work in field astronomy. The ecliptic is then the trace on the celestial sphere of the plane of the earth's orbit, and is a great circle of that sphere.

The eccentricity of the elliptical orbit is small; it is 0.01673, i.e. about $\frac{1}{60}$. Accordingly, the ratio

$$\frac{\text{greatest distance from sun}}{\text{least distance from sun}} \text{ is approximately } \frac{1 + \frac{1}{60}}{1 - \frac{1}{60}} = 1.034$$

This is consequently also the ratio between the greatest and least apparent semi-diameters of the sun's disc, which are given in the Star Almanac as 16'.3 on Jan. 1 and 15'.8 on July 1 for application in observations taken on the sun's 'limb', i.e. edge. In the N.A. the sun's s.d. is given to 0".01 for each day of the year.

SUN'S MOTION ALONG THE ECLIPTIC. The diagram Fig. 54 represents the earth's orbit with the sun S at one focus. The earth is nearest to the sun when at the end A of the major axis; this is *perihelion* and it occurs about January 1 of each year. The further end A' of the major axis is *aphelion* and it occurs half a year later, i.e. about July 2. These dates are subject to a very slow secular change.

The direction of the revolution in the orbit is the same as that of the axial rotation. The apparent annual motion of the sun on the celestial sphere is therefore in the direction contrary to the direction of the apparent daily rotation of the celestial sphere: this is why the solar day is longer than the sidereal day.

The earth's axis is inclined at an angle of about 66° 33' to the plane of the orbit. The ecliptic is therefore inclined at an angle of about 23° 27' to the plane of the equator; this is called the *obliquity of the ecliptic*. The axis remains very nearly parallel to itself while the earth is performing its revolution round the sun. There is a slight change in its direction, which may however be disregarded for the present. It is due to precession and nutation, which do not affect work in field astronomy. The direction of the tilt of the axis is such that its north polar end inclines away from the sun on December 22 and towards it on June 21. The position of the axis will be clear from a consideration of Fig. 54.

The equator and the ecliptic, being great circles, intersect at two diametrically opposite points of the celestial sphere. They are shown in Fig. 55, intersecting at γ and \simeq . At γ , the First Point of Aries or vernal equinox, the sun in its motion along the ecliptic crosses the equator from south to north, about March 21; the sun's declination is then 0° and his R.A. 00^h. At C on June 21 the sun attains his maximum northerly declination of + 23° 27'; this is the northern summer solstice. At the First Point of Libra or

autumnal equinox, about Sept. 23, the sun crosses the equator again, from north to south, when his declination is 0° , and at E he attains his greatest southerly declination $-23^\circ 27'$ on Dec. 22, the northern winter solstice.

The angular velocity of the earth about the sun is not constant, but is such that the radius vector sweeps out equal areas in equal times. This is Kepler's second law of planetary motion, and is a consequence of the fact that the attractive force keeping the earth in its orbit is directed towards the sun. From this it is easily deduced that the angular velocity varies inversely as the square of the distance, or radius vector. The sun's apparent motion along the ecliptic is therefore fastest on January 1 and slowest on July 2.

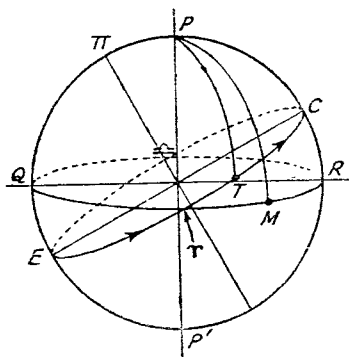


FIG. 55

CONCEPTION OF MEAN TIME. Even if the sun's motion along the ecliptic were uniform, his motion in right ascension would not be so, because a small arc of the ecliptic at E or C subtends a greater angle at the pole P or P' than does an equal arc at φ or \simeq , because the ecliptic is both further from the pole at φ or \simeq than at E or C and oblique to it, and it is the angle subtended at P or P' on which the change of right ascension depends. These then are the two reasons why the true sun's motion in right ascension is not uniform; viz. (a) the motion in the ecliptic is not uniform and (b) the ecliptic is oblique to the equator.

The *mean sun* is accordingly conceived as a point M moving along the equator with the mean or average angular velocity of the true sun T in the ecliptic. It completes one circuit of the equator while the true sun makes one circuit of the ecliptic. Its position on the equator relative to that of the true sun on the ecliptic is so chosen that its right ascension never differs very greatly from that of the true sun, being sometimes ahead of it and sometimes behind it, as regards its hour circle. The hour angle of the mean sun therefore increases uniformly, and this hour angle provides what is called

mean time. *Mean noon*, 12^h M.T., at a place is the instant when the mean sun is on the meridian of that place, and the day is reckoned as beginning at mean midnight, when the mean sun is at its *lower transit*. Accordingly, the mean time is the hour angle of the mean sun, reckoned westwards from upper transit, $+12$ hours; and Greenwich mean time, G.M.T., is the Greenwich hour angle, G.H.A., of the mean sun $+12$ hours, for all places on the meridian of Greenwich, as the day is reckoned as beginning at midnight. G.M.T. is now alternatively called *Universal Time*, U.T.

Mean time increases or flows at a uniform rate equal to the average or mean rate of flow of apparent solar time. The difference, or the amount by which mean time is ahead of or behind apparent time at any instant, is called the *equation of time*. Its value for G.M.T. 00^h of each day of the year is given to 0.01^s in the Nautical Almanac, in the sense apparent time minus mean time, or $A - M = e$. It is therefore the correction to apply to mean time, M.T., to obtain apparent time, A.T.; applied with the opposite sign to A.T. it gives M.T.

The equation of time has two unequal maximum positive values and two unequal maximum negative values, or minima, during the year, as the combined effect of the two causes explained above. The greatest positive value is about $+16^m 22^s$ on November 4 and the greatest negative value about $-14^m 21^s$ on February 12. It is zero four times a year, about April 16, June 14, September 2 and December 25.

The equation of time at any instant is the time equivalent of the angle TPM of Fig. 55, in which T and M represent the true sun and the mean sun, respectively. In the relative positions shown in the figure, the hour angle of T, measured westwards from any meridian, exceeds the hour angle of M by the angle TPM, and the equation of time, in the sense apparent time – mean time, is positive. Clearly, the R.A. of M exceeds the R.A. of T by the same angle. As the year goes on, T moves along the ecliptic with a variable velocity, in the direction shown by the arrow; while M moves along the equator in the same direction with a uniform velocity. The declination circle (hour circle) PT of T consequently swings slowly from one side of PM to the other, through a small angle, according to the value of the equation of time, and whether it is positive or negative.

The local mean times of different meridians differ as in the case of sidereal times, viz. at the rate of 1 hour to 15° of longitude,

because the earth rotates 360° relative to the mean sun in 24 hours of mean time just as it rotates 360° relative to γ in 24 hours of sidereal time. Tables for the 'conversion of time to arc' and vice versa are very convenient; they are to be found in most 'mathematical tables'. There is one in the Star Almanac. Each of the various countries of the world uses the mean time of an appropriate *standard meridian*, which is in most cases based on that of Greenwich, and differs from it by an integral number of half-hours.

The sun makes one circuit of the ecliptic, from γ to γ , in 366.2422 sidereal days, i.e. rotations relative to γ . During this period the earth has accordingly made 365.2422 rotations relative to the mean sun, as the latter has completed exactly one circuit of the ecliptic in the direction contrary to the apparent daily rotation. This period of 365.2422 mean-time days or 366.2422 sidereal days is a year, more strictly a *tropical year*; it is the period of the annual return of the seasons. In it the sidereal clock gains 24 hours on the mean-time clock, or registers 24 hours more than the mean-time clock does; a simple calculation shows that in 1 mean-time day the sidereal clock will register $24^h 03^m 56^s.55$ and in 1 sidereal day the mean-time clock will register $23^h 56^m 04^s.09$. These figures form the basis for the conversion of an interval expressed in mean time (M.T.) into sidereal time (S.T.) and vice versa.

In one mean-time hour the sidereal clock registers $1^h 00^m 09^s.8565$ and in one sidereal hour the mean-time clock indicates one hour less $9^s.8296$. These figures are often useful for the conversion of intervals of time when no tables are at hand. Roughly, the sidereal day is four minutes shorter than the mean-time day, the sidereal clock gaining 10^s per hour on the mean-time clock.

The relation between mean time and sidereal time, as regards *rate*, may also be seen as follows. As the mean sun makes one circuit of the equator, from γ to γ , in 365.2422 mean-time days, its daily motion in R.A. is $360^\circ \div 365.2422$, i.e. $0^\circ.98565$. Therefore, between one mean noon and the next, γ has been separated $0^\circ.98565$ further from the mean sun, and its hour angle is that amount greater than it was the day before, which, at 15° per hour, corresponds to $3^m 56^s.555$, and is the gain of the sidereal clock in 1 M.T. day. Further, the gain in 1 M.T. hour is

$$\frac{0.98565 \times 60 \times 60}{15 \times 24} \text{ seconds, i.e. } 9^s.8565 \text{ as before}$$

The importance of time in astronomy, in navigation and in surveying by astronomical observations is readily seen from consideration of the following. The horizon of any observer, to which the position of stars is naturally referred by him, is being rotated round an axis parallel to the earth's axis at the rate of 1° in 4 minutes, and the disposition of the stars at a place A is precisely what it was at a place B in the latitude of A and 1° of longitude east of A, 4 minutes earlier. Unless the observer knows the time, he cannot determine at what point on a parallel of latitude he is. He can determine the local time, and, if he knows the G.M.T., he can find the longitude, and *vica versa*, but he cannot find both time and longitude from his observations. Usually he will know the G.M.T. from time signals. There are methods based on the comparatively rapid change of the moon's R.A. by which the Greenwich time can be found by lunar observations; they are rarely used nowadays.

The positions of the true sun T, of the mean sun M and of Aries are independent of the location of any place on the earth; the equation of time is therefore the same at any instant all over the world, as is also the difference between local mean time and local sidereal time. These quantities can therefore be tabulated for suitably spaced time intervals, the argument being the time of a selected meridian; and, as they are essential in field astronomy, they are given in the Star Almanac for the meridian of Greenwich for 6-hour intervals for each day of the year.

It was shown above that for any meridian and for any celestial body

$$\text{Local sidereal time} = \text{R.A.} + \text{H.A.}$$

Applying this to the mean sun, a fictitious body, gives

$$\begin{aligned}\text{L.H.A. Aries} &= \text{R.A. mean sun} + \text{H.A. mean sun} \\ &= \text{R.A.M.S.} + (\text{L.M.T.} - 12)\end{aligned}$$

as the L.M.T. is reckoned from lower transit, and the H.A. from upper transit.

Adding 24 hours to the right-hand side, which may be done without affecting the resulting L.H.A. of Aries, it follows that

$$\text{L.H.A. Aries} = \text{L.M.T.} + (\text{R.A.M.S.} \pm 12)$$

and for the Greenwich meridian

$$\text{G.S.T.} = \text{U.T.} + \text{R}$$

where $\text{R} = (\text{R.A.M.S.} \pm 12)$ and is the R of the sun tables in the Star Almanac, where R is tabulated for 6-hour intervals.

The value of R at U.T. 00^h is the G.S.T. at that instant; and if any other tabular value of R be added to the U.T. of that value, the sum is the G.S.T. at that U.T.

From the definition of the equation of time

$$\text{L.H.A. of sun} = \text{L.H.A. of mean sun} + \text{equation of time}$$

$$\text{or } \text{L.H.A. of sun} = \text{L.M.T.} - 12 + e$$

Adding 24 hours to the right-hand side gives

$$\text{L.H.A. of sun} = \text{L.M.T.} + 12 + e$$

and, for Greenwich meridian,

$$\text{G.H.A. of sun} = \text{U.T.} + E$$

where $E = 12^h + \text{equation of time, } e$, and is the E of the Tables in the S.A.

The values of E , of R , and of the sun's declination for times between the tabulated times, are got by simple interpolation, as the time intervals are short. For the declination and for E , an interpolation table is given at the end of the Almanac, and for R , which increases uniformly, a separate critical interpolation table is given. The work of interpolation is thereby reduced to a minimum, and the three quantities are quickly found for any U.T.

The rate of increase of R is the 'acceleration', i.e. the rate at which S.T. is gaining on M.T. The interpolation table for R can therefore also be used for the conversion of intervals of M.T. to S.T., by the addition of the 'respondent' to the M.T. interval; and its use for the conversion of intervals of S.T. to M.T. by the deduction of the respondent from the S.T. will not introduce an error greater than $0^s.16$ for intervals of 6 hours.

Sun or star observations taken at any station for determination of longitude give, as a result of the solution of a spherical triangle, the local hour angle of the observed object. The difference between this L.H.A. and the G.H.A. (got from the clock and the Almanac) is the longitude of the station, which is east if the L.H.A. is greater than the G.H.A. and west if the G.H.A. is greater than the L.H.A.

For the computation of the L.H.A. the declination of the sun or star is also necessary. This is taken from the Almanac. The procedure is explained in full further on.

For the sun, the G.H.A. is given directly by the equation

$$\text{G.H.A. sun} = \text{U.T.} + E$$

$$\begin{aligned} \text{For a star, } \text{G.H.A. star} + \text{R.A. star} &= \text{G.H.A. Aries} \\ &= \text{U.T.} + R \end{aligned}$$

$$\text{or} \quad \text{G.H.A. star} = \text{U.T.} + R - \text{R.A. star}$$

The R.A. star is to be taken from the star tables in the Almanac and interpolated to the date by means of the interpolation table for stars at the end of the Almanac.

It is often required to find the time of transit of the sun or of a star over the meridian of a survey station.

It was shown above that

$$\text{L.H.A. sun} = \text{L.M.T.} + E$$

and

$$\text{G.H.A. sun} = \text{U.T.} + E$$

At the sun's transit at the station, his H.A. is O, and therefore, at that instant, $\text{L.M.T.} = 24 - E$ and for Greenwich transit $\text{U.T.} = 24 - E$. The transit at any place would be at L.M.T. 12^{h} if the equation of time were O at the time of transit; i.e. if E, which is $12^{\text{h}} +$ the equation of time, were 12^{h} , as the mean sun and the true sun would be on the same hour circle. The L.M.T. of transit at the station would be the same as the U.T. of transit at Greenwich, if there were no change in E in the time interval between the transits at the two places.

Example: Find the U.T. of the sun's transit at Greenwich and the U.T. and L.M.T. of his transit over the meridian of a place in longitude $47^{\circ} 50' \text{ E}$, on 1952, March 17.

The longitude must first be converted into time. The conversion table in the Star Almanac gives:

$$47^{\circ} 45' = 3^{\text{h}} 11^{\text{m}}, 5' = 20^{\text{s}} \\ 3^{\text{h}} 11^{\text{m}} 20^{\text{s}} \text{ E}$$

The longitude is then

$$\text{For the transit at Greenwich, } \text{U.T.} = 24 - E$$

$$\text{At U.T. } 12, \quad E = 1^{\text{h}} 51^{\text{m}} 35^{\text{s}}.8$$

$$\text{U.T. of transit is } 12 \text{ } 08 \text{ } 24.2$$

It may be remarked that the value of E taken is that for the transit of the mean sun, which is not precisely the same as that of the true sun, but the difference cannot amount to more than a small fraction of a second and is negligible in field work. For the transit at the station, $\text{L.M.T.} = 24 - E$.

E is to be taken for the time of transit, which takes place approximately $3^{\text{h}} 11^{\text{m}} 20^{\text{s}}$ before transit at Greenwich, during which lapse of time E has been changing and the Almanac shows that it is increasing at the rate of $4^{\text{s}}.3$ in 6 hours, at which rate the increase in $3^{\text{h}} 10^{\text{m}}$ is given in the Almanac as $2^{\text{s}}.2$. This is to be subtracted from the value of E at U.T. 12^{h} .

E at transit is therefore	11 ^h 51 ^m 33 ^s ·6
and the L.M.T. of transit is	12 08 26·4
and the longitude being	3 11 20 E
the U.T. of transit is	8 57 06·4

Example: Find the U.T. of transit of the star Spica on 1952, March 18 at a place in longitude 5^h 08^m 15^s W. Find also the Standard Meridian Time (S.M.T.) of transit, the Standard Meridian being 5^h W of Greenwich.

In the Almanac, the index of places of stars gives Spica (α Virginis) No. 355, of which the R.A. is 13^h 22^m 41^s·9.

The R.A. of the star is the L.S.T. of transit.

L.S.T.	13 22 41·9	
Longitude, W	5 08 15	
G.S.T.	18 30 56·9	
G.S.T. at U.T. 6 ^h	17 42 58·0	(= R at 6 ^h , + 6)
Sidereal interval	47 58·9	(from U.T. 6)
Deduct for mean time	7·9	
	47 51·0	
U.T. of transit	6 47 51·0	
Longitude of Standard Meridian, W	5	
S.M.T.	1 47 51·0	

EXAMPLES OF TIME CONVERSION. Convert 1952, May 29 U.T. 19^h 43^m 28^s·4 to L.S.T. in longitude 17° 32' 45" W.

At U.T. 18 ^h , R is	16 ^h 28 ^m 48 ^s ·3
∴ G.S.T. (= U.T. + R)	34 28 48·3
i.e.	10 28 48·3
M.T. interval from U.T. 18 ^h	1 43 28·4
Acceleration, from interpolation tables	17·0
G.S.T. at given instant	12 12 33·7
Longitude, in time, W	1 10 11
L.S.T.	11 02 22·7

On 1952, Sept. 10 at a place in longitude 2^h 05^m 21^s·8 E the L.S.T. is 00^h 14^m 26^s·5. Find the U.T. at the instant, and the L.M.T.

L.S.T.	00 14 26·5	
Longitude, E	2 05 21·8	
G.S.T.	22 09 04·7	
G.S.T. at U.T. 00 ^h Sept. 10	23 15 52·7	(= R at U.T. 00 ^h)
Sidereal interval	22 53 12·0	from Sept. 10, 00 ^h
Deduct for M.T.	3 45	
U.T. of instant	22 49 27	Sept. 10
Longitude, E	2 05 21·8	
L.M.T.	00 54 48·8	Sept. 11

The U.T. found above is not quite what was asked for, which was for Sept. 10 at the place, whereas at the instant above it is already Sept. 11 there, although Sept. 10 at Greenwich. The instant wanted is 24 sidereal hours earlier.

24 sidereal hours = $23^{\text{h}} 56^{\text{m}} 04^{\text{s}} \cdot 1$ mean time, to be subtracted from the above
This gives

U.T. Sept. 9	$22^{\text{h}} 53^{\text{m}} 22^{\text{s}} \cdot 9$
Longitude, E	<u>2 05 21 · 8</u>
L.M.T. Sept. 10	<u>00 58 44 · 7</u>

APPARENT PLACES OF SUN AND STARS. The co-ordinates, R.A. and declination, of sun and stars given in the Almanac are what are strictly called *apparent places*; they are so called in the Nautical Almanac and are so referred to in the 'Description of Contents' in the Star Almanac. The significance of the word 'apparent' ought to be understood, as also the reason why the co-ordinates of the 'fixed stars' depend on the date. The co-ordinates change, not because of any motion of the stars, or only to a very limited extent for this cause, but because the celestial equator and the equinox Υ , to which their places are referred, are not stationary on the celestial sphere. Further, the *aberration of light* affects the places of celestial objects by causing a displacement towards the point to which the earth's orbital motion is at the time directed; this displacement has a period of one year. The star is seen in the direction of the velocity of the rays from it relative to that of the moving earth. The apparent star places include this displacement, which affects both right ascension and declination. The maximum value of aberration is about $20'' \cdot 5$; this is for celestial objects 90° from the direction of the earth's orbital motion. The sun is always very nearly in that position, and is accordingly displaced along the ecliptic by the amount stated, viz. $20'' \cdot 5$.

The combined attractions of the sun and moon on the earth's equatorial protuberance impart to the earth's axis a slow conical motion about a perpendicular to the plane of the orbit. This is called *precession*. They also give the axis a small nodding motion, called *nutation*. These movements of the axis involve a corresponding movement of the celestial pole and the equator and a consequent change in the places of the stars. The celestial pole moves in a small circle of radius equal to the obliquity of the ecliptic, about the pole of the ecliptic, in a period of about 25,800 years, during which the First Point of Aries also makes a complete circuit of the ecliptic. The mean annual motion of Υ is about $50'' \cdot 2$; it is not a

uniform motion. This is the precession. Superimposed on this is the nutation, the principal component of which has a period of about 18.6 years; this causes a further variability in the motion of γ . The motion of Aries along the ecliptic is in the direction contrary to the sun's motion therein, i.e. it is a retrograde motion, the velocity being variable and the mean motion being $50''.2$ yearly.

The First Point of Aries is thus not a fixed point on the celestial sphere, but has a variable small velocity along the ecliptic. The *mean equinox* of the Nautical Almanac is a fictitious point moving with a uniform velocity along the equator, with the fluctuations 'smoothed out'. *Apparent places* and *apparent sidereal time* are reckoned from the true equinox of date; and *apparent places* include the displacement due to aberration. In short, the apparent places and the apparent sidereal time are corrected for direct use by observers.

REFRACTION AND PARALLAX. Refraction is discussed in Chapter 4 and it is sufficient, here, to state that for astronomical observations the refraction is proportional to the tangent of the zenith distance and can be written as $r = 58'' \tan z$.

There is a further correction to observed altitudes, applicable only to bodies of the solar system. It arises from the fact that, owing to their comparative proximity to the earth, there is a difference in their apparent places on the celestial sphere as seen from different points on the earth; in other words, the rays to them from different places converge at a definite and measurable angle, which is inversely proportional to their distances from the earth. In this they differ from bodies outside the solar system, which are so distant that the rays to them from all points on the earth may be regarded as parallel. In fact the rays to such bodies from opposite ends of a diameter of the earth's orbit, points about 185 million miles apart, do not in any known instance converge so much as $1''$.

The apparent displacement of objects due to a transfer of the view-point is called *parallax*, and is in exactly the opposite direction to the direction of the transfer which causes it; its angular amount is inversely proportional to the distance of the object observed. The sun is the only member of the solar system for which the co-ordinates are given in the Star Almanac, and to which a parallax correction is applicable. For the moon and planets, the Nautical Almanac or the abridged edition thereof must be referred to. The tabulated places are as they would be if seen from the earth's

centre, and allowance must be made for this in observations on them, which are of necessity made on the surface of the earth.

Fig. 56 represents the earth, assumed spherical, with centre C and on it a place A, the horizon of which is the tangent plane,

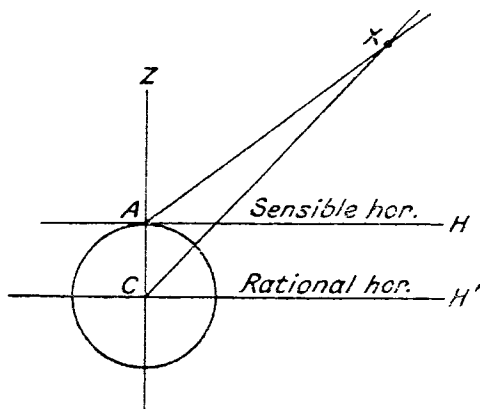


FIG. 56

called the *sensible horizon*. The altitude at A of the object X is the angle XAH and its zenith distance ZAX. Its altitude at C above a plane CH', parallel to AH and called the *rational horizon*, is XCH', and its zenith distance is $\angle ZCX$. Now $\angle ZAX = \angle ZCX + \angle AXC$.

The zenith distance of X at A therefore exceeds that at C by the angle AXC, and this angle AXC is the *parallactic displacement* of X as observed from A. It is called the *geocentric parallax*, and is the angle subtended at X by the radius AC. The parallax correction to an observed altitude is clearly positive and to an observed zenith distance negative.

If X were on the horizon of A, the angle AXC would be $\sin^{-1} \frac{AC}{CX}$, and would be a maximum; it is called the *horizontal parallax*, and its sine is the radius AC divided by the distance CX of the celestial object. The horizontal parallax for all celestial bodies except the moon, for which it is about a degree, is only a few seconds; for such a small angle the sine may be taken as equal to the circular measure of the angle. The horizontal parallax in radians is therefore the earth's radius divided by the distance of the celestial object. The actual meridian section of the earth being elliptical, the equatorial

radius is somewhat greater than the polar radius; the *equatorial horizontal parallax* is the angle subtended at the celestial object by the earth's equatorial radius.

In Fig. 56

$$\frac{AC}{CX} = \frac{\sin AXC}{\sin CAX}$$

$$\sin AXC = \frac{AC}{CX} \cdot \sin ZAX$$

or $\sin \text{parallax} = \sin (\text{horizontal parallax}) \times \sin (\text{zenith distance})$
and as the parallax is a small angle

$$(\text{parallax})'' = (\text{horizontal parallax})'' \times \sin z$$

The horizontal parallax for the various bodies of the solar system is given in the Nautical Almanac, each in its appropriate place. The parallax at any altitude is obtained from the horizontal parallax by multiplying by $\sin z$ or $\cos h$.

The mean value for the horizontal parallax of the sun, which is more frequently observed than any other body of the solar system, is $8''.80$, the extreme values being $8''.66$ towards the end of June when the earth is at aphelion and $8''.95$ in December at perihelion. The sun's parallax in altitude, based on $8''.80 \times \sin z$, is tabulated in most 'mathematical tables'. In the Star Almanac, the addition of $0'.1$ to all altitudes of the sun less than 70° is recommended, which is the same standard of accuracy as that of the tabular declination.

In the preceding paragraphs, the earth has been treated as spherical, for which figure the vertical at A passes through the centre C, and the parallax is entirely in the vertical plane. When the actual spheroidal figure of the earth is taken into account, the vertical at A no longer passes through the centre C, and the parallactic displacement due to transferring the view-point from A to C consequently deviates slightly from the vertical plane, and thereby affects azimuth observations to a small extent, which is quite negligible in field astronomy.

ASTRONOMICAL OBSERVATIONS AND PROBLEMS

Some surveying operations, particularly for precise work, require the preparation of a programme of work on selected stars,

spread out over a great part of the night. Other methods can be applied at any time, suitable stars being picked out just prior to observing. This has obvious advantages.

The angle and level readings, the clock-reading and any other notes relevant to the observation are usually entered in an angle book in which there are spaces for all the necessary data, such as the date, weather conditions, barometer and thermometer readings, so that a correction may be applied to the mean refraction if considered necessary, names of observer and booker, name of station, etc. The computations are done on a form prepared according to the ideas of those concerned.

DETERMINATION OF LATITUDE. There are many ways of determining latitude. Probably the simplest as regards both the observation and the computation or reduction is by *meridian altitudes*.

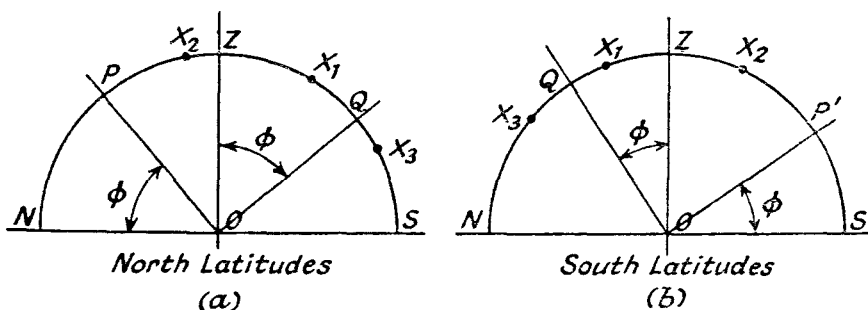


FIG. 57

In Fig. 57a, NPZS represents the celestial meridian of a place O in the northern hemisphere, and in Fig. 57b, NZP'S that of a place O in the southern hemisphere. In Fig. 57a three stars X_1 , X_2 and X_3 are shown at transit of their declinations, δ_1 and δ_2 are north and are by convention positive; δ_3 is south and is accordingly negative. z_1 , z_2 and z_3 are the corresponding zenith distances.

The latitude ϕ of O is the angle PON or ZOQ

$$\begin{aligned}\phi &= \text{arc } QX_1 + \text{arc } X_1Z \\ &= \delta_1 + z_1\end{aligned}$$

From X_2 , $\phi = \text{arc } QX_2 - \text{arc } ZX_2$
 $= \delta_2 + z_2$, if zenith distances north of Z are reckoned
negative

From X_3 , $\phi = \text{arc } ZX_3 - \text{arc } QX_3$
 $= z_3 + \delta_3$, as south declinations are reckoned negative

Consideration of Fig. 57b, for south latitudes, reckoned negative by convention, will show that in this case also the expression

$$\phi = z + \delta$$

holds provided that zenith distances north of the zenith are taken as negative. A sketch showing the relative positions of the elevated pole, the star and the zenith will remove any possibility of error.

For work of an 'approximate' order of accuracy, say to about 10", a single meridian altitude of sun or star, taken with theodolite or sextant, will suffice for a determination of latitude. This method is commonly used at sea, on the sun at apparent noon. The necessary corrections to the observed altitude must, of course, be applied. As the sun's declination is usually changing, the meridian altitude is not exactly the maximum altitude, which may occur a little before or after transit; the effect is negligible for approximate work.

An altitude observation with the sextant, or a pointing on one face with the theodolite, will be affected with the zero or index error, which must be corrected for. The time of transit should be calculated beforehand, and the telescope set to the altitude corresponding to the declination and latitude, so far as the latitude is known.

With a theodolite, the procedure recommended for finding an approximate latitude by meridian altitudes is to start taking altitudes shortly before culmination and to continue taking them F.R. and F.L. alternately until shortly after culmination. The pair, F.R. and F.L., which give the highest maximum altitude (after correction for level, E and O), will give the observed meridian altitude, clear of index error, from which the latitude is derived as shown in the examples which follow. The time is not necessary in the case of a star, but for the sun the approximate time must be known, to permit of the interpolation of the declination.

Example: An observation on the star ζ Aquilae, taken with face left on a theodolite, gave the maximum observed altitude as $52^{\circ} 57' 20''$, the index correction for F.L. being $-5''$. The star's declination was $-13^{\circ} 46' 25''$. Find the north latitude.

Observed altitude	$52^{\circ} 57' 20''$
Refraction	-43
Index correction	-5
Corrected altitude	<u>$52 \quad 56 \quad 32$</u>
Zenith distance	$37 \quad 03 \quad 28$
$\delta \div$	$13 \quad 46 \quad 25$
Latitude ϕ	<u><u>$\div 50 \quad 49 \quad 53$</u></u>

Or thus:

Corrected altitude	$52 \quad 56 \quad 32$
$\delta -$	$13 \quad 46 \quad 25$
colat. c	$39 \quad 10 \quad 07$
$\phi \div$	<u><u>$50 \quad 49 \quad 53$</u></u>

Example: A latitude by meridian altitude of sun:

On March 25th, 1942, the sun's upper limb (U.L.) attained an observed altitude of $41^{\circ} 07' 40''$ at a place near the Greenwich meridian. Find the latitude.

The relevant data are:

$$\delta = 1^{\circ} 40' 27''$$

semi-diameter in arc $16' 04''$

As the observed altitude was that of the upper limb, the semi-diameter (s.d.) has to be subtracted to get the altitude of the centre (\odot).

Observed altitude	$41^{\circ} 07' 40''$
Index correction	— 5
Refraction	— 1 06
Parallax	+ 6
S.d.	— 16 04
Corrected altitude of centre	$40^{\circ} 50' 31''$
Declination δ	+ 1 40 27
Colatitude c	$39^{\circ} 10' 04''$
Latitude θ	$+ 50^{\circ} 49' 56''$

This observation was made at the same station as the preceding one, on ζ Aquilae, but the fairly close agreement, viz. to $3''$ of arc or about 303 feet in latitude, must be regarded as rather fortuitous, in view of the nature of the apparatus used, and other circumstances.

In both of the examples just given the pointing was made on only one face and a correction for index error was applied. This error is always somewhat uncertain. By changing face during the altitude measurement, the index error would have been eliminated, but probably neither the F.R. nor the F.L. pointing would have been made precisely at culmination, certainly not both, and the observed altitude would have been slightly less than the maximum, but it would be near enough for an approximate latitude. This refers, of course, to theodolite observations, with a sextant there is no changing of face.

A single measured meridian altitude would not be expected to give a precise latitude, although mathematically sufficient for that purpose. There are several sources of uncertainty. Two meridian altitudes, one on each side of the zenith, the resulting latitudes from which are averaged, will give greater accuracy. This procedure is called *balancing* or *pairing* of observations, and is commonly used, not only for latitude, but in all observations for geographical position, as it eliminates or at least reduces the effect of constant

instrumental errors, and of errors in the assumed refraction. With balanced observations with a theodolite, it is not really necessary to change face, but each observation may be made on only one face; the mean result will be clear of index error and largely also of errors in the assumed refraction. However, even if the observation on one star is complete in itself, and done on both faces, it is always advisable to balance by a complete observation on another star on the opposite side of the zenith, in all determinations of geographical position where it is possible.

Balancing also eliminates the effect of *droop*, a sort of virtual flexure of the telescope, which is a source of inaccuracy often unsuspected, probably less serious with modern instruments than with the older ones, having longer and heavier telescopes. See a paper 'Modern Methods of finding the Latitude with a Theodolite', by Dr. John Ball, *Geographical Journal*, June 1917.

Suppose that the meridian altitude of a star south of the zenith has been measured with a view to determining the latitude and that the instrument is such, or has been used in such a way (e.g. by observing on one face only), that all altitude readings are too great by an angle θ'' . The observed altitudes are clearly those which would be obtained by a correct reading instrument at a station θ'' more southerly in latitude, which station is θ'' nearer to the *substellar point*, that is, the point which has the star in its zenith. The latitude deduced from the observation is therefore too southerly by an angle θ'' . In the same way, the altitude readings on a star north of the zenith will give too northerly a latitude by θ'' .

The mean of the two deduced latitudes will be correct, apart from any other errors, though the two deduced latitudes will differ by $2\theta''$. The stars selected should culminate at as nearly as possible equal altitudes and as near to the zenith as can be conveniently observed, to reduce refraction errors.

Latitudes may be determined with great precision by means of the *zenith telescope*, an instrument adapted to measure small differences of zenith distance by means of an eyepiece micrometer. Two stars are selected which will culminate, one north and one south of the zenith, at very nearly equal altitudes, such that when one star has been observed, the other star will shortly afterwards come into the field of view of the telescope, which is swung 180° in azimuth to await it. The difference of zenith distance is then measured by the micrometer eyepiece without altering the tilt or elevation of the

telescope itself. The latitude is derived from the difference of zenith distances and the known declinations, without relying on exact measurement of altitude. This is the characteristic feature of the method, which is known as the Talcott or the Horrebow-Talcott method. It requires the selection, from a star catalogue more comprehensive than the Star Almanac, of suitable pairs of stars to form *zenith pairs*. As the field astronomer is not likely to be equipped with a zenith telescope, the method need not be detailed here.

Latitude by Polaris. Polaris, the pole star, provides, in north latitudes greater than about 20° , a good method of latitude determination, being near the north celestial pole, the altitude of which is the latitude. The diurnal circle of the star is a small circle of about 1° radius, the centre being the pole, and a formula expresses the latitude as a function of the star's altitude, its polar distance and its hour angle; or it may be regarded as a correction to apply to the star's altitude to derive the altitude of the pole. Obviously, at upper or lower transit of the star, the correction is equal to the star's polar distance and is negative for upper and positive for lower transit.

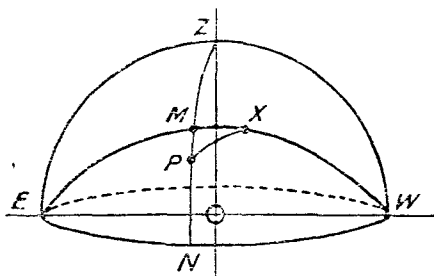


FIG. 58

Let X be the star, at an hour angle $ZPX (=t)$. Draw from X a great circle perpendicular to the meridian ZPN ; this perpendicular is part of a great circle through the east and west points of the horizon and through X .

Let h be the altitude of X

h_M the altitude of M

and h_P the altitude of $P (= \phi$, the latitude)

As XM rises from the east point of the horizon, through X to the meridian, $h_M > h$. Let $h_M = h + x$, or $x = h_M - h$

Now $h_P = h_M - PM$

i.e. $\phi = h + x - PM$, or $\phi = h - PM + x$

It remains to express PM and $x (= h_M - h)$ in terms of the hour angle t , the polar distance p and the star's altitude h .

Taking PM first, and applying the cot formula to the four consecutive parts PX, XPM, PM, 90° of the spherical triangle PXM, gives

$$\cos PM \cdot \cos XPM = \sin PM \cdot \cot PX - \sin XPM \cdot \cot 90^\circ$$

i.e. $\cos PM \cdot \cos t = \sin PM \cdot \cot p - 0$

XPM is the hour angle t , measured east or west from the meridian

$$\therefore \tan PM = \tan p \cdot \cos t$$

or, as PM and p are small,

$$PM'' = p'' \cdot \cos t, \text{ and } -PM'' = -p'' \cos t$$

This is the *first correction* ($-PM$) to the altitude of a close circumpolar star, such as Polaris, to obtain the latitude. The *second correction*, corresponding to $h_M - h = x$, is a little more involved.

$$\text{It reduces to } (h_M - h'') = \frac{1}{2}(p'' \sin t)^2 \tan h \cdot \sin 1''$$

This is the second correction, and the expression for latitude becomes

$$\phi = h - p'' \cdot \cos t + \frac{1}{2}(p'' \sin t)^2 \tan h \cdot \sin 1''$$

In this expression for the latitude, the quantity $-p \cdot \cos t$, the first correction, will be either negative or positive according to whether $\cos t$ is positive or negative, i.e. according to whether the hour angle is less or greater than 90° , reckoned the shorter way from upper transit. It represents the difference between the altitudes of the Pole P and the projection M of the star X on the meridian. The second correction, which is always positive, represents the excess of the altitude of M over the altitude of X.

For a latitude observation on Polaris the local sidereal time has to be known, to derive the hour angle of the star. It can be found by time observations on east and west stars before or after the Polaris observation, by the method explained on p. 188; for this, an approximate latitude has to be assumed in the reduction of the time observations.

The Polaris observation may be carried out as one observation complete in itself, i.e. two pointings on each face for the star's altitude, with the clock time of each, the mean of the altitudes being taken as corresponding to the mean of the clock times. Alternatively, the pointings on Polaris may be made without

change of face; the resulting latitude will be affected with the index error. This can be balanced out by taking a south star on the meridian, on the same face, or by a circum-meridian observation on a south star, as explained below. In any case, a Polaris observation should always be balanced by an observation on a south star at as nearly as possible the same altitude.

Two or four or more pointings should be made on the star, an equal number on each face, say F.L., F.R., F.R., F.L. This, although it involves changing face twice, has the advantage that, if the observation is interrupted for any reason, the data for a latitude have been obtained provided that two pointings have been made. In star work, it is convenient if the observation can be made at any time without waiting for a particular instant, such as transit over the meridian or attaining a set altitude; the Polaris observation can be made at any time. If, however, the observation is made at or about the time of upper or lower transit, it is best to reduce it by the method of circum-meridian altitudes, explained below.

Polaris can readily be picked up in the telescope before nightfall, as its altitude varies only about 2° (twice its polar distance), and its azimuth lies between limits which depend on the latitude, increasing from about 1° E or W in latitude $+10^\circ$ to about 3° E or W in latitude $+70^\circ$. The azimuth of Polaris is tabulated in the Nautical Almanac to $0'.1$, the argument being the star's hour angle, for latitudes $+10^\circ$ to $+70^\circ$, advancing by intervals of 2° . It can also be got readily from the Star Almanac, Pole Star tables, as explained in the introduction to the Almanac.

For Polaris, in the Star Almanac, tables are given by means of which a rough or approximate latitude may be got quickly without the use of log tables and with no computation other than the addition of the tabular quantities a_0 , a_1 , and a_2 , as explained in the introduction to the Almanac.

If the best use is to be made of the observation, which is really an accurate one, it should be reduced by the formula; the approximations used in deriving the latter will not introduce an error of as much as $1''$ in the case of Polaris, or in the southern hemisphere in the case of σ Octantis, which is about $52'$ from the south celestial pole. The circumstances are less favourable in south latitudes, as σ Octantis is not a conspicuous star (magnitude 5.5). The co-ordinates, R.A., and declination, for five circumpolar stars are given in the Star Almanac for every tenth day throughout the year.

*Example of a latitude observation on Polaris*Sidereal watch, fast $24^s.8$ on L.S.T.Formula: $\phi = h - p \cdot \cos t + \frac{1}{2}(p \cdot \sin t)^2 \tan h \cdot \sin 1''$

Face	Watch reading	Vert. circle	Polaris
R.	17 ^h 35 ^m 01 ^s .9	50° 17' 15"	R.A. 1 ^h 41 ^m 14 ^s .6
L.	38 23.0	18 28	$\delta + 88^\circ 57' 31''.8$
L.	40 48.4	19 05	$p \quad 1^\circ 02' 28''.2$
R.	43 48.7	19 12	<u><u>$= 3748''.2$</u></u>
	4) 158 02.0	74 00	
Watch time	17 39 30.5	50 18 30	observed alt.
Fast on L.S.T.	24.8	48.3	refraction
L.S.T.	17 39 05.7	50 17 41.7	corrected alt. h
R.A.	1 41 14.6	logs: p	3.57382
H.A.(W)	15 57 51.1	$\sin t$	1.93515
H.A.(E)	8 02 08.9	$p \sin t$	3.50897
$t(E)$	<u>120° 32' 13''.5</u>	$(p \sin t)^2$	7.01794
		$\frac{1}{2}$	1.69897
logs: p	3.57382	$\tan h$	0.08073
$\cos t$	1.70595 n	$\sin 1''$	6.68557
log $p \cos t$	3.27977 n	log 2nd correction	1.48321
$p \cos t$	- 1904''.5	2nd correction	<u>+ 30''.4</u>
$= - 31' 44''.5$			
1st correction		h	50° 17' 41''.7
$(-p \cos t) = + 31' 44''.5$		1st correction +	31 44.5
		2nd correction +	30.4
		ϕ	<u>+ 50 49 56.6</u>

The degree of precision to be expected depends on many circumstances, such as the skill of the observer, the kind and condition of the apparatus and the accuracy with which the sidereal time and the refraction are known. At the most, the star's altitude may change by $1''$ in about 4 secs. of time. With a good modern theodolite reading direct to single seconds a Polaris observation or one on σ Octantis, paired with a meridian or circum-meridian star on the opposite side of the zenith, should give a latitude correct to within about $2''$, and probably closer.

In this observation a Tavistock theodolite and a sidereal watch were used. The observing and booking were both done by one person; this makes the observation somewhat protracted. There is no level correction, the bubble being set for each pointing if

necessary. The symbol n after $\log \cos t$ and $\log p \cos t$ indicates that the quantities $\cos t$ and $p \cos t$ are negative.

Latitude by circum-meridian altitudes. This is an extension of the meridian altitude method. In it the altitude of the sun or star is observed from about 10 minutes before until about 10 minutes after transit. The time of each pointing is booked, and, as the time of transit is known or is easily found, the small hour angle at each pointing is known. A correction, which is a function of the hour angle, the declination and the latitude, is then applied to the altitudes to deduce the meridian altitude. In finding the correction, an approximate latitude, readily got from the altitude readings, is used. Each pointing, so corrected, provides a meridian altitude, and the mean of these is taken; it is more reliable than a single observed altitude.

This correction is called the *reduction to the meridian*, and it is clearly always additive for a star near upper transit, to obtain the meridian altitude. For a star near lower transit the correction to altitudes, i.e. the reduction, is negative.

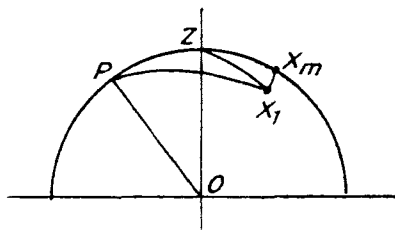


FIG. 59

Its derivation and form are as follows:

In Fig. 59, let X_m be the position of a star at upper transit

z_m its meridian zenith distance ZX_m

X_1 its position shortly after transit, when its H.A. ZPX_1 is t secs.

z_1 the zenith distance ZX_1 of X_1 , which exceeds z_m by $z_1 - z_m$, the amount of the reduction to the meridian

If the declination has not changed in the time t , $PX_m = PX_1$. In the spherical triangle PZX

$$\cos ZX_1 = \cos PX_1 \cdot \cos PZ + \sin PX_1 \cdot \sin PZ \cos ZPX_1$$

$$\text{i.e.} \quad \cos z_1 = \cos p \cdot \cos c + \sin p \cdot \sin c \cdot \cos P$$

$$= \cos p \cdot \cos c + \sin p \cdot \sin c \left(1 - 2 \sin^2 \frac{P}{2} \right)$$

$$= \cos (p - c) - 2 \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2}$$

Now

$$(p - c) = PX_m - PZ = ZX_m = z_m$$

$$\therefore \cos z_1 = \cos z_m - 2 \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2}$$

$$\cos z_m - \cos z_1 = 2 \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2}$$

$$- 2 \sin \frac{z_m + z_1}{2} \cdot \sin \frac{z_m - z_1}{2} = 2 \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2}$$

$$\text{or} \quad \sin \frac{z_1 + z_m}{2} \cdot \sin \frac{z_1 - z_m}{2} = \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2} \quad . \quad . \quad (a)$$

Now $z_1 - z_m$ is a small angle and, approximately,

$$\begin{aligned} \sin \frac{z_1 - z_m}{2} &= \frac{z_1 - z_m}{2} \text{ in radians} \\ &= \frac{(z_1 - z_m)''}{2} \cdot \sin 1'' \end{aligned}$$

$$\text{Also} \quad \frac{z_1 + z_m}{2} = z_m, \text{ approximately}$$

$$\therefore \sin z_m \cdot \frac{z_1 - z_m}{2} \cdot \sin 1'' = \sin p \cdot \sin c \cdot \sin^2 \frac{P}{2}, \text{ from (a)}$$

$$\text{whence} \quad (z_1 - z_m) = 2 \cdot \frac{\sin c \cdot \sin p}{\sin z_m} \cdot \sin^2 \frac{P}{2} \cdot \frac{1}{\sin 1''}$$

$$\text{and} \quad h_m = h_1 + \frac{\cos \phi \cdot \cos \delta}{\cos h_m} \times \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

putting t for the hour angle $P (= ZPX_1)$.

The expression $\frac{\cos \phi \cdot \cos \delta}{\cos h} \times \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$ for the reduction to the meridian is usually written in the form $A \times m$; the second factor

$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$ can be tabulated for a suitable range of values of the small hour angle t , the time interval between the transit of the celestial object, sun or star, and the instant of the observation of

its altitude. The watch time of transit is computed beforehand; a series of pointings is made from about 10 minutes before to about 10 minutes after transit; the watch time and the altitude reading for each pointing are booked. The meridian altitude h_m , corresponding to an altitude h at an hour angle t , is given by

$$h_m = h + Am$$

where h is the altitude corrected for index error and refraction. Accordingly, for a series of pointings

$$\begin{aligned} h_m &= h_1 + Am_1 \\ h_m &= h_2 + Am_2 \\ h_m &= h_3 + Am_3 \\ h_m &= h_n + Am_n \\ &= \frac{\Sigma h}{n} + A \frac{\Sigma m}{n} \\ &= h_o + Am_o \end{aligned}$$

where h_o and m_o are the mean values of h and m for all the pointings. The refraction correction may be applied to the mean of the altitudes h_o instead of to each individual observed altitude.

The factor A is the same for all the pointings. For it the pair of pointings, one on each face, which give the highest altitude, will give h_m and ϕ with sufficient accuracy.

The factor m is given in the *Star Almanac*, in the form of a critical table in which the respondent m is tabulated for every single second of m from $0''$ to $199''$, corresponding to $10^m 04^s.8$. The tabulated values of the argument t are the limits between which each value of the respondent is applicable. For hour angles t from 10^m to 20^m , advancing by 5^s , the values of m are given in a table of the ordinary form. Tables for the value of m from 0 to 20^m , advancing by single seconds of time, are given in the *Text Book of Topographical Surveying*, Close and Winterbotham, to an accuracy of $0''.1$.

More pointings can be made within the period favourable to accuracy if face is changed only after the first, third, fifth, etc., pointings; there should be an equal number on each face before and after transit; 8 in all is mostly considered sufficient. If the observation on one star is to be balanced by an observation on another star on the other side of the zenith, as is advisable for precision, the whole series of pointings may be made on one and the same face, without any change thereof. The latitudes deduced

from the pair of north and south stars will of course differ by twice the amount of the index error (collimation in altitude) and the flexure, if any; the mean will be correct. More pointings can be got near the meridian, at a smaller hour angle; the $A \times m$ correction is thereby kept small and there is a less error due to the approximation used in deriving the formula, as the latter assumes the

correction to be small. Further, the factor $A = \frac{\cos \phi \cdot \cos \delta}{\cos h}$ should

be kept small by observing only stars which transit at say 15° or more from the zenith, for which the denominator $\cos h$ is large. Very low altitudes must, of course, be avoided because of uncertainty of refraction. Altitudes from about 35° to 70° are accordingly favourable, and the pair of north and south stars ought to be at about the same altitude.

For a determination of latitude by circum-meridian altitudes the clock time of transit must be known, as the hour angles t are to be obtained from the difference between it and the clock times of the individual pointings. The L.M.T. can be found by forenoon and afternoon sun observations, and the L.S.T. by east and west star observations, if they are not known from the time signals and the longitude. The sun transits at L.M.T. = $24^h - E$, and a star at L.S.T. = R.A. of star. The clock error must be applied to get the clock time of transit.

The pointings on the sun, for which a dark glass must be used, may be made on either the upper limb (U.L.) or the lower limb (L.L.), or preferably on both, alternately. This reduces any error due to consistent inaccuracy in making the tangential contact of wire and limb, which has to be done with the slow-motion screw. If each pointing is to be reduced separately, the sun's semi-diameter must be applied, $+$ or $-$, to each altitude, for the altitude of the centre. It is given in the S.A. for the first and for the second half of each month, to $0'.1$. In the N.A. it is given to $0''.01$ for each day throughout the year.

The image of the sun formed in the plane of the cross-wires by the object-glass is inverted both up-and-down and sideways, and with the ordinary straight eye-piece a completely reversed image is seen. With the diagonal eye-piece one of these reversals is rectified; the image is therefore seen reversed in only one direction – usually sideways, i.e. in azimuth. It is advisable to check this for

any particular eye-piece, lest there should be any doubt as to which limb has been observed; it will usually be obvious from the readings.

The hour angles t are to be sidereal intervals in the case of a star. For the sun they should strictly be the apparent time intervals; these are not exactly equal to the mean-time intervals given by the mean-time clock, as the equation of time changes. Usually this can be neglected. Instead of h_o being taken, the mean of all the altitudes, and the mean reduction Am_o applied to it, each observed altitude may be treated separately to give a meridian altitude, by correcting for level and applying the corresponding Am reduction. This shows the closeness of agreement between the pointings, and a poor one may be rejected and the probable error computed.

The sun's declination may change by nearly as much as $20''$ during the 20 minutes or so covered by the observation, and this has to be considered. Strictly, each pointing should be computed separately, using the hour angle and declination for the instant of the pointing, from which the meridian altitude for a body of that declination is got by applying the reduction to meridian, and thence the latitude. Further, any gain or loss of the true sun on the mean sun, which may amount to about 0.4 seconds during 20 minutes, ought also to be taken into account, as also the *rate* of the watch. Such precision is not necessary and would not be called for in most field astronomy work. But it can be shown that the declination for the mean of the times of the pointings should be taken, and this mean of the times should be as nearly as possible L.A.N. Taking this declination avoids the necessity for interpolating the declination for the time of each pointing.

For suppose that at the first pointing the declination is δ_1 and the altitude h_1 , the corresponding meridian altitude h_m will be $h_1 + Am_1$, and for the zenith distance, $z_m = 90 - h_1 - Am_1$.

Above, it was shown that

$$\phi = z_m + \delta$$

Accordingly, from the pointing h_1 ,

$$\phi = 90^\circ - h_1 - Am_1 + \delta_1$$

and

$$\phi = 90^\circ - h_2 - Am_2 + \delta_2$$

$$\therefore \phi = 90^\circ - h_o - A.m_o + \delta_o$$

where h_o is the mean of all the altitudes

m_o is the mean value of m

and δ_o the mean of the declinations at all the pointings, which is the same as the declination at the mean of the times of all the pointings.

assuming only that the declination changes uniformly during the period of the observation.

If it is desired to compute each pointing separately for comparison, to each observed altitude the defect of the declination from the declination at transit may be added, or its excess subtracted therefrom, as at or very near transit the altitude is affected by the whole amount of a change of declination.

Determination of longitude and local time. The determination of local time can be made in several ways, and from the local time, provided that the Greenwich time is known, as it ought to be, the longitude is at once obtained, being the difference between the Greenwich and local times of the same kind, whether mean, sidereal, or apparent solar.

As has been explained, local time is the hour angle of the relevant celestial object, viz. φ for sidereal and the mean sun ($\pm 12^h$) for mean time. What is meant by a time determination is really a determination of the error of the clock at the instant of the observation, as the clock has to be depended on to carry the time on until the next observation. The *rate* of the clock, i.e. how much it gains or loses per day, is also determined from two or more time observations made at suitable intervals. The constancy of the rate is the important characteristic of a good time-keeper.

In fixed observatories, the direction of the meridian is known accurately, and the time is determined by star transits across the meridian by a transit instrument set in that meridian, as the local sidereal time is the right ascension of the star. This method is also in use in the field; it avoids dependence on measured altitudes, which are always liable to be affected with errors, e.g. in the assumed refraction. But methods based on star altitudes are generally preferred in field work, as they do not call for a previous determination of the direction of the meridian; in fact both local time and meridian can be determined in one observation with sufficient accuracy for many purposes.

The most usual method of determining clock error on local time, both in surveying and in navigation, consists in measuring the altitude of the sun or a star, with a simultaneous reading of the clock time. The celestial object to be observed should be one that is changing its altitude fairly rapidly, so that the instant of its attaining the altitude shown on the instrument can be noted with precision. An object near the meridian would obviously not serve. It

will be shown later that objects on the prime vertical have the greatest rate of change of altitude; the best stars for a time or longitude observation are those of azimuth near 90° or 270° , and at a convenient altitude for observing, preferably without using the diagonal eyepiece.

The latitude is supposed to be known at least approximately. The measurement of the altitude then enables the spherical triangle PZX, Fig. 60, to be solved for ZPX, the hour angle. The polar distance PX being known from the Star Almanac, all three sides are known.

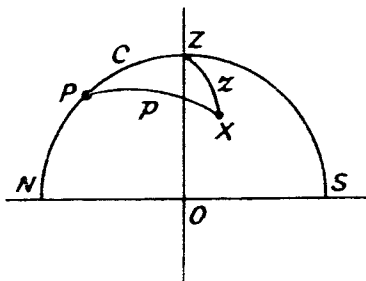


FIG. 60

There are several formulae by which the triangle ZPX may be solved, the three sides being known. One of the following is generally used:

$$PZ = c = 90^\circ - \phi \text{ known}$$

$$ZX = z = 90^\circ - h \text{ from the observation}$$

$$PX = p = 90^\circ - \delta \text{ from the Star Almanac}$$

$$\text{If } p + c + z = 2s$$

$$\sin \frac{P}{2} = \sqrt{\frac{\sin(s-c) \sin(s-p)}{\sin c \cdot \sin p}}$$

$$\cos \frac{P}{2} = \sqrt{\frac{\sin s \cdot \sin(s-z)}{\sin p \cdot \sin c}}$$

$$\tan \frac{P}{2} = \sqrt{\frac{\sin(s-c) \sin(s-p)}{\sin s \cdot \sin(s-z)}}$$

The choice of which formula to use rests with the computer. The tan formula is usually preferred, as the tangent of an angle changes faster than does the sine or the cosine, enabling the angle to be determined more precisely from its tangent than from the sine or cosine. This advantage is not very great at the values of the hour angle P at which the observation is usually taken.

The fundamental formula

$$\cos z = \cos p \cdot \cos c + \sin p \cdot \sin c \cdot \cos P$$

from which is derived $\cos P = \cos z \cdot \sec \phi \cdot \sec \delta - \tan \phi \cdot \tan \delta$

is sometimes used if a number of observations are taken on the same

star at the same station. Five-figure logs are often sufficient in the computation, but six or seven are sometimes used, according to circumstances. If the body observed be in the eastern sky, the computed hour angle P is an eastern or negative one, and its amount is to be subtracted from 24^h to give the H.A. reckoned westward, according to convention. It may be remarked here that, at the time of the observation, the star has the same altitude at another place in the same latitude. At one place it is east and at the other it is west of the local meridian; the two places differ in longitude by twice the star's hour angle taken the shorter way from the meridian.

It was stated above that at any given station the rate of change of altitude is greatest for stars in the prime vertical. This can be proved thus:

$$\cos z = \cos p \cdot \cos c + \sin p \cdot \sin c \cdot \cos P$$

$$\therefore -\sin z \cdot \frac{dz}{dt} = 0 - \sin p \cdot \sin c \cdot \sin P \frac{dP}{dt}$$

$$\frac{dz}{dt} = \frac{\sin p}{\sin z} \cdot \sin c \cdot \sin P \cdot \frac{dP}{dt}$$

$$= \frac{\sin Z}{\sin P} \cdot \sin c \cdot \sin P \cdot \frac{dP}{dt}$$

$$= \sin c \cdot \sin Z \cdot \frac{dP}{dt}$$

Now $\frac{dP}{dt} = 15''$ per second of time

$$\therefore \delta z'' = 15'' \cdot \sin c \cdot \sin Z \cdot \delta t^s$$

This is a maximum and equal to $15'' \cdot \sin c \cdot \delta t^s$ when $Z = 90^\circ$ or 270° . It can also be shown thus:

The rotation of the celestial sphere about the axis OP at a uniform angular velocity of $15''$ per second can be resolved into two rotations, one about ON at an angular velocity of $15'' \cos \phi$ per second and one about OZ at an angular velocity of $15'' \sin \phi$ per second. The latter produces motion in azimuth only; the former produces in general motion in both altitude and azimuth, but for stars on the prime vertical all the motion due to it is in altitude, and its angular velocity is $15'' \cos \phi$ per second, or $15'' \sin c$. It is easy to show similarly that for a star at an azimuth Z the rate of

change of altitude per second of time is $15'' \sin c \cdot \sin Z$. All stars on the same vertical circle at any instant have therefore the same rate of motion in altitude at that instant. Time determinations by ex-meridian altitudes are better in low than in high latitudes, as

$\frac{dz}{dt}$ is greater, being proportional to the cosine of the latitude.

A further reason why stars on the prime vertical are most favourably placed with a view to determining the longitude is as follows. The curvature of the earth's surface is such that a distance of about 101 feet on the surface subtends an angle of $1''$ at the centre. A movement or transfer of position of that amount accordingly tilts the observer's horizon by $1''$ downwards in the direction of the movement and upwards in the opposite direction; i.e. the horizon tilts about a horizontal axis at right angles to the direction of the movement. Therefore if a star is on the prime vertical a small movement along the meridian, being at right angles to the prime vertical, will not affect the altitude of the star. This is equivalent to the statement that a small error in the assumed latitude has no effect on an ex-meridian time observation made on a star on the prime vertical. The meridian is in fact a tangent to the *position circle* or locus of points at which the star has the same altitude at the instant, and a short length of the meridian is a *position line* on which the observing station is situated. If the star observed is θ° away from the prime vertical, the position line is a line inclined at θ° to the meridian.

Only stars between certain limits of declination cross the prime vertical. It was shown with reference to Fig. 50, that a star on the equator rises and sets in the east and west points respectively, all over the earth. It is then on the prime vertical but moving away from it, towards the south in the northern hemisphere and towards the north in the southern. To cross the prime vertical the star must have declination of the same name or sign (i.e. N or S) as the latitude. A star of declination δ equal to the latitude ϕ will at culmination pass through the zenith, where its path is a tangent to the prime vertical. Accordingly δ must lie between 0° and ϕ . But it is not at all essential that the star for a time observation by altitude should be on the prime vertical; it is sufficient that it should be in a nearly east or west direction. Every star which crosses the prime vertical must, of course, do so twice in a sidereal day, the

lapse of time between the two passages depending only on the latitude and on the declination.

It is always advisable to balance the observation by taking two stars, one in the east and one in the west. The usual practice is to take, for each star, two altitude-readings on each face, with simultaneous clock-readings, to eliminate index error. The mean of the four altitudes is taken as the observed altitude at the mean of the clock times. This is not strictly accurate, because the motion in altitude is not quite uniform; the more quickly the four pointings are put through, the less will be the error. It is best to take the pointings in two pairs, e.g. F.L., F.R., F.R., F.L. This entails changing face twice, but has the advantage that, if the star becomes obscured by cloud before the observation is complete, there is a chance of having one pair of pointings, F.L. and F.R.; also the two pairs can be computed independently if time permits, thus reducing any small error due to non-uniform rate of change of altitude.

Procedure in an ex-meridian time observation. The star having been picked up in the telescope, the horizontal wire should be placed so that the star will cross it in a few seconds, and the booker is warned to that effect; the hand can then be removed from the tangent screw. The telescope should be set so that the star will cross the horizontal wire near the intersection of the wires. When the star is dead on the horizontal wire the observer calls out 'Tip' sharply, and the booker records the clock-reading, the seconds and fraction thereof first, followed by the minute and hour. Then the observer gives him the bubble reading of the altitude level, eye end E and object end O, followed by the readings of the two verniers or micrometers of the vertical circle. With modern theodolites such as the Tavistock, bubble readings are not necessary, as if it is not central it may be made so even after the pointing without changing the direction of the line of sight. The bubble correction is, in fact, transferred to the circle reading which is altered by adjusting the bubble. The observer will then change face, saying so to the booker, who will give him the setting of the vertical circle or both circles so that the star may be picked up without delay. This is usually more convenient than using the sights, which are not always very satisfactory. The pointing on the star on the changed face is then done as before, preferably at the same part of the horizontal wire. The booker should remind the observer when to

change face during the observation. Alternatively, for the times of intersection the observer may use a stop-watch; as in the example given below.

Time observations on the sun. With these the accuracy is rather less than for a star observation. The declination is taken for the G.M.T. corresponding to the mean of the clock times of the pointings, for which purpose the G.M.T. must be known approximately. It must, of course, be known precisely for a longitude determination. The sun's declination changes most rapidly at the equinoxes, the rate being then about $59''$ per hour. The usual four pointings, two on each face, may be made on upper and lower limbs alternately, but it is preferable to make all the pointings on the trailing or following limb, because the instant of tangency can be noted more precisely than with the leading limb. Therefore, for a forenoon sun observation use the lower limb, and for an afternoon one the upper limb. Corrections have to be applied to the observed altitude of the limb, for refraction and parallax for the altitude, taken from tables, and for the sun's semi-diameter, taken from the Star Almanac, for the date. In high latitudes in either hemisphere in winter, the sun is never very favourably placed, never being near the prime vertical.

A good sun observation should give the L.A.T. to within about 2 secs.

The most convenient way of picking up the sun in the telescope is to receive the shadow of the sights or other part of the instrument on a sheet of paper held in the hand. Balancing can only be done by taking both forenoon and afternoon observations.

For a complete time observation, it is advisable to balance by taking both east and west stars. Each star is usually taken on both faces, but some observers take both stars on only one, the same, face. In the case of meridian or near-meridian observations for latitude on north and south stars, this gives perfect balancing, but in the case of ex-meridian time observations the balancing is not perfect unless the east and west stars are equidistant in azimuth from the meridian. This is because the error in the altitude, taken on one face only, though the same for each of the two stars, does not correspond to equal time intervals, i.e. the resulting error in the computed time is different for the two stars.

It was shown in the foregoing that

$$\delta z'' = 15'' \sin c \cdot \sin Z \cdot \delta t^s$$

$$\text{or} \quad \delta t^s = \frac{\delta z''}{15 \sin c \sin Z}$$

If $\delta z''$ be the error in measured altitudes, i.e. the difference between the reading of the instrument and the true altitude of the line of sight, δt^s represents the time interval between the recorded clock reading and the instant when the star really reaches the altitude read on the instrument, and δt^s will only be the same in amount but of opposite sign, if $\sin Z$ is the same numerically for both stars. However, if both stars are near the prime vertical, $\sin Z$ and consequently δt^s will differ only to a negligible extent, and the mean of the computed longitudes may be taken as correct. It should be correct to within a fraction of a second of time, in favourable circumstances and with good observing and time-taking. More than one pair of E and W stars may be taken with advantage.

Examples of ex-meridian time observations

I. *On a star.* In this observation face was not changed, six pointings for altitude were made in just over 2 minutes, with a Tavistock $3\frac{1}{2}$ " theodolite, for determination of longitude.

Barometer 29 inches. Thermometer 93° F.

Chronometer, slow on G.M.T. $6^m 29^s.8$. Latitude $+23^{\circ} 22' 20''$

East star. α Pegasi. Declination $+14^{\circ} 53' 36''$

R.A. $23^h 01^m 52^s.5$

Chrom. reading	vert. circle		
18 ^h 55 ^m 36 ^s	128 ^o 46' 40"	Observed alt. (v.c. -90°)	$39^{\circ} 01' 38''$
56 09	128 54 00	Refraction, mean $1' 11''.7$	
56 31	128 59 09	Correction for barom. -2.4	
56 55	129 04 48	" " temp. -6.0	$1' 03''$
57 17	129 09 54	Corrected alt. h	$39 00 35$
57 41	129 15 17	$z.d.$ or $z.$	$50 59 25$

6). $34^o 09'$ 6) $54^o 09' 48''$

18^h 56^m 41^s.5 129^o 01' 38"

(The refraction tables used were not those of the Star Almanac)

mean of clock times 18^h 56^m 41^s.5

slow on G.M.T. 6 29.8

G.M.T. of Observation 19 03 11.3

S.T. at G.M.T. 00^h 22 43 00.1

accelern. 19^h 3 07.3

3^m 00.5

11^s.3 00.0

G.S.T. of observn. 17 49 19.2

$$c = 90^{\circ} - \phi = 66^{\circ} 37' 40''$$

$$p = 90^{\circ} - \delta = 75^{\circ} 06' 24''$$

z	$50^{\circ} 59' 25''$	$\log \sin$
c	$66 37 40$	1.962818
p	$75 06 24$	1.985160
$2s$	$192 43 29$	1.947978
s	$96 21 44$	0.052022
$s - c$	$29 44 04$	1.695465
$s - p$	$21 15 20$	1.559342
		2.1306829

$\frac{P}{2}$ $26 45 25$ 1.653414

P $53^{\circ} 30' 50''$

P $3^h 34^m 03^s.3$ (the eastern hour angle)

R.A. $23 01 52.5$

L.S.T.₀ $19 27 49.2$

G.S.T.₀ $17 49 19.2$

long. E $1^h 38^m 30^s.0$

II. *On the sun*

Latitude: + 50° 49' 57"

M.T. watch, fast on G.M.T. 1^h 00^m 03^s·2

Sun's lower limb L.L.

Face	Watch	Stop-watch	Vert. circle	Appt. alt.	
R	9 ^h 25 ^m 15 ^s	16·7	52° 45' 09"	37 14 51	
L	9 28 00	10·3	127 41 43	37 41 43	
	18 53 15	27·0		2)56 34	
	-27		Observed alt.	37 28 17	$\delta : +20^{\circ} 41' 01''$
	2)18 52 48		Refraction	- 1 16	E 11 53 49·5
mean	9 26 24		Parallax	+ 07	sin
fast	1 00 03·2		Semi-dia.	+ 15 46	$P = \frac{\sin(s-c)\sin(s-p)}{\sin c \cdot \sin p}$
GMT	8 26 20·8		Corrected alt.	37 42 54	2
			<i>z.d.</i>	52 17 06	
<i>z</i>	52° 17' 06"		log sin	U.T.	8 ^h 26 ^m 20·8 ^s
<i>c</i>	39 10 03	1·80044	E	11 53 49·5
<i>p</i>	69 18 59	1·97106	G.H.A.	20 20 10·3
	25 160 46 08		1·77150	L.H.A.	20 19 31·1
<i>s</i>	80 23 04		0·22850	long. W.	39 ^s ·2
<i>s-c</i>	41 13 01	1·81883		
<i>s-p</i>	11 04 05	1·28324		
			2)1·33057		
$\frac{P}{2}$	27° 33' 37"	1·66528		

Note: Four pointings would have been better.

P 55 07 14
H.A.(E) 3^h 40^m 28^s·9
H.A.(W) 20 19 31·1

In the foregoing sun observation, the observer did his own booking and he used a stop-watch. The latter is started at the instant of tangency of the limb with the horizontal wire, or, in the case of a star, at the instant of its bisection by the wire, and stopped when the clock gives a convenient reading as soon as possible thereafter. From this clock reading that of the stop-watch is subtracted to get the clock time of the tangency or of the bisection. The mean refraction and the use of five-figure logarithms were considered accurate enough for the purpose of the observation.

The method of time determination by ex-meridian altitudes is the one most generally employed in field astronomy. It has the advantages that it can be applied at any time when stars or sun are visible, and it does not require a previous knowledge of the direction of the true meridian, nor a previously worked out programme, nor a long wait until one or more stars reach any particular

altitude. It does call for a knowledge of the latitude, but, as the time observation for longitude would naturally be accompanied by latitude observations to fix the geographical position, this is no drawback. Further, as will be explained in dealing with determination of meridian, the ex-meridian time observation may quite well be used for that purpose also, though it is in general best to determine the direction of meridian by a separate observation to that end.

DETERMINATION OF TRUE MERIDIAN. It is often necessary to determine the true bearing of a survey line at a station, i.e. the horizontal angle between the terrestrial meridian of the station and the survey line, measured clockwise from true north to the survey line. There are various ways in which this may be done; all of them involve observing the horizontal angle between the survey line and a celestial object, as well as such other data as will enable the azimuth of the celestial object to be computed for the instant at which it was observed. The azimuth or true bearing of the line is then readily deduced, and from it the bearings of any other survey lines meeting at the station.

The observational data most frequently used in field astronomy as a basis for the computation of the azimuth of the celestial body are either (*a*) its altitude, or (*b*) the local time of the observation on it. The observation, as commonly carried out in field astronomy, consists in taking one or more horizontal circle readings, F.R. and F.L., on the survey line, and also one or more horizontal circle readings, F.R. and F.L., on the celestial object. In addition, either the altitude must be read on the instrument, if altitude is to serve as basis for the computation of azimuth, or the clock time of the pointing must be read carefully, if the time is to serve as basis; from the time, the hour angle ZPX is derived, to be used in the solution of the spherical triangle ZPX.

A mark or object, clearly visible in all ordinary circumstances from the station where azimuths are to be observed, is necessary. This mark, called the *reference mark* (R.M.) or *reference object* (R.O.), must be such that an accurate pointing can be made on it; it need not necessarily be on one of the survey lines. It should be well clear of the ground, and it should be at such a distance that refocussing of the telescope will not be necessary when changing the pointing from it to a star.

Let OA be the line from the station O to the R.M., NOS the true meridian of O , the exact direction of which is of course unknown. The horizontal circle reading on OA is taken.

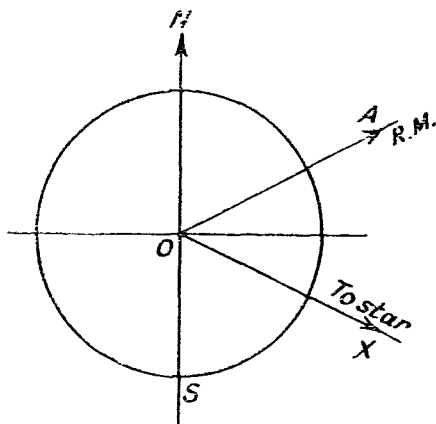


FIG. 61



Let OX be the direction to a star X ; this is of course changing, and it is therefore necessary to book such particulars as will make it possible to compute its azimuth at the instant of observing. This computed azimuth is the angle NOX , and the true bearing of OA is the difference between the computed angle NOX and the observed angle AOX . It is always advisable to make a sketch plan showing the relative directions of the R.M. and the star, and the approximate direction of true north.

Azimuth by ex-meridian altitudes. As already stated, in this method the altitude of the celestial object is to be used, with its polar distance and the known co-latitude, in the solution of the astronomical triangle PZX ; the altitude must therefore be observed and booked. The observation is essentially the same as an ex-meridian time observation, and the same conditions are favourable for it. But, in addition to the star's altitude, the horizontal circle readings on the star and on the R.M. have to be taken, so that, when the azimuth of the star is computed, the bearing of the R.M. can be deduced as explained above. As with an ex-meridian time observation, it is advisable to balance an ex-meridian azimuth observation on one star by another on a star similarly situated on the opposite side of the meridian; stars near the prime vertical are to be preferred.

The object of the observation is to obtain one or more simultaneous values of the star's altitude and of the horizontal angle between it and the R.M. There will usually be two pointings, one F.R. and one F.L., on the star, to eliminate instrument errors. The mean of the two altitudes is taken as the altitude at the instant of observation, and the mean of the horizontal circle readings as being the horizontal circle reading at the same instant. This would be strictly correct if the motion in azimuth were proportional to the motion in altitude, but this is not quite the case. The error is, however, negligible in ordinary field work. The mean of the two altitudes, F.R. and F.L., is taken; the star's azimuth is computed for this altitude, and taken as being the azimuth when the horizontal circle reading on the star was the mean of the F.R. and F.L. horizontal circle readings.

If the observation is to be made on a star, the usual procedure in observing is to point carefully on the R.M. on one face and direction of swing, and read the horizontal circle. Then get the star on the vertical wire and set the telescope so that the star is approaching the horizontal wire. Keep the star on the vertical wire by the tangent screw and remove the hand from the tangent screw the instant the star is at the cross of the wires. Give the booker the readings E and O, if any, of the altitude level and the readings of the vertical and horizontal circles. This should always be done in the same order. Change face and observe the star again, giving the new readings. Return to the R.M. on the changed face and swing and give the reading of the horizontal circle. If the booker records the clock-times of the star pointings, the clock error on L.S.T. can also be computed from the observation.

If the observation is to be made on the sun, it is necessary to have the Greenwich time to within, say, a minute, as the sun's declination has to be interpolated. The sun presents a disc of about half a degree in diameter; a rough determination of azimuth, to about 1' of angle, can be made by bisecting the disc with the vertical wire, and following it up until it is also bisected by the horizontal wire, just as in the case of a star. A skilled observer will in this way get results good enough for many purposes. But for accurate work the pointings should be made so that the wires touch the edge or 'limb' of the disc, one pointing in each of two

opposite quadrants, thus: I  II 

By taking the mean of the altitude readings so observed, one being on the upper limb U.L. and one on the lower limb L.L., the altitude of the centre is got without any correction in altitude for the sun's semi-diameter, and it may be taken, without serious error, as being the altitude reading when the sun's centre is at the mean of the horizontal circle readings, one of which is of course on the right limb and one on the left. Actually, the difference in azimuth between the sun's centre and the limb is not quite the same for the two pointings, as it varies with the sun's altitude, being in fact $s \cdot \sec h$, where s is the sun's angular semi-diameter, and h his altitude, as will be shown presently. Consequently, the sun's centre is nearer in azimuth to the vertical wire for that pointing which is made at the lower altitude than for the other and higher pointing. The difference is negligible for ordinary altitudes of observation. *Observing for azimuth.* A suitable order of observing is:

For a star

F.R. swing right on to R.M.; read horizontal circle.

F.R. on star at intersection of wires; read altitude level E and O, vertical circle and horizontal circle.

F.L. on star as for F.R.

F.L. swing left on to R.M.; read horizontal circle.

For the sun

F.R. swing right on to R.M.; read horizontal circle.

F.R. on sun thus $\begin{array}{c} \circ \\ | \\ - \end{array}$; read level E and O, vertical circle and horizontal circle.

F.L. on sun thus $\begin{array}{c} | \\ | \\ \circ \end{array}$; read as on F.R.

F.L. swing left on to R.M.; read horizontal circle.

The clock time should also be read, for the interpolation of the sun's declination.

Sometimes, for greater accuracy, a complete observation is made on each face, e.g.

$\left. \begin{array}{l} \text{F.L. on R.M.} \\ \text{F.L. on sun } \begin{array}{c} \circ \\ \\ - \end{array} \\ \text{F.L. on sun } \begin{array}{c} - \\ \\ \circ \end{array} \\ \text{F.L. on R.M.} \end{array} \right\}$	followed by	$\left\{ \begin{array}{l} \text{F.R. on R.M.} \\ \text{F.R. on sun } \begin{array}{c} \\ \\ \circ \end{array} \\ \text{F.R. on sun } \begin{array}{c} \circ \\ \\ - \end{array} \\ \text{F.R. on R.M.} \end{array} \right.$
---	-------------	---

This takes longer and involves more computation, but is more accurate.

The azimuth of the star at the instant of observation is computed by the solution of the triangle PZX for the azimuth angle PZX or Z. The three sides of the triangle are known, for

PZ = c , the co-latitude, supposed known

PX = p , the star's polar distance = $90^\circ - \delta$ from the S.A.

ZX = z , the star's zenith distance = $90^\circ - h$, from the observation

For the solution, any of the formulae

$$\sin \frac{Z}{2} = \sqrt{\frac{\sin(s-c) \sin(s-z)}{\sin c \cdot \sin z}}$$

$$\cos \frac{Z}{2} = \sqrt{\frac{\sin s \cdot \sin(s-p)}{\sin c \cdot \sin z}}$$

$$\tan \frac{Z}{2} = \sqrt{\frac{\sin(s-c) \sin(s-z)}{\sin s \cdot \sin(s-p)}}$$

may be used. None of them has any great advantage over the others, but the tan formula is perhaps the best, all things considered.

The difference in azimuth between the sun's centre and the limb is $s \cdot \sec h$, where s is the angular semi-diameter. It is proved thus:

In Fig. 62

ZX is the vertical circle through the centre X

ZA is the vertical circle touching a limb at A

XA is the semi-diameter s

XZA is the difference ΔZ in azimuth between centre and limb.

$$ZX = z = 90^\circ - h$$

$$ZAX = 90^\circ$$

$$\frac{\sin XA}{\sin \Delta Z} = \frac{\sin ZX}{\sin ZAX}$$

$$\therefore \frac{\sin s}{\sin \Delta Z} = \cos h, \text{ and } \sin \Delta Z = \sin s \cdot \sec h$$

or approximately, as ΔZ and s are small angles,

$$\Delta Z'' = s'' \cdot \sec h$$

This is analogous to the correction $c'' \cdot \sec h$ for collimation error.

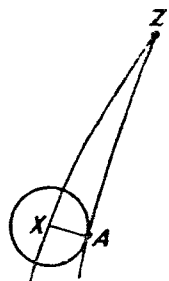


FIG. 62

Example of an azimuth determination by altitude of a star

Latitude + 29° 10' 50"	Star α Lyrae (Vega)	δ + 38° 43' 50"
Data from angle book:		
Mean reading of hor. circle, on R.M.	359° 59' 36"	p 51° 16' 10"
ditto on star	30 38 48	ϕ 29 10 50
Mean reading of vert. circle on star	30 37 47	c 60 49 10

$$\text{Formula: } \tan \frac{Z}{2} = \sqrt{\frac{\sin(s-c) \sin(s-z)}{\sin s \cdot \sin(s-p)}}$$

					$\log \sin$
Observed alt.	30° 37' 47"	s	85 34 35	.	$\overline{1.9988002}$
Refraction r	1 38	$s-p$	34 28 25	.	$\overline{1.7528365}$
Corrected alt. h	30 36 09	$\log \sin s \cdot \sin(s-p)$.	$\overline{1.7516367}$
z	59 23 51	colog	.	.	0.2483633
c	60 49 10	$(s-c)$	24° 55' 25"	.	$\overline{1.6247043}$
p	51 16 10	$(s-z)$	26 20 44	.	$\overline{1.6471715}$
$2s$	171 29 11			2)	$\overline{1.5202391}$
s	85 44 35	$\log \tan$	29° 55' 29"		$\overline{1.7601195}$
		$\frac{Z}{2}$	29° 55 29		
		Z	59 50 58		

Reading on R.M.	359° 59' 36"
" " star	30 38 48
Angle, R.M. to star	30 39 12
Angle, star to north (Z)	59 50 58
Angle R.M. to north	90 30 10
True bearing of R.M.	269 29 50

Azimuth by hour angle. Azimuth may be determined from a star or from the sun without measuring the altitude, provided that the local time be known with an accuracy corresponding to the required precision of azimuth. From the local time the hour angle of the star or the sun is deduced, and from this, with the known co-latitude and star's polar distance, the triangle PZX is solved for the azimuth, viz. the angle PZX.

The observation is easier to make than the azimuth observation by altitude, in that the celestial body has to be observed on only the vertical wire, with a simultaneous clock reading. It is free from uncertainty of refraction, as altitudes do not enter into it. The most likely source of error is in the time. Such other sources of error as may affect the result are present also in observations for azimuth

by altitude, e.g. instrumental and personal errors and inaccuracy of the assumed latitude. Errors arising from inexact time and inexact latitude have least effect when the observed celestial body is far from the meridian. Balancing by east and west stars, or by morning and afternoon sun, is also advisable, though it will not eliminate an error due to inaccurate time.

The order of observing may be, for example,

For a star, F.L. on R.M., read horizontal circle

F.L. on star, read clock and horizontal circle

F.R. on star, read clock and horizontal circle

F.R. on R.M., read horizontal circle

For the sun, F.L. on R.M., read horizontal circle

F.L. on sun \odot , read clock and horizontal circle

F.R. on sun \odot , read clock and horizontal circle

F.R. on R.M., and read horizontal circle

The procedure is to point carefully on the R.M. and read the horizontal circle. Then swing to the star, setting the telescope so that the star will cross the vertical wire in a few seconds, near the cross of the wires. Remove the hand from the instrument, warn the booker to stand by at the clock, and give him the word 'up' the instant the star is right on the vertical wire. He will book the time, starting by putting down the seconds and the decimal thereof if any, and will then be ready to book the horizontal circle reading or readings, according to the type of instrument.

Change face and observe the star again as described. Then return to the R.M. on this face, on the opposite swing to the first pointing on the R.M. The stars selected should be at a low altitude, and such as have only a slow motion in azimuth. In each case the mean of the horizontal circle readings on sun or star is taken as being the circle reading at the mean of the times of the pointings.

It is assumed that the latitude and longitude and the U.T. are known. From the U.T. the Greenwich hour angle of the sun or star is deduced, as

$$\text{G.H.A. sun} = \text{U.T.} + E$$

and

$$\text{G.H.A. star} = \text{U.T.} + R - \text{R.A. star.}$$

Then the local hour angle is got from the G.H.A. by adding or subtracting the longitude, according to whether the station is east or west of Greenwich. The azimuth of the star is then got by solving

the triangle ZPX, Fig. 60, in which $PX = p$, $ZP = c$ and ZPX the hour angle are known.

In Fig. 63, XM is part of the great circle through X perpendicular to the meridian. XM when produced passes through the east and west points E and W of the horizon, as these points are the poles of the meridian. The azimuth of X, reckoned the shortest way from the meridian, is $PZX (= Z)$. It can be computed in various ways. A convenient formula is

$$\cot Z = \frac{\sin (c - x)}{\sin x} \cdot \cot P$$

where x is the arc PM and is derived from

$$\tan x = \tan p \cdot \cos P$$

The proof of this is as follows:

Applying the cot formula to the four consecutive parts $p \cdot P \cdot x \cdot 90^\circ$ of the triangle PMX

$$\begin{aligned} \cos P \cdot \cos x &= \sin x \cot p - \sin P \cot 90^\circ \\ \therefore \tan x &= \tan p \cdot \cos P, \text{ as } \cot 90^\circ = 0 \end{aligned} \quad \text{I}$$

The cot formula, applied to the four consecutive parts Z.ZM, 90° , MX of the triangle ZMX, gives

$$\begin{aligned} \cos ZM \cdot \cos 90^\circ &= \sin ZM \cdot \cot MX - \sin 90^\circ \cot Z \\ \cot Z &= \sin ZM \cdot \cot MX \end{aligned}$$

and, as $ZM = PZ - PM, = c - x$

$$\cot Z = \sin (c - x) \cot MX \quad \text{II}$$

The same formula, applied to the four consecutive parts $P \cdot x \cdot 90^\circ$ MX of the triangle PMX, gives

$$\begin{aligned} \cos x \cdot \cos 90^\circ &= \sin x \cdot \cot MX - \sin 90^\circ \cot P \\ \text{or } 0 &= \sin x \cdot \cot MX - \cot P \\ \cot MX &= \frac{\cot P}{\sin x} \end{aligned} \quad \text{III}$$

and, from II and III,

$$\cot Z = \sin \frac{(c - x)}{\sin x} \cdot \cot P$$

x is, as already stated, first evaluated from I, viz.

$$\tan x = \tan p \cdot \cos P$$

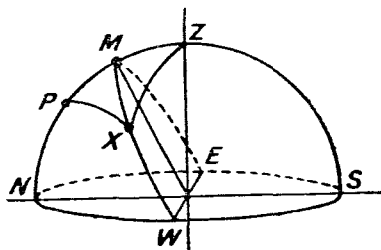


FIG. 63

If $x = c$, M coincides with Z and the star X is on the prime vertical EZW, and $\cot Z = 0$ and $Z = 90^\circ$

If $x > c$, M lies on the opposite side of Z to P, $(c - x)$ is negative, $\cot Z$ is negative and $PZX > 90^\circ$

Example of an azimuth from sun's hour angle

Watch slow on U.T. $3^m 17^s \cdot 2$				Latitude of station $+ 50^\circ 52' 52''$	
				Longitude, W. $7^m 08^s \cdot 9$	
Face	Object	Watch h m s	Hor. circle ° ' "	Sun's declination, $\delta + 23^\circ 21' 25'' \cdot 2$	
R	R.M.	—	11 34 52	,, N. polar dist., $p 66 38 34 \cdot 8$	
R.	O	8 55 56.8	257 17 27	Co-latitude, $c 39 07 08$	
L.	O	8 58 48.6	77 15 57	E $11^h 57^m 17^s \cdot 5$	
L.	R.M.	—	191 34 56		
Mean hor. circle, on R.M. 11 34 54				The values of δ and E are interpolated from the tabular values	
Mean hor. circle, on sun 257 16 42					
Angle, sun to R.M. <u>114 18 12</u>				Formula: $\tan x = \tan p \cdot \cos P$	
				$\cot Z = \frac{\sin(c - x)}{\sin x} \cdot \cot P$	
Mean of watch $8^h 57^m 22^s \cdot 7$				log tan p 0.364669	
slow on U.T. 3 17.2				log cos P 1.831349	
U.T. of observation 9 00 39.9				log tan x 0.196018	
E 11 57 17.5				x $57^\circ 30' 44''$	
G.H.A. sun 20 57 57.4				$c - x$ $- 18 23 36$	
long. W. 7 08.9				log sin($c - x$) 1.4990527	
L.H.A. sun 20 50 48.5				log cot P 1.965127	
Sun's H.A. east 3 09 11.5				log cosec x 0.073912	
ZPX(P) <u>47° 17' 52".5</u>				log cot Z 1.5380917	
				Z $109^\circ 02' 44''$	
				Angle, N to sun 109 02 44	
				Angle, sun to R.M. 114 18 12	
				Bearing of R.M. <u>223 20 56</u>	

The effect of the dislevelment or tilt of the horizontal axis is shown as follows, with reference to Fig. 64

HAR is the horizon and Z the zenith.

H'OR' is the trunnion axis, inclined at an angle $H'OH = b$ to the horizon when a point X is observed on the cross-wires. It

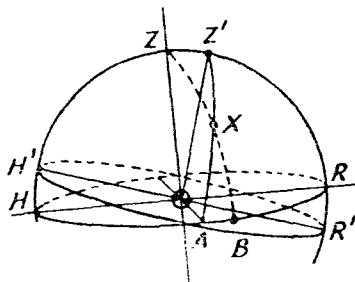


FIG. 64

is assumed that there is no collimation error. When the telescope is turned about the trunnion axis, the axis of collimation sweeps out the great circle AXZ' , cutting the horizon in A. Z' is the pole of the great circle $H'AR'$.

The vertical circle through X cuts the horizon in B, which point determines the real azimuth of X. But, owing to the tilt of the trunnion axis, the point X is referred to the horizon at the point A. The arc AB accordingly represents the error δZ in the measured horizontal angle between the reference mark and the point X.

In the spherical triangle XAB

XA = the altitude h' of X above the 'false horizon' $H'AR'$

XB = the true altitude h

$$\angle XBA = 90^\circ$$

$$\text{and } \angle XAB = \angle XAR' - \angle BAR' = 90^\circ - b$$

Four consecutive parts are XB, XBA, BA, BAX.

Applying the cot formula or Napier's rules to these gives

$$\cos AB \cdot \cos XBA = \sin AB \cot XB - \sin XBA \cdot \cot BAX$$

$$\text{or } \cos \delta Z \cos 90^\circ = \sin \delta Z \cot h - \sin 90^\circ \cot (90^\circ - b)$$

$$0 = \sin \delta Z \cot h - \tan b$$

$$\text{and } \sin \delta Z = \tan b \cdot \tan h$$

As δZ and b are both small quantities

$$\delta Z'' = b'' \cdot \tan h$$

As explained, b'' is measured by the striding level (direct and reversed); and the correction $b'' \tan h$ is additive to the horizontal circle reading when the higher end of the trunnion axis is on the observer's left, and subtractive from the circle reading when the higher end is on his right, as in Fig. 64, where the actual reading is that of the point A, whereas it ought to be that of B which is less.

Further, in the spherical triangle AXB

$$\frac{\sin AX}{\sin ABX} = \frac{\sin BX}{\sin BAX}, \text{ or } \frac{\sin h'}{\sin 90^\circ} = \frac{\sin h}{\sin (90^\circ - b)}$$

$$\sin h = \sin h' \cdot \sin (90^\circ - b) = \sin h' \cdot \cos b$$

$$= \sin h' \left(1 - \frac{b^2}{2} + \dots \right)$$

Accordingly $h = h'$, neglecting small quantities of the second order; that is, a small tilt of the trunnion axis has no appreciable effect on measured altitudes.

It was stated that the effect of a collimation error c'' on an

azimuth reading taken on a point at an altitude h is $c'' \sec h$. What is meant by the effect of a collimation error on a horizontal circle reading is: the difference between the actual reading on a point, and what that reading would be if there were no collimation error. That this is $c'' \sec h$ is shown thus:

In Fig. 65, let HOR represent the position of the horizontal or trunnion axis when a point X is sighted on the cross-wires of a theodolite having a collimation error c , the line of sight lying c'' to the observer's left of the true collimation axis, which is perpendicular to HR. The vertical circle ZA is swept out by the true collimation axis as the telescope is turned about HR, but the line of sight describes the small circle BZ' parallel to ZA and at a distance c'' from it,

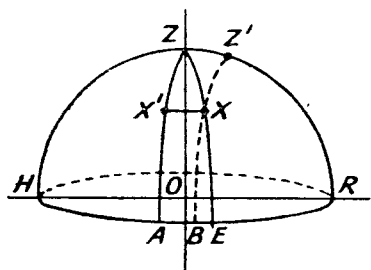


FIG. 65

where $AB = c''$. Draw XX' perpendicular to ZA ; then XX' also $= c''$.

In the spherical triangle ZXX'

$$\frac{\sin XX'}{\sin XZX'} = \frac{\sin ZX}{\sin ZX'X}$$

or

$$\frac{\sin c}{\sin \delta Z} = \frac{\sin z}{\sin 90}$$

and

$$\sin \delta Z = \frac{\sin c}{\sin z} = \sin c \cdot \sec h$$

As c is a small angle, δZ is also small and

$$\delta Z'' = c'' \sec h$$

The error in the horizontal circle reading on a point X at an altitude h is therefore $c'' \cdot \sec h$. When the horizontal angle between a reference mark R.M. and a star is being measured, the correction $c'' \sec h$ is, of course, applicable to both horizontal circle readings; if the R.M. is on the horizon, $h = 0^\circ$ and the correction is c'' . It is additive to the circle reading if the line of sight lies to the observer's right of the true collimation axis, and subtractive from the reading if the line of sight lies to his left, as in Fig. 65, with the usual clockwise graduation of the circle. It is easily seen that a small collimation error has no appreciable effect on measured altitudes. Usually,

both errors b and c will exist simultaneously; the corrections for them are applied separately, as their mutual effect is of the second order of smallness.

Change of face, by changing $+c''$ to $-c''$, eliminates the effect of collimation error, except in so far as a considerable change in the altitude of the celestial object affects $c \cdot \sec h$. This is usually negligible.

The symbols b and c for the tilt of the trunnion axis and the collimation error respectively originate in the use of the transit instrument for star transits over the meridian, for which purpose the trunnion axis ought of course to lie in the east-west line. There is usually a small error or departure from this E.W. position. For this error the symbol a is used; b and c are as already explained.

AZIMUTH BY A STAR AT OR NEAR ELONGATION

A favourable condition for azimuth determination is that the motion in azimuth of the body observed should be slow, because the effect of an error in the measured altitude or in the recorded time, whichever is used as the basis for the computation, is then correspondingly small.

Stars near the elevated celestial pole have, at two points in their diurnal paths, no motion at all in azimuth, their motion being

entirely in altitude. They are accordingly very suitable for azimuth determinations. This will be understood from the diagram, Fig. 66, in which ZN represents part of the celestial meridian, P the elevated pole, and the dotted small circle the diurnal path of a circumpolar star of polar distance PA or PB . The vertical circles ZAC and ZBD are tangential, at A and B respectively, to the dotted circle, and at these points the motion of the star, coinciding with the vertical circles ZA and ZB , is entirely in altitude, its azimuth being respectively the arc NC or $360^\circ - ND$, its whole

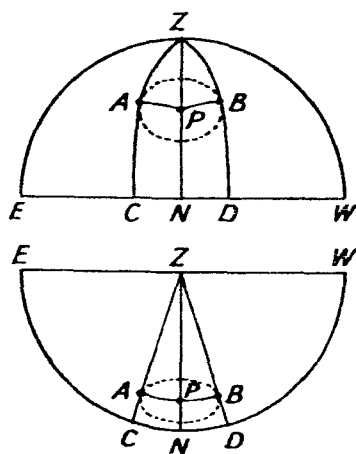


FIG. 66

range of motion in azimuth being the arc CD, or the angle CZD. At A the star is said to be at *Eastern elongation*, and at B, at *Western elongation*. Any star which has its upper transit between P and Z elongates, but usually only those near the celestial pole are called circumpolar stars.

The azimuth, hour angle and altitude of a star are easily computed for the instant of elongation, when the star's co-ordinates and the latitude of the station are known. The angle ZAP of the spherical triangle ZAP is 90° , as ZAC is tangential to the small circle.

PA = p , the star's polar distance

PZ = c , the co-latitude

$$\frac{\sin PA}{\sin PZA} = \frac{\sin PZ}{\sin 90^\circ}, \text{ i.e. } \sin PZA = \frac{\sin p}{\sin c} = \frac{\cos \delta}{\cos \phi} = \cos \delta \sec \phi \quad (a)$$

which gives the azimuth at elongation.

For the hour angle, the four consecutive parts PZ, ZPA, PA, 90° give:

$$\cos PA \cdot \cos ZPA = \sin PA \cdot \cot PZ - 0$$

$$\text{or} \quad \cos p \cdot \cos ZPA = \sin p \cdot \cot c$$

$$\therefore \cos (\text{hour angle}) = \tan p \cdot \cot c$$

$$\text{or} \quad \cos t = \cot \delta \cdot \tan \phi \quad . \quad . \quad . \quad (b)$$

For the altitude h , or zenith distance z

$$\cos PZ = \cos PA \cdot \cos ZA$$

$$\text{i.e.} \quad \cos c = \cos p \cdot \cos z$$

$$\sin h = \cos z = \frac{\cos c}{\cos p} = \frac{\sin \phi}{\sin \delta} = \operatorname{cosec} \delta \sin \phi \quad . \quad (c)$$

It is evident that $\angle ZPA$ (Fig. 66), the east or west hour angle at elongation, will always be less than 90° or 6^h , the defect from 90° increasing with increase of latitude ϕ and with increase of the star's polar distance p . The Nautical Almanac gives a table of the 'Azimuth of Polaris' to 0.1 for intervals of 10^m hour angle, for latitudes from $+10^\circ$ to $+70^\circ$ advancing by suitable intervals; from this table the approximate hour angle and azimuth of elongation, i.e. of maximum azimuth, can be seen by inspection.

From the Pole Star Table in the Star Almanac the azimuth of Polaris at any L.S.T., in any latitude from 0° to 66° N and on any date in the year, can be found quickly and conveniently, to an accuracy of about 0.2 .

An example of the computation follows:

Find the Standard Meridian Time (S.M.T.) of the western elongation of Polaris for a station in latitude $+27^{\circ} 12' 35''$ and longitude $31^{\circ} 30' 30''$ E on 1952, Nov. 30. The standard meridian is 30° E. Find also the azimuth at elongation.

From the Star Almanac the co-ordinates of the star for the date are, R.A. $1^{\text{h}} 51^{\text{m}} 31^{\text{s}}$, declination $+89^{\circ} 02' 56''$.

$\cos t = \cot \delta \cdot \tan \phi$	H.A. (computed)	$5^{\text{h}} 58^{\text{m}} 03^{\text{s}}$
$\log \cot \delta \quad \bar{2} \cdot 22015$	R.A.	1 51 31
$\log \tan \phi \quad \bar{1} \cdot 71108$	L.S.T.	7 49 34
$\log \cos t \quad \bar{3} \cdot 93123$	Longitude, E, in time	2 06 02
$t \quad 89^{\circ} 30' 39''$	G.S.T.	5 43 32
H.A. (W) $5^{\text{h}} 58^{\text{m}} 03^{\text{s}}$	G.S.T. at U.T. 00 ^h	4 35 13.7
	Sidereal interval	1 08 18.3
	Deduct for M.T.	11.2
	U.T.	1 08 07.1
	Long. of S.M. (E)	2 00 00
	S.M.T.	3 08 07.1

The azimuth at elongation, PZA, is given by

$$\begin{aligned} \sin \text{PZA} &= \cos. \delta \cdot \sec \phi \\ \log \cos \delta &\quad \bar{2} \cdot 22008 \\ \log \cos \phi &\quad \bar{1} \cdot 94907 \\ \log \sin \text{PZA} &\quad \bar{2} \cdot 27101 \\ \text{PZA} &= 1^{\circ} 04' 10'' \end{aligned}$$

For a star near the celestial pole, the motion in azimuth, which is zero at elongation, is very slow for several minutes before and after elongation. Accordingly, as the elimination of instrumental errors is so important, an observation may be made on both faces, one pointing being before and one after elongation, without introducing any serious error. The azimuth of the star at elongation is then computed by (a) above, and the mean of the F.L. and F.R. readings on the reference mark is taken in conjunction with the azimuth so computed; from them the bearing of the R.M. is then deduced. This will be accurate enough for most field purposes; the procedure in observing and the computation are both simple. For a star within, say, 20° of the pole, i.e. if $\delta > 70^{\circ}$, the effect of any likely error in the latitude is negligible in ordinary field work.

Error due to an inaccurate latitude for an azimuth by a star at elongation may be eliminated by observing at both the eastern and western elongations, as both computed azimuths will be too near to, or too far from, the meridian by the same amount on opposite sides of the meridian. It may be inconvenient or impossible to observe the same star at both elongations; the error may, however, be reduced by observing two stars, preferably of nearly the same

declination, one at eastern and the other at western elongation. In high latitudes elongation may take place at an inconveniently high altitude, as the altitude at elongation is always greater than the latitude, i.e. the altitude of the pole. The $b \tan h$ striding level correction becomes correspondingly important.

An azimuth observation on a circumpolar star may be made at any time by the altitude or hour angle method. The most favourable time is at or near elongation, when the star is at its furthest from the meridian. The computation will then be done by the methods already shown.

A series of F.R. and F.L. pointings may be made on the star. The true azimuth for each pointing may be computed separately and applied to the R.M. reading to find the bearing of the R.M., and the mean of the resulting bearings taken as being the observed bearing. If the altitude is to be taken as data for computing the azimuth, the star must be observed on the intersection of the wires, and the altitude read off on the vertical circle and altitude level; if the hour angle is to be used as data, only the clock time need be taken, its error being known or determined by E and W stars near the P.V.

But to reduce the amount of computing in the hour angle method, the azimuth may be computed for the mean of all the hour angles; the azimuth computed for this mean of the times is always slightly further from the meridian than the mean of the separately computed azimuths, and a correction called the *curvature correction* ought to be applied to it to allow for this. This correction is always negative; it is:

$$- \tan Z \cdot \frac{1}{n} \cdot \sum \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

where Z is the azimuth computed for the mean of the times

n is the number of pointings

and t is the interval between the time of each pointing and the mean of the times of all the pointings.

The quantity $\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$ occurs also in the *reduction to meridian* of

circum-meridian altitudes. It is to be found in tables. The necessity for the curvature correction arises from the proximity of the star to the pole, causing its path to be appreciably curved, and its motion in azimuth to vary accordingly.

METHOD OF EQUAL ALTITUDES. At equal hour angles before and after transit, the altitudes of a star are equal, as are also the horizontal angles between it and the meridian, except for such slight change as may take place in the star's co-ordinates, which change is quite negligible for most purposes. This circumstance provides a simple means of determining both the clock error on L.S.T. and the direction of the meridian. The mean of the clock times of equal altitude is the clock time of meridian passage, at which instant the L.S.T. is equal to the star's R.A.; and the direction of the meridian is midway between the two directions of equal altitude.

Equal altitudes of a star. The method is simple in principle. The actual altitude of the star does not enter into it, the only condition being the equality of the east and west altitudes. There is no need to reverse the instrument nor to change the setting, but only to see that the altitude level shows by its bubble position that the line of sight has the same inclination. It is assumed that the refraction has not changed. The method is independent of latitude and also of declination, which is nearly enough constant, for a star. There are no vertical circle readings; for an azimuth determination no watch is necessary, and for a time determination there are no circle readings at all; and there is little or no computation in either case.

There are, however, certain practical drawbacks which cause the method to be less frequently used, with the theodolite at least, than might be expected. Unless the star is crossing the horizontal wire fairly quickly, it is difficult to mark the precise instant of the altitude being attained; also the rate of change of azimuth ought to be slow. This implies that the star should be near the prime vertical. Further, there will be a long wait between the east and west pointings unless the star is at a high altitude, which is not always convenient for observing and may cause a number of errors which are not eliminated when the observation is all done on one face. The striding level correction $b \tan h$ and the collimation correction $c \sec h$ can of course be applied.

Equal altitudes of the sun. The method of equal altitudes can be applied to the sun for both time and azimuth, in which case, owing to the changing declination, a correction has to be applied to the mean of the clock times of the east and west passages over the horizontal wire to obtain the clock time of meridian passage. A different correction is applicable to the mean of the horizontal

circle readings to obtain the reading for the meridian. These corrections are:

(a) For the time of meridian passage

$$c^s = \frac{-\Delta\delta \cdot \tan \phi}{15 \sin t} + \frac{\Delta\delta \cdot \tan \delta}{15 \tan t}$$

where $\Delta\delta$ is the increase of δ from transit to west observation, i.e. $\frac{1}{2}(\delta_W - \delta_E)$, and t is half the elapsed time between the observations. This is equivalent to

$$c^s = -\frac{(\delta_W - \delta_E)}{2 \times 15} (\tan \phi \cdot \operatorname{cosec} t - \tan \delta \cdot \cot t)$$

(b) For the horizontal circle reading at meridian passage, the correction to the mean of the two horizontal circle readings is:

$$c'' = -\frac{1}{2} (\delta_W - \delta_E)'' \sec \phi \cdot \operatorname{cosec} t$$

The above corrections are derived on the assumption that $(\delta_W - \delta_E)$ is small, as it is in the case of the sun. But it may be stated that, for time and azimuth determinations by the sun, the general preference for other methods is fully justified. A rough determination of the meridian may be made by marking the directions of the forenoon and afternoon equal-length shadows of a vertical rod or of a vertical gnomon on a plane table, and bisecting the angle between these equal-length shadows. This is an ancient method.

Azimuth by equal altitudes of two stars. To avoid the long wait between the east and west passages of a star over the horizontal wire, the azimuth can be determined very well by observing two stars at equal altitudes, one east and one west of the meridian. The procedure, after a reading has been taken on the R.M., is to observe an east star on the intersection of the wires, at a convenient altitude, read the horizontal and vertical circles, then swing round to the west, without changing the inclination of the telescope. If necessary, bring the altitude level bubble back to its original place, and select a star which will shortly cross the horizontal wire, preferably at about the same distance from the meridian. Intersect it with the vertical wire as it does so; then read the horizontal and vertical circles again—the latter in order to make sure that the inclination has not been inadvertently altered. There are no watch readings, nor is a change of face necessary; a small error in the altitude will not greatly affect the result.

A formula for computing the azimuth is derived as follows:

In Fig. 67, X_1 and X_2 are two stars at equal zenith distances

$$ZX_1 = ZX_2 = z$$

$$h = 90^\circ - z$$

δ_1 and δ_2 are their declinations

ϕ is the latitude

A_1 and A_2 are their azimuths from north,

both considered positive.

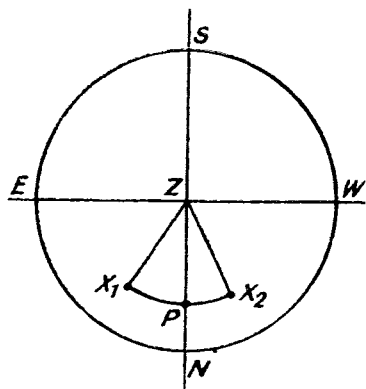


FIG. 67

We have $\cos PX_1 = \cos PZ \cdot \cos ZX_1 + \sin PZ \cdot \sin ZX_1 \cos A_1$

$$\cos PX_2 = \cos PZ \cdot \cos ZX_2 + \sin PZ \cdot \sin ZX_2 \cos A_2$$

i.e. $\sin \delta_1 = \sin \phi \cdot \sin h + \cos \phi \cdot \cos h \cdot \cos A_1$

$$\sin \delta_2 = \sin \phi \cdot \sin h + \cos \phi \cdot \cos h \cdot \cos A_2$$

$$\sin \phi_1 - \sin \delta_2 = \cos \phi \cdot \cos h \cdot (\cos A_1 - \cos A_2)$$

$$2 \cos \frac{\delta_1 + \delta_2}{2} \sin \frac{\delta_1 - \delta_2}{2} = \cos \phi \cos h \left(-2 \sin \frac{A_1 + A_2}{2} \sin \frac{A_1 - A_2}{2} \right)$$

$$\sin \frac{A_1 - A_2}{2} = \frac{-\cos \frac{\delta_1 + \delta_2}{2} \sin \frac{\delta_1 - \delta_2}{2}}{\cos \phi \cdot \cos h \cdot \sin \frac{A_1 + A_2}{2}}$$

Now $A_1 + A_2$ is known from the horizontal circle readings, being the reading on X_1 (increased by 360° if necessary) — reading on X_2

$\frac{A_1 - A_2}{2}$ can therefore be computed, $\frac{A_1 + A_2}{2}$ being known

A_1 and A_2 are respectively the sum and difference of these two

angles. It is believed that this method was introduced by the late Dr. John Ball of the Egyptian Survey Department.

When the difference between the declinations is small, the correction to the mean of the readings on X_1 and X_2 reduces to that given above for equal altitudes of the sun.

It is evident from Fig. 67, that if p_1 is greater than p_2 , i.e. if $\delta_1 < \delta_2$, A_1 will be greater than A_2 . This will apply equally to northern and southern latitudes, provided that the azimuths A_1 and A_2 and the polar distances p_1 and p_2 are in each case reckoned from the elevated pole.

ASTRONOMICAL POSITION LINES. The finding of geographical position by *astronomical position lines* ought to be understood by the surveyor.

A position line, in general, is a line on the map or chart such that the surveyor, from the observed bearing from him of a fixed identifiable point shown on the chart, or otherwise, knows that his position is on that line. If he has two position lines, he must lie at their intersection. Such a determination of position is of course liable to errors due to various causes well known to him, but in principle it is accurate. For such purposes the curvature of the earth's surface is negligible over small areas.

An astronomical position line is one obtained from an observed and corrected altitude of a star or of the sun, at a known instant of

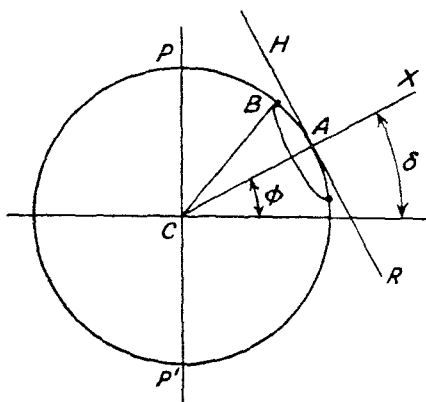


FIG. 68

Greenwich time. The basis of the method is simple. To explain it, the earth will be assumed spherical, with the horizon at any point

a tangent plane at that point to the sphere; the radius produced will pass through the zenith of the point.

A straight line from C the centre of the earth, Fig. 68, towards a star X will meet the surface in a point A, and X is in the zenith of A. The latitude ϕ of A is identical with the declination δ of the star X. A is called the *substellar point*, or the geographical position, of X at the instant; the rotation of the earth will carry each point of the parallel of latitude ϕ in turn under the star X, but at the instant represented the zenith distance of X is 0° at A, and its altitude 90° . A place B such that AB subtends say 20° at C has its zenith 20° from X; the zenith distance of X is therefore 20° at B, and its altitude 70° . As B may be taken in any direction from A, it is clear that the locus of points at which the zenith distance of X is 20° is a small circle on the earth, with centre at A and a radius subtending 20° at C; this is a *position circle*. This is of course general and applicable to any zenith distance, z . The position circle is perfectly definite when the star's altitude and the Greenwich time are known. At any point within the position circle, the altitude of X is greater than at B; outside the position circle the altitude is less. The geographical co-ordinates of the sub-stellar (or sub-solar) point are not required in working out position lines.

They are: latitude = declination of the celestial object
 longitude = G.H.A. of the same.

This applies whether the earth be regarded as spherical or spheroidal.

If the star's azimuth were known, in addition to the altitude and the Greenwich time, it would be possible to compute the geographical position of the observer, as the bearing and distance of the substellar point would both be known, the bearing being identical with the star's azimuth. The observation, however, does not supply enough data for computing the azimuth; further, the result would be liable to considerable error due to even a small error in the azimuth, as the radius of the position circle is likely to be several thousand miles, since there is 1 nautical mile for each minute of arc of zenith distance.

A single altitude of a star will tell the observer that he is on a certain position circle, but it will tell him nothing more, and even for this he must have the Greenwich time of his altitude observation. To fix his position a second position line is required, and this can be got by taking the altitude of a second star, preferably at

such an azimuth that the two position circles cut at such an angle as will give a well-defined point of intersection on the chart; i.e. the two stars should be about 90° apart in azimuth. The observer's position must be at one or other of the two points of intersection of these two position circles.

The determination by computation of the points of intersection of two small circles, the position circles, would be tedious, and is in fact unnecessary. Further, owing to the fact that the earth is not quite spherical, but is spheroidal, the position lines would not be truly circular. A different procedure is accordingly adopted, a semi-graphical one, in which a short portion of the astronomical position line is plotted as a straight line on the map or chart, which is nearly enough correct provided that the radius, i.e. the zenith distance of the star, is not a small one. The map or chart may be merely a grid or graticule drawn by the surveyor for his immediate purpose, and on it the minutes of latitude and longitude will be plotted to a suitable scale of distance.

The position line, representing a short portion of the position circle, must be at right angles to the direction to the star observed, i.e. to the radius of the position circle. There are two ways in which the position line may be found, the longitude method and the St. Hilaire method; an outline of each will be given.

Position lines by the longitude method. In finding the position line by the longitude method, a latitude, known to be nearly correct, is assumed; the corresponding longitude is then computed just as in an ex-meridian observation for time. The azimuth is also computed. The position line is then drawn, from the assumed latitude and the computed longitude, at right angles to the star's azimuth. The position line so drawn will cut each adjacent parallel of latitude at the longitude which would have been found if the latitude of that parallel had been assumed for the longitude computation.

This circumstance was first noticed and appreciated by Capt. Sumner of the U.S. Navy, in 1837; hence the position lines are often called *Sumner lines*. Two such position lines intersecting at a suitable angle will fix the position, the latitude and longitude being scaled off the chart. To reduce the effect of errors it is advisable to have four position lines, one from each of the four quadrantal points, NE, SE, SW and NW; and to estimate the true position therefrom. This is the usual practice with the prismatic astrolabe.

The four position lines will not usually intersect at a point, but will enclose a quadrilateral. This is because of unavoidable errors.

Position lines by the St. Hilaire method. In the St. Hilaire method, both latitude and longitude are assumed (the position being known approximately), and the zenith distance z_c of the celestial body is computed for that assumed position for the known instant of the observation. This will in general not agree exactly with the observed and corrected zenith distance z_o ; if it does not, it means that the assumed position is either too far from the substellar point or too near it, according to whether the computed zenith distance is greater than or less than the observed and corrected z.d. This is easily seen by consideration of Fig. 68. Motion towards A lessens the zenith distance of X and receding from A increases it. For instance, in Fig. 69 let P be the assumed position, z_c the computed

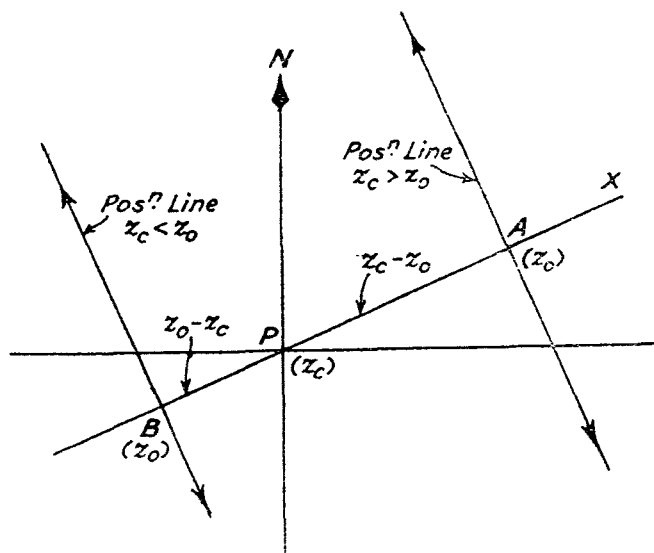


FIG. 69

z.d. of X at P at the instant of observation, and z_o the corrected z.d. got from the observation. The position line is at right angles to PX, the direction to the star (which may be found either by computing the star's azimuth for the time or roughly, from its

observed compass bearing, duly corrected), and it passes through a point A on PX such that PA represents the arc $z_c - z_o$. If z_c is less than z_o a point B is set off on XP produced to represent the intercept $z_o - z_c$ to scale, and the position line is as shown in Fig. 69.

The same position line should be arrived at whether the longitude or the intercept method is used.

Alternative methods for finding the azimuth are

(a) by altitude and azimuth tables,

(b) by Weir's azimuth diagram,

if these are available.

The following *example* shows the method of working out and plotting by the intercept method.

The position of a station is known to be approximately latitude S $37^\circ 48'$, longitude E $9^h 39^m 40^s$. An observation taken on 1952, Nov. 5, at Standard Meridian Time $22^h 44^m 46^s$ gives the altitude of Rigel β Orionis (S.A. No. 136) as $30^\circ 41' 59''$ after correction for refraction. The Standard Meridian is 10^h east of Greenwich. Plot the position line obtained from the observation.

S.M.T.	22 ^h 44 ^m 46 ^s	Rigel No. 136
Long. of S.M.	— 10	R.A. 5 ^h 12 ^m 17 ^s .9
U.T.	12 44 46	δ — $8^\circ 15' 02''$
G.S.T. at U.T. 12 ^h	14 58 38	p $81^\circ 44' 58''$ (south p.d.)
M.T. interval	44 46	c 52 12 00 (co-lat.)
Acceleration	7.4	$\cos z = \cos p \cdot \cos c + \sin p \cdot \sin c \cdot \cos P$
G.S.T. of observation	15 43 31.4	
Long. E	9 39 40	
L.S.T.	01 23 11.4	
R.A.	5 12 17.9	
H.A.(W)	20 10 53.5	
H.A.(E)	3 49 06.5	
$\angle ZPX(P)$	57 $^\circ$ 16' 37"	
	Computed z.d.	
	Observed z.d.	
	Intercept, from	

The intercept is *from*, or *away*, as the station is further from the substellar point than was assumed.

To compute the star's azimuth the formula

$$\sin \frac{Z}{2} = \sqrt{\frac{\sin(s-c) \sin(s-z)}{\sin c \cdot \sin z}}$$

will be used.

p	81°	44'	58"			
c	52	12	00	.	.	$\log \sin c$ 1.89771
z	59	17	30	.	.	$\log \sin z$ 1.93439
$2s$	193	14	28			1.83210
s	96	37	14			$\csc s$ 0.16790
$s - c$	44	25	14	.	.	$\log \sin$ 1.84505
$s - z$	37	19	44	.	.	$\log \sin$ 1.78275
						2) 1.79570
				$\log \sin$	$\frac{PZX}{2}$	1.89785
				$\frac{Z}{2}$	52° 13' 26"	
				Z	104 26 52	

This is the angle from south, the elevated pole, towards east, as the star is east of the meridian. The azimuth clockwise from north is accordingly

$$Z = 75^\circ 33' 08''$$

Fig. 70 shows the plotting of the position line. In the original thereof the scale is 1.5 mm. to 1" of great circle arc, in all directions, i.e. 1.5 mm. to about 101 feet. The assumed position is in lat. $37^\circ 48' S$, and long. $9^h 39^m 40^s E$ ($= 144^\circ 55' E$). PX is plotted at the computed azimuth of $75^\circ 33'$ from north, and PA the intercept away is $31 \times 1.5 = 46.5$ mm., giving the position line CD. In order to fix the actual position, at least one other position line is needed.

Suppose a star in the direction PX_2 gave an intercept of $24''$ towards. Mark off $PB = 36$ mm. EF at right angles to PX_2 is the second position line and the intersection O is taken to be the actual position. To get its latitude, OM scales 57 mm. and represents $38''$ of arc; the latitude of O is therefore $37^\circ 48' 38'' S$.

For its longitude PM, which scales 32.5 mm., represents its departure from P which is accordingly $32.5 \times \frac{2}{3}$ of g.c. arc (at 1.5 mm. to 1") or 21.6 along the parallel of $37^\circ 48' S$. This represents a difference of longitude of $21.6 \times \sec 37^\circ 48'$, i.e. 21.6×1.26 or $27''$. The longitude of M and of O is accordingly $144^\circ 54' 33'' E$.

The graticule of latitude and longitude has been added in Fig. 70; the latitude lines for $30''$ difference are 30×1.5 mm. = 45 mm. apart and the meridian lines for $30''$ of longitude are $45 \times \cos 37^\circ 48' = 45 \times 0.790$ mm. = 35.5 mm. apart.

In the foregoing example the z.d. was computed from the fundamental formula.

$$\cos z = \cos p \cdot \cos c + \sin p \cdot \sin c \cdot \cos P$$

CHAPTER 8

HYDROGRAPHIC SURVEYING

HYDROGRAPHIC SURVEYING is the art of delineating accurately and adequately the submarine levels, contours and features of seas, gulfs, estuaries, rivers and lakes and relating them to suitable data as well as to shore control points, determined by sound methods of triangulation and connected to a truly oriented geographical system.

This chapter has been written as a brief guide to basic principles for surveyors and engineers who may be entrusted with the duty of producing small hydrographic surveys for the safe navigation of harbours and sea approaches, or engineering projects relating thereto.

BASIC DATA AND METHODS OF CONTROL

Where land-survey departments have provided local tertiary and minor trig control, it is a simple matter to build up a good network of conspicuous sounding marks by the usual methods and show intimate topographical details, especially defining the spring high water contour and margin of cultivation. Nevertheless, if the final plan is being considered for purposes even remotely concerned with navigation or the setting out of dredged channels, it is of paramount importance that the orientation of the shore control system be checked carefully by astro-azimuths. Navigators rightly demand that the plan upon which ships' tracks are plotted, or operations planned, should enable them to steer safe courses through the channels or past the submarine hazards shown thereon.

In localities where the directional force of the navigator's compass is diminished, or where tidal streams or currents are of considerable velocity, and variable in direction and rate, the navigator will desire to use his hydrographic chart or plan as a basis for determining his position amongst dangers by observing two simultaneous sextant angles on three suitable salient points.

In such circumstances the surveyor should test data by measuring a good check base, extending it by triangulation to two suitably spaced intervisible trig points, and then determine the true azimuth

between them by astronomical methods. The method by time azimuths, employing a good theodolite and radio checked watch or chronometer, is perhaps the simplest for this purpose.

SHORE CONTROL, WHERE NO TRIANGULATION EXISTS. Where no land-survey triangulation exists, the fundamental procedure for triangulating the area must be carried out. Base measurement must have an order of accuracy of at least $1/50,000$ and be reduced to Mean Sea Level. For small harbours, a length of 1,500 feet should suffice, but in the case of larger coastal areas this can be increased to 5,000 as requisite. A triangulation covering the shore control for the area requiring survey can now be built up by modern theodolites with great accuracy and 'longsides' oriented to ± 1 second of arc using microptic theodolites reading to 1 second. Observations on four zeros at each observing station (two face right, two face left) should suffice, and suitably chosen triangles should close to within 3 seconds. The adjustment of observations is dealt with elsewhere.

SOUNDING MARKS. After the provision of the preliminary minor trig framework outlined above, it is now necessary to provide a more dense concentration of accurately determined coastal points, either naturally prominent or made conspicuous by beacons, structures or colouring which will enable the off-shore observers afloat, using simultaneous double sextant angles, to maintain an intimate plot and control of the track of the ship or launch engaged in running planned patterns of sounding lines. Such marks may be determined as extensions of the triangulation or by traverse methods, but it is often found convenient to use resection by the three-point method with suitable checks by intersecting rays from accepted stations. Semigraphic plots showing the intersecting rays or arc tangents on large scales, say 1 inch to the foot, are frequently resorted to, especially where a co-ordinate grid system is adopted.

The following tentative method for determining the position of a sounding point by a good set of theodolite observations on three suitably chosen trig points is given:

Assume, as in Fig. 71, that points A, B and C provide the three-point minor trig control for a point near D requiring resection as a sounding mark. The trial point D has been determined approximately either by sextant and station pointer or from good topographical detail on, say, an Ordnance or land survey map,

but it is desired to obtain closer determination by theodolite. The following principle is valuable:

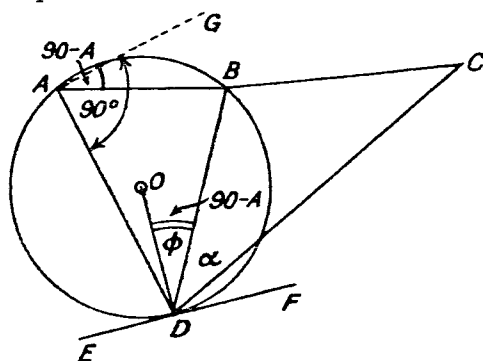


FIG. 71

Assuming D as a trial point, the angles ϕ and α are calculated. Now in the triangle ABD the shift of the arc tangent along the line to centre OD, for small differences, is given very closely by the following simple formula:

$$\text{Shift in feet per minute of arc towards or away from O} = \frac{ab}{3438 AB}$$

where $a = BD$, $b = AD$

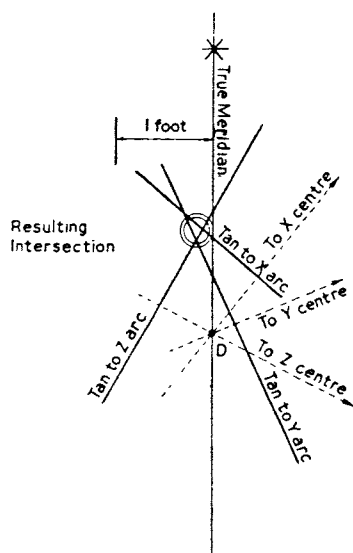


FIG. 72

The direction to centre referred to the line DB is given by $(90^\circ - A)$, such direction from D being anti-clockwise from D when $(90 - A)$ is positive and clockwise when negative. The position of the trial point with directions to centres for triangles ABD and DBE can be plotted on a large-scale semigraph (say 2 feet to one inch) and taken to the site. Theodolite determination of ϕ and α will show the differences between observed and computed angles for D, which can be converted rapidly into the linear shift of tangent arcs by values derived from the formula. The true arc tangents can quickly be scaled off, and their intersection will be the true position of the point from which re-computed angles will agree with the observed angles. Fig. 72 shows a semigraphic plot related to a trial point T.

PLOTTING SHEETS. It is essential for accurate work that the shore control of the marine area of survey be plotted on gridded standard sheets for easy reference and authoritative comparison. Where a harbour terminal is well established the trigonometrical control should be plotted on thin metal sheets, which, if possible, should be sprayed over with a suitable white enamel surface for drawing and erasure. The gridding should be done at intervals not exceeding six inches, if possible, and this principle should be applied to all fair tracings to enable distortion to be located and original results replotted after tracings and sheets have been filed away for long periods.

Material. Linen-backed drawing paper should be used for main office plot, and antiquarian in double elephant size is the most satisfactory. There will undoubtedly be distortion due to variations in temperature and humidity, but, apart from the metal sheets alluded to, no manageable and portable opaque material has yet been developed. A promising transparency is, however, now available in Astrafoil,* which appears to possess a high order of dimensional stability after rigid tests over wide ranges of temperature and humidity, and is used by H.M. Ordnance Survey of the United Kingdom. For small plots Whatman Boards are very useful, and these are often used for boat or working sheets which are equivalent to the land surveyor's field sheets.

Standard Plotting Sheet. Harbour and engineering surveys should always, if possible, be referred to a standard gridded sheet or series

* A new transparency known as COBEC has now been produced which is dimensionally stable and provides a better basis for waterproof inks.

of sheets connected to the main land survey triangulation if such is available for the site of survey. Provided the area covered is no more than 15×15 miles, a simple system of plane rectangular co-ordinates on a single origin can be employed without involving plottable errors in linearity and relative position; but convergence from the true meridian of origin must be taken into account to enable accurately oriented meridians to be inscribed on navigational plans or charts. Simple local tables can be computed, giving convergence in terms of easting or westing from meridian of origin,* and a geographical graticule constructed covering the standard sheet in a distinctive colour. All trig and sounding points should be shown distinctively and a good diagonal scale drawn on the sheet. A carefully dated magnetic meridian shown in distinctive colour (say green), with secular change shown, is valuable.

In the United Kingdom the National Grid 1/1250 Ordnance plans employing the Transverse Mercator (Gauss conformal) projection are now coming into common use, and the high order of accuracy of the recent triangulation affords an excellent basis for the hydrography of off-shore areas; this should be preserved by using modern microptic theodolites for extending the minor trig to sounding marks. The excellent system of revision points and their density on the new plans enables standard sheets and data to be prepared with great precision, and also affords a sound and intimate basis for detailed topography, whether by land or air survey methods.

Where triangles have sides of less than 4 miles, correction from great circle to loxodrome ($T - t$) can be disregarded without reducing essential accuracy. On the other hand, the distances computed from T.M. co-ordinates will differ from those measured in the field, the differences depending upon the distance from O scale factor eastings. A simple scale-factor table can readily be computed for the area covered, using the easting and mean difference in northing as arguments. It is reiterated that, where land survey data are of a high order of accuracy, such should be maintained by the use of instruments and methods capable of reasonably sustaining precise control over all offshore areas with which the surveyor or engineer is concerned, and particularly where important deep-water sea termini are involved. Rough approximations for

* $C' = \sin \text{departure (in arc)} \times \tan \text{Mid. Lat.}$
 $= \frac{\text{Departure (in feet)} \times \tan \text{Mid. Lat.}}{\text{Length of } 1'' \text{ at Earth's radius } (r)}$

dredging limits and submarine quantities are inexcusable where such facilities are available.

CO-ORDINATE LISTS AND SURVEY COMPUTATIONS. In important sea terminal areas, or where engineering works extend over large areas, it is essential that the results of intersection, resection or triangulation of points should be arrived at by computation from observed angles and not merely derived by protraction on the scale of the main plotting sheet. Results can thus be plotted on very large scales for a variety of purposes and still connected to the main plot by co-ordinate reference points and a grid meridian. Distortion of filed sheets occasioned by temperature, humidity or lapse of time can then be detected by recomputation or reconstruction of grid-ding, which should not exceed 6-inch intervals even on large-scale plots where authoritative reference may be required.

To this end a co-ordinate list of all survey points and salient objects related to the point of origin selected should be prepared with alphabetical index for easy reference, the convergency from the true meridian or origin being shown for each listed station.

If this system is adopted, an accurate large-scale detailed plot of any site can readily be prepared for any purpose with accurate references for location and orientation. Resections, intersections or direction of traverses using external control points may be computed precisely from the co-ordinated values and the observed angles.

TOPOGRAPHY BY AIR SURVEY. When a reasonably dense system of minor trig points and ancillary sounding marks has been achieved, a great deal of time and expense can be avoided by means of air survey. Provided a careful specification has been prepared, and conspicuously marked at a common level, say Mean High Water Springs, careful photography and the use of rectified enlargements will give intimate and accurate detail. In addition, if a suitable flight can be made at Spring low water and other intermediate tidal conditions, valuable and intimate hydrographic contours can be determined, including those of creeks in soft alluvium which are often difficult to survey either by shore or marine parties. Where tidal ranges are large, the value of this method is increased; but it is important that an accurately levelled tide pole should be read throughout the whole time of survey.

TIDAL RECONNAISSANCE. In a relatively unsurveyed area, whilst trig and levelling were proceeding on shore, the hydrographic surveyor would have established a series of tide poles at suitable points controlling the survey and so sited as to be sheltered as much as possible from wave and swell effect and capable of recording the lowest levels without drying out. It is not necessary that sounding should await the final setting of the zeros of these tide poles, but continuous day and night readings at hourly intervals should be commenced as soon as any new site is under consideration. Temporary tide pole zeros should, however, be connected by levelling to near and stable temporary bench marks, but the final adjustment of zeros and soundings can be made when a suitable common land survey plane of reference is established, and data from observations can be analysed. From such accurate and continuous readings, say for one lunar month (with the obvious precautions for accurate time synchronism and level checks), the surveyor in charge will gain a practical knowledge of general tidal features prevailing throughout the area under examination and so determine in particular whether variations in range require intermediate tide-pole stations to ensure that readings are representative of levels at all sites where soundings are taken.

To qualify the above it should be remembered that, whilst this applies in the North Atlantic area where tides are of a semi-daily or synodic type, varying dominantly with the moon's phases, in other areas, e.g. Do Son (Indo China) and Victoria, B.C., the tide may be of the dominantly diurnal or daily type, the range varying dominantly with the declinations of the sun and moon. In such cases maximum ranges will depend mostly on the maximum declination of the moon.

By modern methods of harmonic tidal analysis, thirty days of continuous hourly readings will provide data for a good rough preliminary working knowledge of the tidal constituents for predictions. Analysis of, say, one year's continuous recordings by a competent tidal institute using machine predictors will afford a basis for very accurate yearly predictions, which can be made very rapidly and provided, say, two years in advance.

Reliable portable automatic tide gauges are now obtainable at reasonable prices, and can be used with profit for such purposes, but timing, levelling and mechanical performance must be tested at least daily against an adjacent visual tide pole.

Stationing the tide gauges. The stationing of tide poles throughout an estuarine area should err on the side of excess, and the records of varying ranges and shallow water features between entrance and headwaters will guide the surveyor.

In Southampton Water, for example, readings can be obtained every mile along an eight-mile area of survey, and this is essential for the accurate computation of dredging quantities based on soundings agreeing to within 6 inches throughout.

For such work the surveyor is recommended to rely on visual tide poles rather than automatic tide gauges, care being taken to synchronize watches, the time, if possible, being confirmed by R/T checks, at frequent intervals.

DETERMINATION OF MEAN SEA LEVEL. The following table gives a convenient and accurate method for obtaining Mean Sea Level from an hourly series of 38 observations using a system of multipliers. The results, of course, do not exclude meteorological perturbations.

TABLE 19
MULTIPLIERS FOR MEAN SEA LEVEL
DIVISOR = 30

<i>t</i>	Multiplier	<i>t</i>	Multiplier
0	1	20	2
1	0	21	1
2	1	22	1
3	0	23	2
4	0	24	0
5	1	25	1
6	0	26	1
7	1	27	0
8	1	28	2
9	0	29	0
10	2	30	1
11	0	31	1
12	1	32	0
13	1	33	1
14	0	34	0
15	2	35	0
16	1	36	1
17	1	37	0
18	2	38	1
19	0		

This table is deduced from a valuable formula, which reduces errors in main harmonic constituents to the third decimal place, and is given in the Admiralty Manual of Tides, p. 111, using 38 hourly intervals, a system of multipliers and a division of 30 into the summation of products for M.S.L. (Table 19).

ENGINEERING AND HYDROGRAPHIC DATUM PLANES. The type of survey we are considering may be required either for engineering works or harbour approach navigation, and the datum level for survey should be selected accordingly.

(a) Engineering surveys in dock areas should be referred to a datum well below the level of all possible engineering works, and if possible below submarine borings, to enable all working values to have the same sign. A datum 100 feet below the general working tidal quay level would be suitable, say, for a harbour where the Spring tidal range was about 30 feet. It is, however, essential that the standard local level of reference—a dock sill or special mark cut in some prominent and sheltered dockyard—should be precisely connected both to the land survey datum (or selected local level plane of reference governing the area) as well as the navigational low water datum of soundings chosen for the reduction of soundings on Admiralty or official local charts or plans.

(b) Marine survey datum has been generally defined by the International Hydrographic Bureau (based on the consent of navigators) as 'the level below which the tide *seldom* falls'. This enables a mariner to look at a marine chart or plan and see the depths and drying banks, rocks or shoals, at roughly the lowest low-water level experienced at the place shown. At extraordinary Springs he knows the tide will fall perhaps a foot or so lower, and so, on occasions he can foresee, he will be prepared for this situation, but in general his chart will show the shallowest situation by soundings and contours. On the more recent Admiralty charts contours are reinforced by colouring. The French datum is that of the *lowest possible* low water. This, in the opinion of British seamen, gives a picture for the worst situation which rarely occurs and is thus unduly restrictive. It will be appreciated that the Spring low water datum chosen for British and Atlantic seabords is not applicable to areas where the tides are diurnal, and mean lower low waters occur at the time of the moon's maximum declination. Navigational low water planes will, of course, vary according to

range, and may differ by one or two feet over a few miles owing to physical conditions, being in reality mean tide or sea level minus semi-Spring range in semi-diurnal areas.

LEVELLING. In hydrographic work it is generally regarded as sufficient to work to an accuracy of $\sqrt{\frac{D}{4}}$ inches where D is the distance in nautical miles over which levels are transferred. In levelling tide pole zeros, where possible use two bench marks levelling from B.M. $\overline{\Delta}$ (A) to tide pole *and near water level*, thence to B.M. $\overline{\Delta}$ (B) connecting to (A) for confirmation.

In all hydrographic work the surveyor should take water levels wherever possible, noting the time accurately for reference to near tide pole readings or checks by tidal computation. Where only a single B.M. is available, levels to and from the B.M. to tide pole and water level should be taken, the latter being compared with adjacent tide pole readings for consistency. Where surveys are used for navigation, heights of hills and salient objects should be referred to the level of local mean *high* water Springs, as this will safeguard navigators using vertical subtense methods for offshore distances by under-estimation.

When transferring levels to offshore tide poles by ordinary levelling, refraction and curvature must of course be considered, but where distances from the shore are between a quarter and half a mile, transfer of water levels at high water slack with no streams, little wind and no swell will generally be found more satisfactory. Where the distance of such an offshore tide pole extends several miles out from the shore, and where no superimposition of river-bed flow drastically interferes with the true tidal curve so as to 'truncate' it near low water times, good results will be obtained by adjusting to local mean tide level. This can be determined by taking 38 continuous hourly readings simultaneously both at the tide pole for which levels are required and *at the nearest shore-levelled* tide pole, care being taken that the two sites are not subject to different hydraulic or meteorological conditions. The resultant mean tide levels can then be calculated from the table of multipliers given above (page 228) under 'Determination of Mean Sea Levels'.

The difference between the land survey datum and the computed mean tide level on the shore-levelled tide pole of reference,

Let M = computed M.T.L. reading at tide pole of reference.
 m = " " " " offshore tide pole
 S = level of standard land survey datum on tide pole of reference
 x = level of standard land survey datum on offshore tide poles

If the simultaneous 38 hourly observations are neither practicable nor possible, obtain approximate half tide levels by taking four successive low water readings with those of the three intervening high waters.

Mean the sets of high and low waters respectively and by meaning M.H.W. and M.L.W. obtain half tide level at the tide pole of reference and the offshore tide pole. Then $x = m + (S - M)$ as before. Comparative low waters or low water datum below half tide level can be transferred to the offshore tide pole by ratio of ranges as requisite. The latter approximate method is not suitable where the tide is appreciably diurnal in character, but the method using 38 simultaneous consecutive hourly readings gives a safe co-ordination, especially with the moon at zero declination at the equinoxes.

NAVIGATIONAL ACCESS TO SITE OF OPERATIONS. It frequently falls to the surveyor to deal with relatively unknown or war-damaged and obstructed harbours, or to prospect seaplane bases. In such cases preliminary reconnaissance of the submarine area is essential for the safeguarding of survey craft and personnel, as well as expediting the works. In areas where the tide is semi-diurnal or synodic, as generally off the British and North Atlantic coasts, the time of the occurrence of Spring high and low waters on the days of the moon's full and change is a valuable constant and should be known.

Where Spring low water occurs in daylight, salient dangers should be located by intersecting rays from shore angles, and the time of their covering on the flood carefully noted and referred to the local tide pole. Transit posts can be erected indicating clear passage at low water times, and, if the draught of survey craft is less than two-thirds of the Spring tidal range, these can safely use

such transit marks in coincidence near high water times. Where circumstances and economics permit, air survey at low water Springs related to the tide pole reading at flight will give a useful L.W. Spring contour.

Entrance after those precautions should be made on the last half of the flood rise *taking care not to risk working on the ebb* and withdrawing accordingly in good time. Entering cautiously on the flood, if possible, along reaches previously marked by transits, one may buoy dangerous features as observed, or even touched, such buoyage assisting safe withdrawal before the commencement of the ebb.

SOUNDING. When astronomical location and orientation, shore trig control and extended sounding marks in adequate density have been provided, together with suitable plotting sheets and facilities, and accurately levelled tide poles so stationed as to govern the survey, plans for running systems of sounding lines can be put into

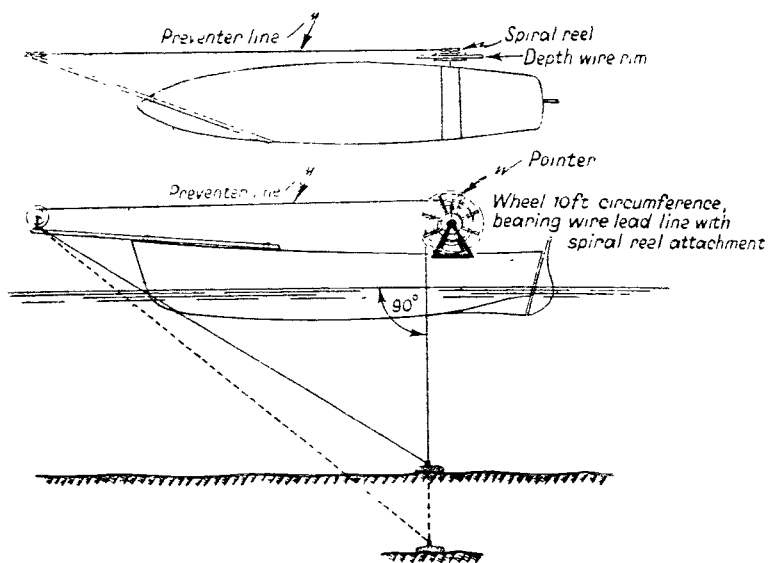


FIG. 73

operation. Here we may review, briefly, methods of sounding, always remembering that the fundamental standard steel tape must be the criterion of all submarine measuring devices.

From antiquity up to the beginning of this century, tangible methods alone were used, including the weighted sounding pole for very shoal depths and the lead and line (or chain). Leadline catenary error and soft bottoms, taken together with the shrinkage of cordage, linear instability of chain and difficulties in handling small gauge wire, have always made precise sounding difficult, even at the slow speed essential for harbour work. The Sutcliffe sounding device (Fig. 73) is the most ingenious method yet devised to overcome this in depths of up to 40 feet, but fouling by obstructions is a serious disadvantage.

Yet, when running short lines on measured wires off quays in relatively shallow water, the lead is necessary and valuable, and for this purpose many of the difficulties have been overcome by using a ten-pound conical zinc or iron lead with a cable laid hemp line into which a fine wire heart has been worked (Fig. 74).

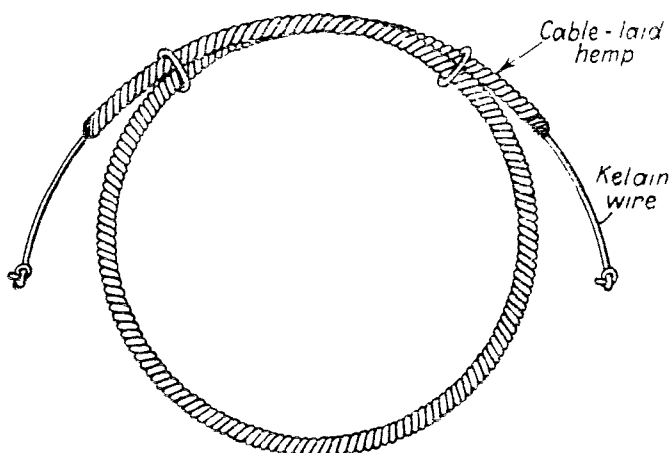


FIG. 74

Almost perfect linear stability is achieved by the use of such leadlines, and handling by leadsmen is easy, but progress is necessarily slow.

DOCK SURVEYS BY LEADLINE. The method employing lead and line is still valuable in dealing with dock approaches extending seaward or riverward for about 700 feet. In such cases it is customary to use an 18-20 foot pulling boat or 20-25 foot launch carrying a measured wire stowed inboard on a reel, so mounted as to pay out

over-all under cross tidal conditions without binding the stern of the craft and so hindering her from maintaining the survey transit line.

The zero end of the measuring wire is secured on shore at each parallel line and steering transit marks provided, spaced at a distance of no less than $\frac{D}{6}$ where D is the maximum distance out from the mark nearest the quay wall (Fig. 75). The measuring wire is calibrated at ten-foot intervals, and it is usual to proceed under oars or power at a speed of no more than two knots at or near slack water, plumbing each cast as the ten-foot marks pass the sounding position.

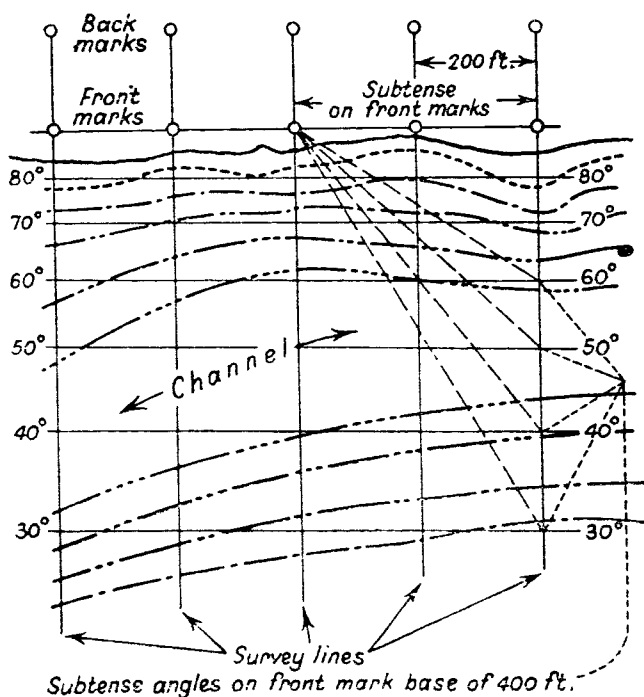


FIG. 75

It is important to determine wire catenary error by sextant angles on shore base at the end of each line with steering marks 'on', as an error of ten feet can easily occur over a 700-foot length, and this can occasion serious errors in the quantities computed for dredging slopes on offshore channel boundaries.

Such fundamental sounding and boatwork are essential foundations in the training of a surveyor specializing in hydrographic work, and the elements of the supervision of a survey party under way are best learned in this manner.

Personal instructions to a surveyor for this work are here detailed:

1. (a) Synchronize your watch with the tide watchers, and make sure the tide pole is accurately calibrated, levelled and set vertically.

2. (a) Keep a stoutly-covered sounding book for record.

- (b) Carry a sextant and enter index error at the beginning and end of survey; also subtense angle at end of each line.

- (c) Log times at start and end of each line. Enter soundings seriatim at 5- or 10-foot intervals as requisite, checking leadsman's plumbing and value of casts throughout, adjusting speed and maintaining steering transit in coincidence without drastic 'corrections' which might cause the lead to 'trail' and so make sounding impossible. Note any bottom fouling or obstructions.

- (d) Note any sounding marks that coincide, i.e. transits that 'come on'.

- (e) Note and if possible anticipate variations in direction and strength of tidal stream or current.

- (f) Record at intervals (say every 200 feet) whether the boat is *right* or *left* of, or *on*, steering marks ashore.

- (g) Display a signal indicating that you *are* surveying, and don't collide with anything in your preoccupation with precise survey.

- (h) Take full charge of the party, correct mistakes boldly and audibly, and, *if you are dissatisfied, cancel and recommence the line*. Train the leadsman to 'feel' the top of silt or mud, never to let the lead settle and indicate at once if speed is likely to cause 'trail'.

- (j) Prominently record date, locality, method of survey and site of tide pole, facilitating future reference by others.

- (k) If possible, run a cross-test line on good transit marks diagonally across your planned and parallel survey lines to check sounding, location and tidal reductions.

- (l) Where slack water periods are short and tidal range large, start about 2 hours before low water at Springs if possible and use neap tides to full advantage.

(m) Check leadline by standard steel tape before and after survey.

(n) Check measuring wire before survey by steel tape.

3. (a) When lines of soundings have been run satisfactorily, check tide watcher's readings by plotting the curve of heights, reduce soundings, correct measured distance for catenary error and ink in soundings accordingly on a rough collector tracing.

(b) If 'cross-test' line shows discrepancies, repeat lines of soundings as requisite.

(c) Contour soundings in pencil and note discontinuities, places where authoritative contours cannot be drawn, isolated and drastic shoal casts or spots where bottom fouling has been experienced. Over such suspicious areas set up steering marks for interlining at closer intervals to develop the suspicious feature. Whenever doubtful about results discard them and repeat *de novo*.

4. When all the work harmonizes from all possible points of view and will stand criticism, ink in final contours, emphasizing those of navigational importance (say the 30-foot contour at mean low water Springs), delimit any foul ground for wire bottom sweep and diving investigation and prepare final plan as follows:

(a) Prepare a fair tracing from the standard plotting sheet showing trig points and topography covering the area. Insert the title and date of survey in a prominent position.

(b) The sounding plan should show the following:

Units. The unit of the soundings and their datum must be clearly defined and their datum shown. The soundings should be reduced to a datum below which the tide seldom falls, and this datum should be connected to some clearly marked bench mark (or level surface) inscribed (or defined) on an object of a permanent and durable nature for recovery and comparison with future surveys.

Soundings. Soundings should be sufficiently plentiful as to leave nothing to the imagination in contouring.

Scale. The natural scale of the chart should be given and a working scale shown, and the authority from which such scale is derived. Failing this, the long-side of two prominent objects derived from an accurate base measurement extended by simple triangulation should be defined in bearing and distance.

Meridian. A true meridian should be prominently shown over at least three-quarters the length of the plan, starred at its north end with the authority for its orientation. If no existing survey authority can be quoted, a true bearing by sun's azimuth of the long-side should be obtained.

Magnetic meridians should be subordinated to true and are better left out, as they are generally of inferior accuracy and lead to many errors after lapse of time. If they are shown the variation value, year of observation and rate of annual secular change should be inscribed on the plan.

Topography. The topography of the berths or coastline adjacent to the survey should be shown in good detail and prominent land marks listed for reference. Outstanding transit or clearing marks should be shown, and if necessary erected to mark shoal edges or clear dangers.

Authority. Names of surveyors and method of survey should be given in a brief memoir; also signature of officer in charge.

Seamarks. Navigational marks, beacons or buoys, lighted or unlighted, with their characteristics, should be shown prominently.

LEADLINE METHODS IN GENERAL. Whilst leadline methods have been superseded by modern supersonic devices, it is a mistake to believe that a hydrographic surveyor should not be able to appreciate and use them, as the fundamentals of sounding are best learnt by their use, and mechanical or electrical failures far away from base may sometimes necessitate reversion to the old techniques. Furthermore, science has not yet provided any alternative method for determining the hardness or nature of bottom objects apart from investigation by the 'feel' of the lead or by 'arming' it with tallow or grease for bed-surface sampling and identification.

(a) The use of leadline in large-scale dock surveys, as given in the preceding section, is confined to depths of no more than 40 feet in good weather near low water times, and at slack water. The lead is barely lifted above the bottom between soundings and plumbed at each cast whilst proceeding at very slow speed and generally under oars. If a powered launch is used, it is difficult to maintain steerage way with such methods, but over clear sandy bottoms the Sutcliffe sounding device as illustrated above is the best practical solution. The author has used this device with good results in the

port of Liverpool. In more exposed sites, in estuaries or coastal offshore areas, it is not possible by this technique to cover large areas economically on scales of say $1/10000$ to $1/20000$ except by stopping for each cast, which is tedious and unseamanlike, and in such circumstances the honoured nautical procedure of 'heaving the lead' at a speed of about 4-5 knots is the only suitable method of using the lead and line.

It is essential to train men carefully in this technique, which consists of standing on a 'chain' or lead platform, rigged outboard, swinging the lead fore and aft clear of the water to gain momentum and finally heaving it well ahead of the leadsman (say about 25 to 30 feet), first paying out the line until the lead dips, then quickly hauling in the slack line in time to plumb the cast 'up and down' when the ship is directly over the lead. Conspicuous marking in fathoms and feet enables the surveyor to check the leadsman's calling (see Fig. 76).

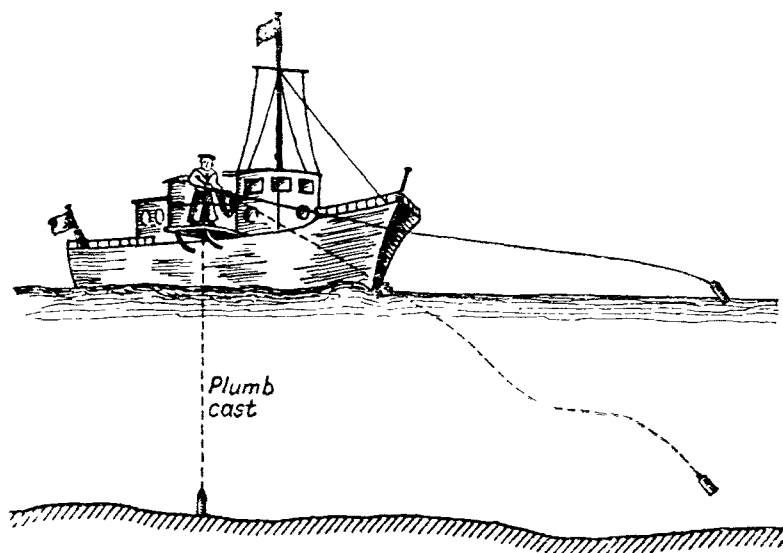


FIG. 76

Great proficiency has in the past been developed by seamen employed on hydrographic survey work using this method, but it is clear that intervals between casts are necessarily considerable, and, as these vary with depth, the plotting of the linear intervals cannot be precise enough for large scales.

Furthermore the catenary of the leadline varies with submarine currents, and the lead itself sinks into soft bottoms. Both features together, with failure to plumb precisely, conspire to give excess depth. It is possible to use long platforms on the port and starboard sides of a survey launch for this method, but for port-side heaving it is necessary to train a left-handed leadsman. Sometimes these are very expert.

(b) In very shoal areas, at high water slack, in fine weather, where creeks or channels and swashways require survey, it is possible to use a sounding pole with a 7-8 inch weighted base to advantage.

(c) In dealing with offshore areas, as in (a) and (b) above, it is of course essential to have a closely levelled and representative tide pole near the site of survey, and planned lines of soundings can only be run accurately by fixing with double simultaneous angles at short time intervals on prominent shore sounding marks.

Fixes are plotted by station pointer in pencil on a working sheet or sounding board carried on board the survey launch or, if possible, on a sextant arc-graph to be described later.

The sounding book is carefully kept, the objects used, times of fixes and the value of the angles being recorded as follows:

21st January, 1952

SURVEY IN MID-CHANNEL - OFF CALSHOT

<i>Time</i> <i>G.M.T.</i>	<i>Fix</i> <i>No.</i>	<i>Object</i>	<i>Left</i> <i>Angle</i>	<i>Object</i>	<i>Right</i> <i>Angle</i>	<i>Object</i>	<i>Sdg at</i> <i>Fix</i>
1016	1	Stansore Bn. Δ	41°30'	Tank Tp. Δ	43°50'	Lutt. Tr. Δ	39/0
(Intermediate in feet)	39/6	39/0	36/0	34/6	33/6	33/0	32/6

The principle of keeping the sounding book under way in off-shore areas using three-point sextant fixing and station pointer or sextant graph is fundamentally the same as for dock surveys, all circumstantial features being noted including transits and in particular soundings at fixes and intermediate soundings as called by leadsmen and checked by the surveyor. Normally the surveyor-in-charge determines the fixes by calling 'stand by' and 'fix', takes one angle and plots the fixes. The assistant surveyor records and takes the 'second angle' simultaneously with that of the surveyor-in-charge, and records the soundings.

It is important to ensure that fix numbering in the working sheet or echo-sounding recorder is in step with that recorded in the

sounding book. It should be noted that a line of lead soundings indicates only a succession of isolated points, and intermediate shoaling or dangers are missed. Also, as it is customary to fix after an odd number of intermediate casts (to enable the middle sounding to be plotted halfway between fixes), say five or seven, it is important for plotting that a constant speed be maintained. This means that when depths change from deep to shoal or vice versa the time interval between casts varies and the plotted linear intervals are accordingly adjusted by estimate.

Fixing should be carried out where depths change suddenly in this manner but when using the lead the change must necessarily precede fixing. It is clear that a device which gives a continuous bottom profile at a constant speed overcomes these ancient difficulties, and such a device is made available by modern echo-sounding principles.

MODERN ECHO-SOUNDING DEVICES. Echo-sounding engineering, arising out of the feverish anti-submarine and other subaqueous research initiated during the First World War, has now developed shallow-water devices capable of giving a continuous bottom trace on recording paper on a scale of over $\frac{1}{16}$ inch to 1 foot of depth.

It is not proposed in this section to describe in technical detail the electrical and mechanical features of the many and various reliable echo-sounding recorders now available, but rather to place the method of measuring submarine depths by reflected supersonic sound waves within the context of the well-tested techniques of two centuries of world-wide hydrographic surveying and an unprecedented output of marine charts, notably by the Hydrographic Department of the British Admiralty. Echo-sounding is simply the conversion into linear values of the recorded time intervals between the transmission of sound waves and their return to the point of origin after reflection from the sea bed, by applying the mean unit speed of sound at the time.

The accuracy of the results obviously depend on:

- (i) The accurate estimation of the speed of sound in water at the time of sounding.
- (ii) The precision with which the time interval can be measured and converted to linear values.
- (iii) The safeguard of a *practical test of performance* applied over the range of depth experienced on the site of survey.

With regard to (i), the standard practice is to construct instruments for a reasonable average speed of sound in sea water, the values for the British and metric calibrations being 4,800 f.p.s. and 1,500 metres p.s. respectively.

Recording devices must include methods for adjusting to the ranges of the speed of sound over all possible variations likely to be met with in fresh or sea water. This increases directly with the mean temperature and specific gravity respectively, and tables can readily be prepared for instrumental adjustment to these values at the time of survey. With reference to (ii), several devices have been developed, and in considering them it is important to bear firmly in mind the requirement to convert the echo interval in time to linear functions.

All methods have used an electrical contact stylus traversed at a uniform rate over (wet or dry) chemically recording paper, moving at right angles to the direction of stylus traverse for serial recording, to give a 'bottom trace'. At the instants of the transmission of the sonic pulse and the reception on return respectively an electric circuit is closed, energizing the stylus and causing a visible recorded mark on the paper. A series of such recordings will give a continuous transmission trace, also a continuous bottom trace, some point on the former being the zero and effective plane of the origin of transmission. Provided the points of transmission and reception are identical, the echo time interval, divided by twice the mean speed of sound, will give the correct linear distance from the sea bottom, i.e. least depth.

It is, of course, clear that the speed of traverse of the stylus will determine the linear scale of the depth-recording device. For example, let

T = traversing speed of stylus in inches per second

R = extreme range of depth in feet required for scope of recording paper

L = length of stylus traverse over recording paper in inches corresponding to R

S = average speed of sound in sea water

I = sonic time interval - transmission/return cycle - for R

Then

$$I = \frac{2R}{S} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

$$L = TI \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

From Fig. 77 it will be seen that the scale must be graduated on the chord of the stylus arc BC employed.

Recording of soundings. The Admiralty pattern recorders, and other well-known makes, use a paper moistened by a solution of starch and iodine, the electrical discharge through the stylus causing a brown precipitation mark.

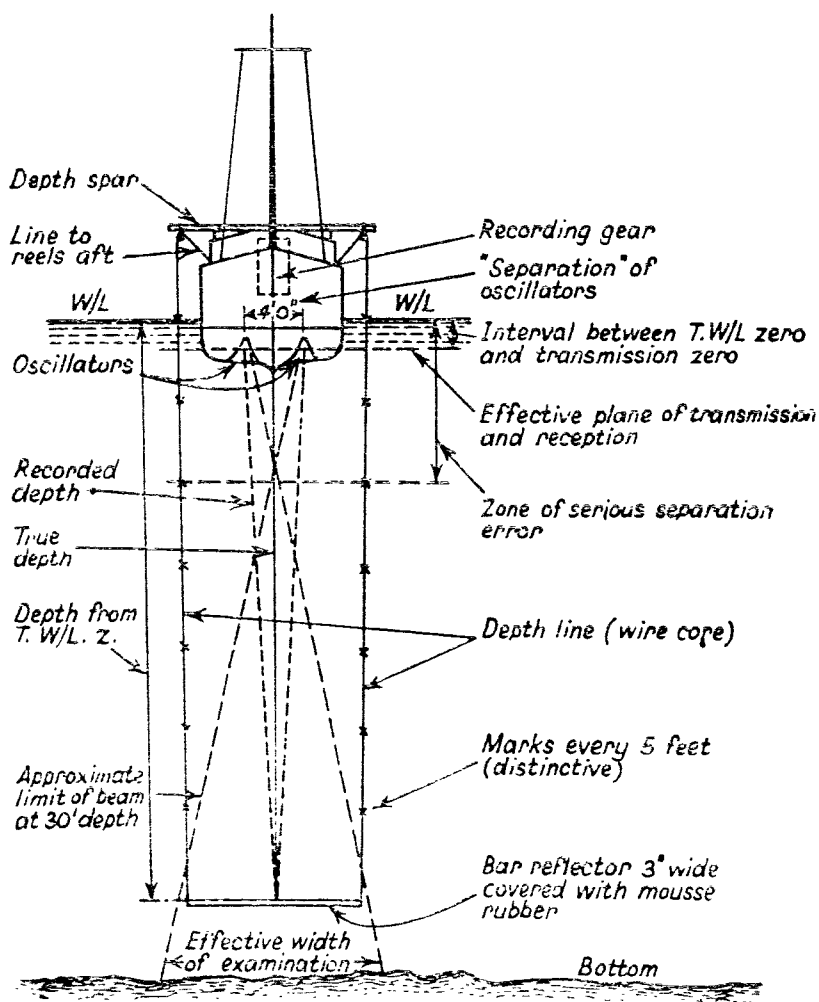


FIG. 78

It is Admiralty practice for the E.-S. operator to call out the soundings for booking at say 10-second intervals. On the other hand the writer has found it more convenient when working close to base to calibrate the dry record. As the paper may shrink up to 4%, it is necessary to use a dry scale. For a given paper the shrinkage, which can be tested, is very uniform and dry scales bearing four calibrated values enable the most appropriate dry paper scale to be selected by comparison with the standard intermediate depth lines borne on the record after marking in its wet condition. If this system is adopted, it is imperative to pencil in the bottom record immediately after drying in case of fading.

Accuracy, calibration and errors. As, at present, it is necessary to have separate transmitting and receiving oscillators, it is obvious that the 'separation' of these in the horizontal plane will affect shoaler depths.

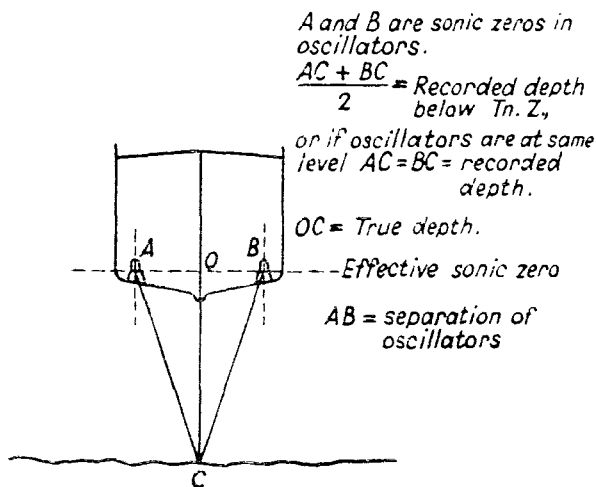


FIG. 78a

This is illustrated in Figs. 78 and 78a, the formula for depth error being:

$$C = 1.2 \sqrt{\left[D^2 - \left(\frac{S}{2} \right)^2 \right]}$$

where

D = recorded depth in feet

C = subtractive correction in inches to recorded depths

S = separation of oscillators in feet in the horizontal plane.

Fig. 79 enables separation error to be found by inspection.

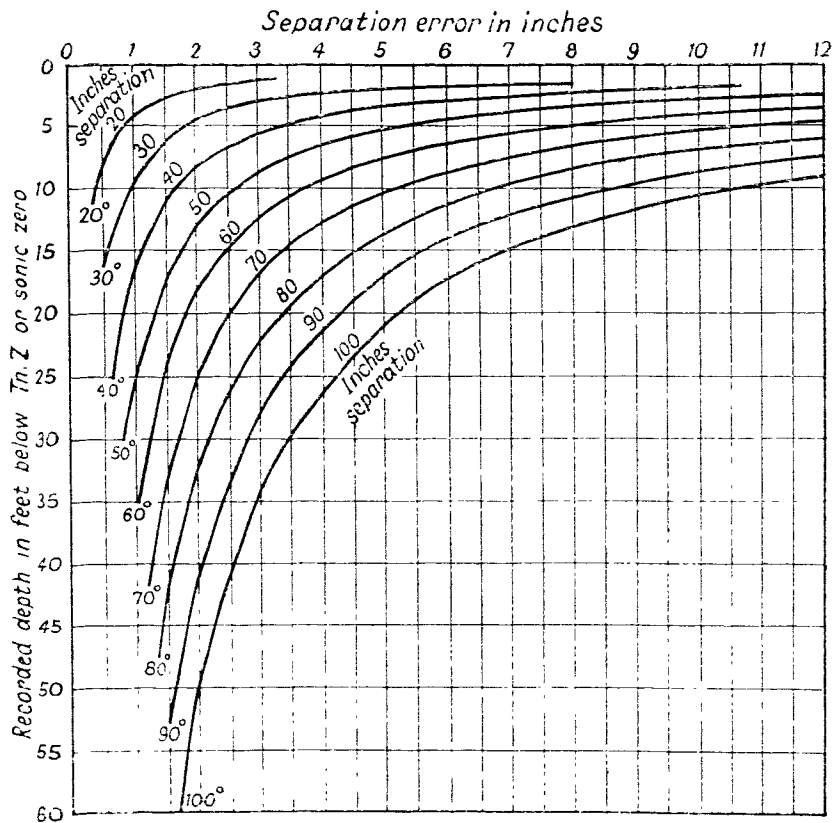


FIG. 79

Normal separation is about 40 inches, in which case the error is 1.0 and 3.4 inches at depths of 14 and 5 feet respectively.

The calibration and determination of the zero of soundings is done by lowering a reflector bar below the sounding oscillators suspended from carefully taped wire stabilized depth lines and making a scale of 5 or 10 foot steps (see Fig. 80) on the actual

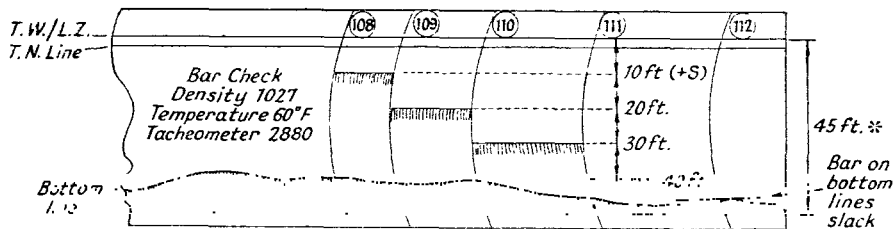


FIG. 80

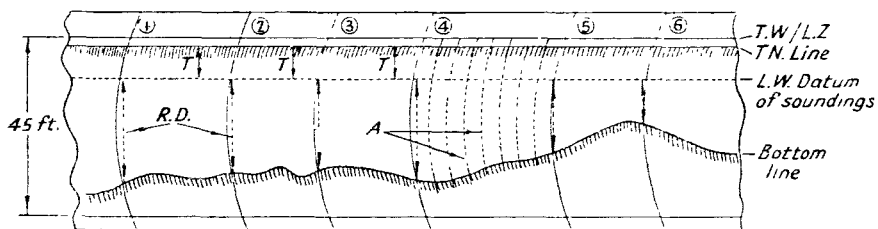


FIG. 81

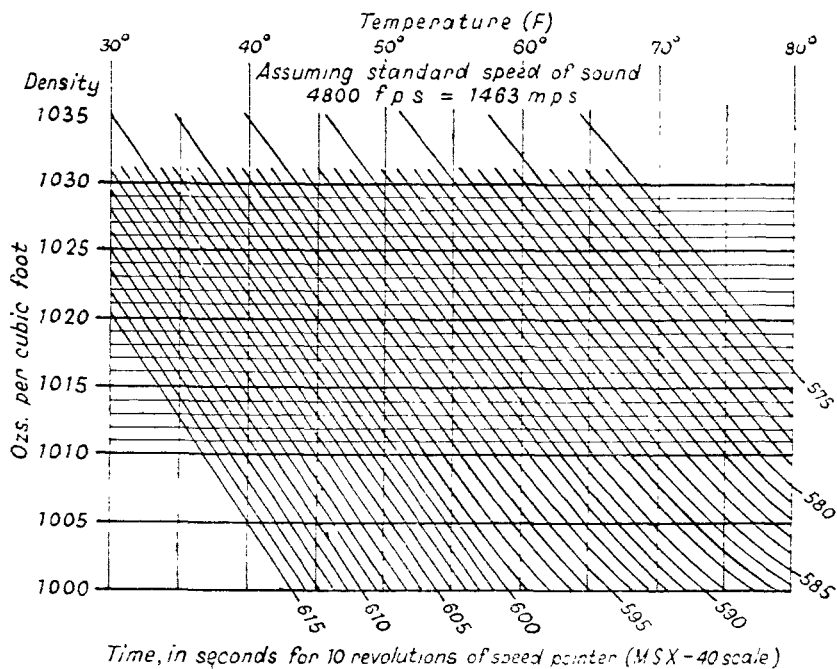


FIG. 82

record (Fig. 81). Prior to the operation in smooth water at a site representative of survey conditions, the machine should be adjusted for density and water temperature at mean depth.

Fig. 82 shows a diagram computed from the Admiralty publication HD.282 for the Kelvin Hughes M.S. X recorder where the true time interval for ten revolutions of the 'minute' dial is given by entering with density and temperature. The salinity values from HD 282 may be converted to relative density values by the use of hydrographic tables published by H.M. Stationery Office. It will be sufficient, however, to give the data in Tables 1 and 2 which indicate percentage of increase or decrease in the speed of sound for given ranges of density and temperature and thus the amount of increase or decrease of instrumental speeds for agreement with calibrated depth scales. Tables 20 and 21 are for British and metric calibrations respectively.

TABLE 20

RELATING TEMPERATURE (F), DENSITY OF SEA-WATER AND SPEED OF SOUND, GIVING PERCENTAGE CORRECTION TO INSTRUMENTAL SPEED BASED ON 1463 M.P.S. (BRITISH CALIBRATION)

	1035	1030	1025	1020	1015	1010	1005	1000
30°	- 0.8	- 1.2	- 1.8	- 2.3	- 2.9	- 3.3	- 4.0	- 4.5
40°	+ 0.9	- 0.4	- 0.0	- 0.8	- 1.2	- 1.9	- 2.4	- 2.9
50°	+ 2.4	+ 1.9	+ 1.3	+ 0.7	+ 0.3	- 0.2	- 0.7	- 1.3
60°	+ 3.8	+ 3.3	+ 2.8	+ 2.2	+ 1.7	+ 1.3	+ 0.7	+ 0.2
70°	+ 5.0	+ 4.5	+ 4.0	+ 3.5	+ 3.0	+ 2.6	+ 2.0	+ 1.5
80°	+ 6.0	+ 5.6	+ 5.1	+ 4.7	+ 4.2	+ 3.7	+ 3.2	+ 2.7
90°			+ 6.1	+ 5.7	+ 5.2	+ 4.7	+ 4.2	+ 3.7

1. Enter with temperature and density of sample at mean depths to obtain percentage factor F.

2. $\frac{F \times \text{standard r.p.m.}}{100} = \text{increase or decrease r.p.m. according to sign.}$

e.g. Let $\left. \begin{array}{l} \text{standard r.p.m.} = 300 \\ \text{density} = 1025 \\ \text{temperature } F = 50^\circ \end{array} \right\} F = +1.3\%$
 $\text{r.p.m. required} = 300 + \frac{(300 \times 1.3)}{100} = 303.9$

TABLE 21

RELATING TEMPERATURE (C), DENSITY OF SEA-WATER AND SPEED OF SOUND, GIVING PERCENTAGE CORRECTION TO STANDARD INSTRUMENTAL SPEED BASED ON 1,500 METRES/SEC. (METRIC CALIBRATION)

	1035	1030	1025	1020	1015	1010	1005	1000
C 0°	-3.0	-3.4	-4.0	-4.4	-5.0	-5.6	-6.2	-6.7
5°	-1.4	-1.9	-2.5	-3.0	-3.5	-4.0	-4.7	-5.0
10°	-0.1	-0.6	-1.1	-1.6	-2.1	-2.6	-3.1	-3.7
15°	+1.0	+0.5	+0.1	-0.3	-0.9	-1.3	-1.9	-2.4
20°	+2.1	+1.7	+1.2	+0.8	+0.3	-0.2	-0.7	-1.2
25°	+3.1	+2.7	+2.3	+1.8	+1.3	+0.8	+0.3	-0.2
30°	—	+3.5	+3.1	+2.7	+2.2	+1.8	+1.3	+0.75
35°	—	—	+4.1	+3.7	+3.2	+2.7	+2.2	+1.7

If, therefore, a series of depth steps are made, a scale can be constructed giving the true waterline zero (T.W.Z) which can be drawn on the record, say in blue colour (see Fig. 81).

Below this datum of reference the tidal readings can be plotted as points in relation to the fix numbering and a red line drawn through them showing the datum of soundings for the record.

In modern recorders a tacheometer will facilitate close instrumental timing and it is possible to achieve accuracy to ± 3 inches on the largest scales of recorder now in use, for example the 0-45 foot scale of the Kelvin Hughes M.S. XXIA.

Fitting-out and speed of sounding. Theoretically the only limitation on the speed of E.-S. work would be the intimacy and accuracy with which the track of the survey craft could be plotted. It is possible to obtain a good bottom record at speeds of 1 up to 20 knots but a simultaneous double angle fix every 500 feet (which is wide on large harbour scales) would involve continuous observations at 15-second intervals.

For general reconnaissances of the axis of navigable rivers to locate siltage such high speeds have been employed, but, if accurate depths are required, the alteration in the levels of the plane of

transmission reception due to settlement and squat of the survey ship must be determined beforehand by levelling from a pier with level staff upright over the oscillator position both with the ship at rest and when passing at survey speed.

Here it might be mentioned that, whilst the longitudinal position of the oscillators could with advantage be sited just slightly forward of the centre of water plane, this position often gives rise to aeration due to turbulence from forward skin friction. A good general rule is one third the length from the stem and thus on the fore side of the centre of waterplane of the survey ship. The latter can be determined experimentally by cutting out a cardboard shape from measurement or plans of the waterplane and balancing on a knife edge. Incidentally it is necessary on large scales to ensure that the sextant observers are stationed immediately above the oscillator position selected, for obvious reasons.

SEXTANT-GRAPH PLOTTING. The use of sextant graphs employing intersecting arcs or lines of position is the only visual method by which the higher speeds of sounding now available can be utilized for large-scale survey work to best advantage.

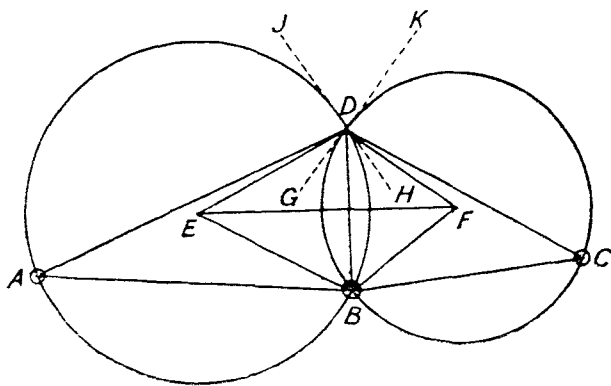


FIG. 83

It can be shown in the case of the three-point fix (see Fig. 83) that where the included angle ADC at the centre object of the two extreme points and the total angle at the observing position ABC add up to 270° , the angle of intersection of the arc tangents is 90° .

The use of the three-point fix enables the two selected intersecting subtense arcs to be plotted either by beam compasses or calculation over the areas required for survey as shown in Fig. 83. It is not, of course, necessary to derive the position from three points as two pairs may sometimes provide two more suitable arcs of greater sensitivity and cutting more nearly at right angles.

Let A, B, and C be three shore beacons and D the observer's position.

Let $\angle ABD$ and $\angle DBC$ be the two observed angles.

GK and JH are the tangents of the arcs of position intersecting at D.

Joining the centres of the circles to the points D and B:

$$\angle A = \angle DEF = \angle BDH \text{ (complements of } \angle EDB)$$

$$\angle C = \angle DFE = \angle GDB \text{ (complements of } \angle FDB)$$

In quadrilateral ABCD:

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

But $\angle A + \angle C = \angle BDH + \angle GDB = \angle GDH$

therefore $\angle B + \angle D + \angle GDH = 360^\circ$

and $\angle GDH = 360^\circ - (\angle B + \angle D)$

Thus when $\angle B + \angle D = 270^\circ$

$$\angle GDH = 90^\circ$$

giving optimum angle of tangent intersection.

Apart from the angle of intersection it is of course necessary to determine the linear sensitivity of the subtense angle per minute of arc. The simple formula already given gives a close approximation for the movement of position per 1 minute of arc towards or away from the centre as in Fig. 71.

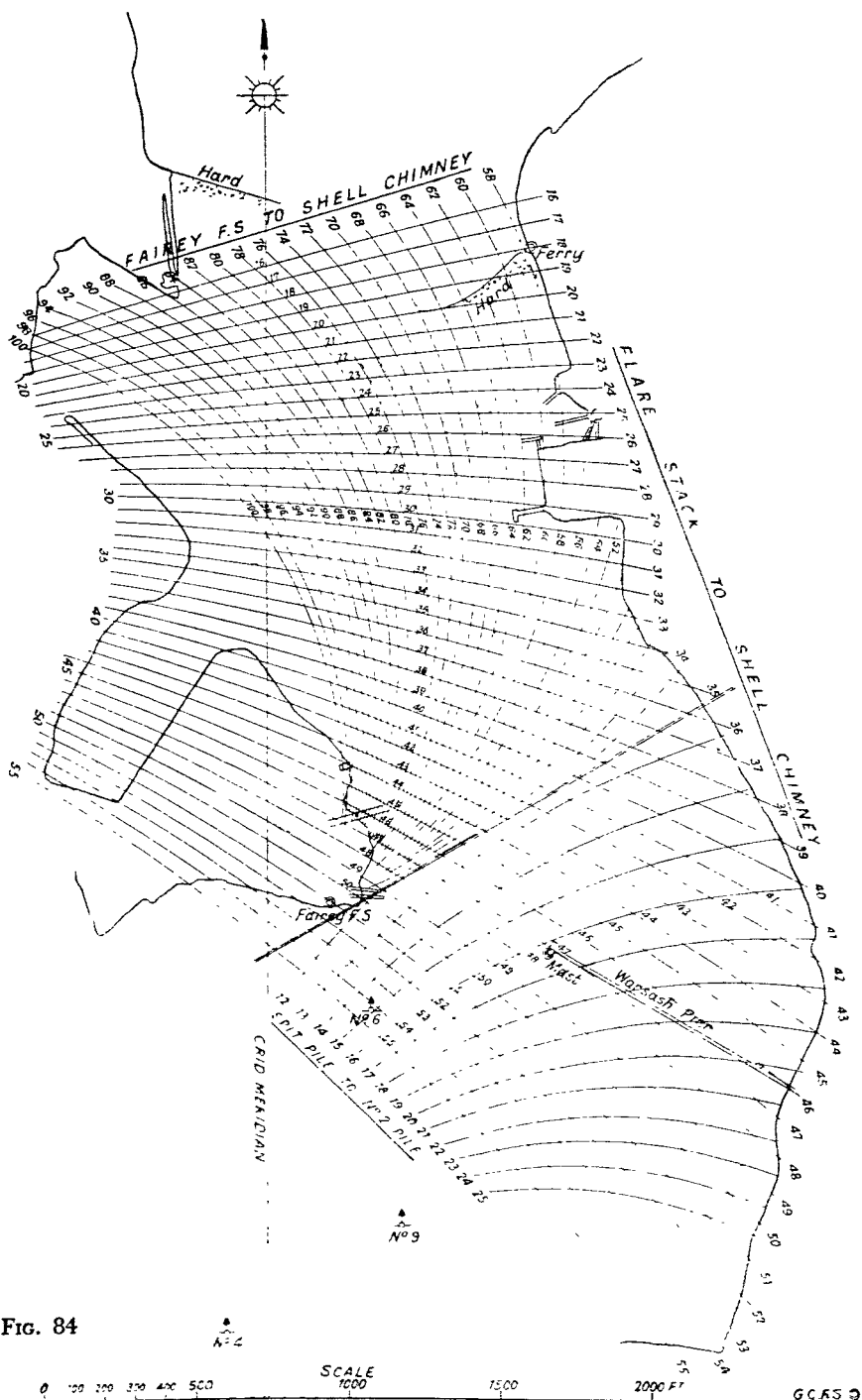
$$d = \frac{a \times b}{AB \times 3438}$$

where d = distance in feet towards or away from centre on the line of radius for a change of 1 minute of arc.

If these principles are considered, it will be seen why the following practical rules for the use of the station pointers have come into use:

In all the following cases the fix is a good one:

- (i) If the three points are in the same straight line.
- (ii) If two of the points are in transit.
- (iii) If the central point is nearest to the observer and the angles are not too small.



(iv) If the observer's position is within the triangle formed by the three points.

(v) When the points are about the same distance from the observer and the angles are not less than 70° .

(vi) When one angle is large and the other is small, and the small angle is made with the outer object far behind the central point.

Using the three-point fix and working on these principles, arcs can be plotted on the loci of the circles say for each $\frac{1}{2}^\circ$ using the

$$\text{formula } d = \frac{D}{2} \cot \theta$$

where D = distance between subtense points

d = distance to centre measured along a perpendicular line erected halfway between the subtense points and the locus of centres as shown in Fig. 85.

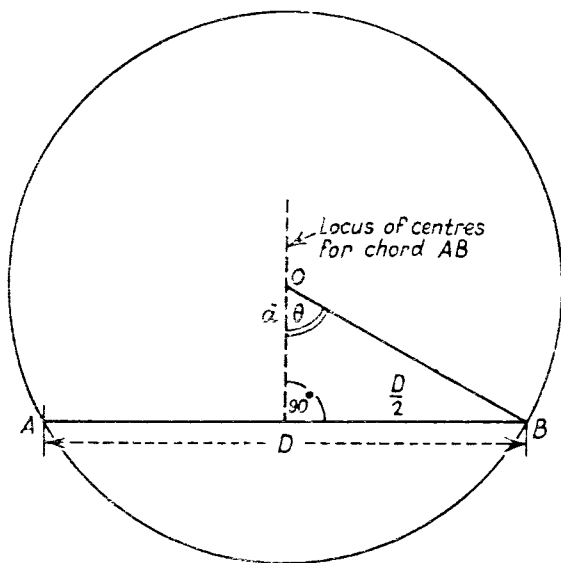


FIG. 85

It is sometimes advantageous to use two separate subtenses or a four-point fix as shown in Fig. 86.

Where the scale is too large for the control points to be borne on the sheet and beyond the limits of beam compasses, it is fairly simple to compute the arcs using a co-ordinate system and plotting

three points on each arc from three radii chosen to bring them on the sheet as shown in Fig. 87. Standard curve templates can be used for this purpose.

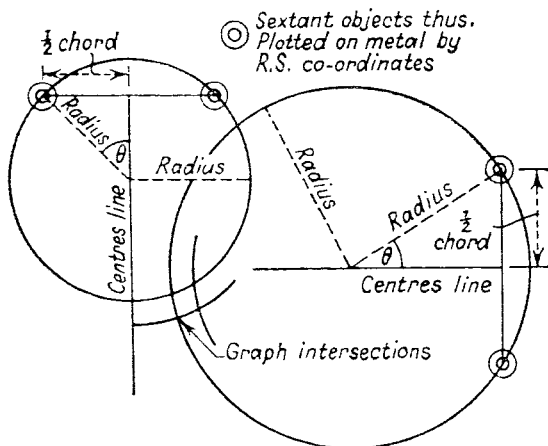


FIG. 86

The sheet should show a reference point connecting the area to the local trig origin, and related rectangular gridding at about

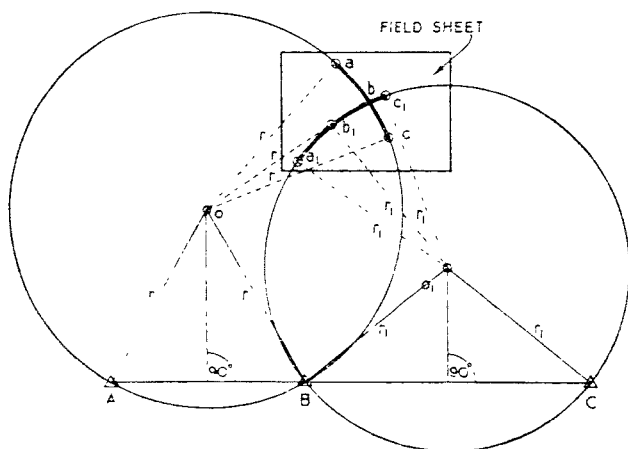


FIG. 87

6-inch intervals should be imposed as a check on distortion. A true meridian and scale should be added (Fig. 84).

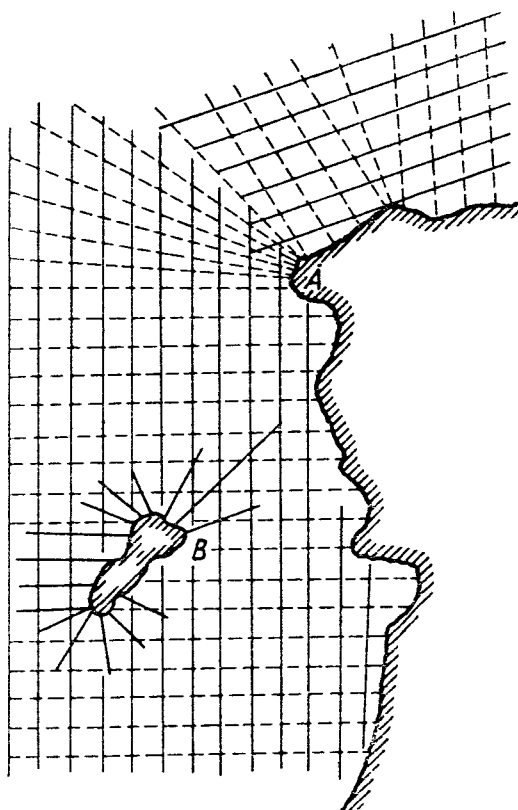


FIG. 88

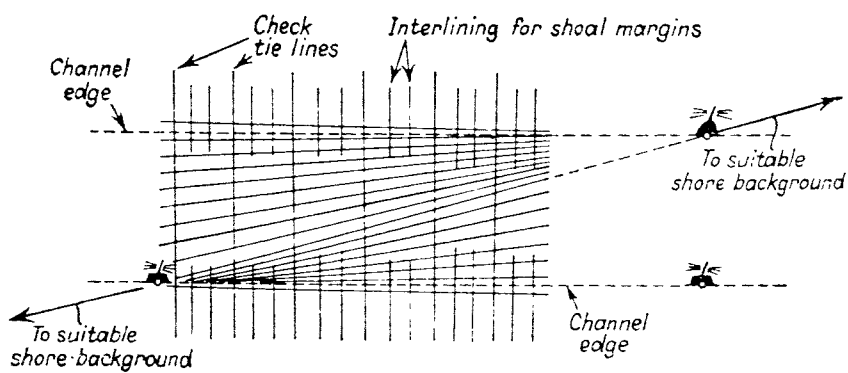


FIG. 89

PLANNING SOUNDINGS. The scale of useful hydrographic plans used for approaches to docks and in estuarine areas may be said to range from about $1/500$ for dock-wall surveys to $1/20,000$ for the outer coastal approaches to harbours. The traditional Admiralty rule is to plot sounding lines at intervals of about five to one inch and at scales of $1/1250$ and less. This rule should not be relaxed if close examinations are required.

On larger scales this rule can be somewhat relaxed provided the pernicious practice of running lines 100 feet apart drawn through traditionally fixed and unvaried points be avoided. Obstructions have frequently been missed in this manner, and it cannot be too much stressed that the principle of closely spaced parallel lines with further examination of discontinuous contours (or suspicious features) by more intersecting lines is absolutely necessary for safeguarding navigation.

Sounding lines off a straight coastline or contour may often with advantage be run at right angles, but this need not become a fixed rule. With outlying shoals extending from the shore, especially when echo-sounding methods are being used at moderate speeds, lines parallel to the shore or channel contours will often be found the best to start with, the usual cross lines being run afterwards.

Figs. 88 and 89 illustrate the treatment of various areas. It is again emphasized that cross check lines should *always* be run over any system of parallel or radiating lines, not only to define discontinuities and steep bottom gradient, but also to confirm consistency of fixing and sounding, also tidal reduction. Doubtful lines or soundings should be rejected at once and lines repeated.

Sounding routine.

- (i) Synchronize watches ashore and afloat and station tide-watcher.
- (ii) Steel tape, leadlines or depth wires for bar-check.
- (iii) Take pre-survey bar-check.
- (iv) Check adjustment of sextants and all instruments.
- (v) Plot all fixes with details and time, numbering fixes to agree with E.-S. operator's log and sounding book.
- (vi) Note time of all heavy-draught ships passing tide gauge near channels for assessment of interruption of normal tidal levels during survey. (Tide watchers should also note this.)
- (vii) After survey repeat bar-check over range of depths surveyed.

(viii) Dry record and pencil in depth line against fading; also draw true waterline zero.

(ix) Relate fix numbering on record to tidal levels and plot datum of sounding (zero of tide gauge) below true waterline zero. Ink in.

(x) Attach all tidal data to echo-sounding record for cross reference.

PLOTTING

Admiralty practice is to plot all fixes afloat, and there should be no deviation from this rule. For engineering or harbour soundings close to a permanent base it is generally more convenient to take off each day's numbered fixes on a 'rough collector', tracing in pencil lining in the ship's track, and to sub-divide the fix intervals for sounding points. An odd number of such intermediate points is chosen, as this facilitates the sub-division for 5, 7 or 15 intermediate sounding points by simple bisection of intervals as in Fig. 90.

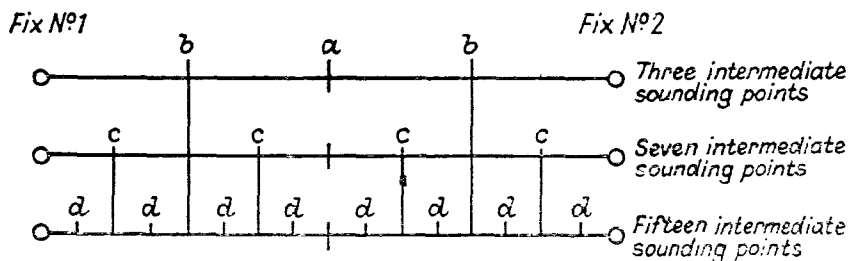


FIG. 90

THE ROUGH-COLLECTOR TRACING. The assumption of uniform speed over the ground is made, and where this is varied the surveyor will adjust intervals accordingly. The sub-division of the corresponding numbered fixes on the echo-sounding record is then made to agree with the 'rough collector' plot. This is best done by sub-dividing at the datum line between the fix arc sub-divisions and transferring to the bottom line by an arc template (see Fig. 81 above).

When record and collector tracing are in correspondence, soundings are inked in, different colours being used for each day and the following annotations made in the particular colour.

- (a) Date of soundings.
- (b) Number of sounding book.
- (c) District.
- (d) Echo roll number and stowage.

Shoal features between sounding points should, of course, be given prominence and discrepant lines or discontinuities marked, indicating repetition or more detailed examination. Shoals observed covering or uncovering at certain tidal levels should be confirmed until revealed by close soundings, and agreement of crossing lines should be well within 1 foot. If this limit is exceeded, re-examination of shore control points, strength of fixes and intimacy of representative tide-pole stations should precede further investigation by sounding. In this connection it is possible to experience Bernoulli effect at times other than slack water if sounding is carried out in narrow channels and sections with side constrictions or shallow side boundaries and the local tide pole is situated clear of and unaffected by such constrictions. With a tidal stream of 2 knots an error of 1 foot has been experienced in such a case. Conversely, if a near tide pole is situated close to a constricted channel section, and sounding is carried out in an area of relatively unrestricted flow, the same situation will arise.

In such circumstances, if the stationing of a more representative tide pole is impossible, sounding or tide near narrow sections will not become truly related unless the tidal streams near such sections fall below about 0.5 knot.

THE FAIR TRACING. When the rough collector tracing has been satisfactorily completed for the sounding site, and all discrepancies have been resolved *in the field* (not by estimation or *prima facie* judgment from the documents) and contouring is satisfactorily completed, it will be possible to transfer all to a fair collector tracing of a good transparency, which can then be printed for use. As, apart from metal or Astrafoil, no paper or tracing is strictly dimensionally stable, it is important to control all by fundamental and intimate gridding by dimensions or to standard metal plots throughout all the work referring to fundamental control points fitting locally when tracing off. Admiralty practice is necessarily bound to the drawing of the fair chart on linen-backed sheets, but frequency of coastal, estuarine, dock or engineering

surveys is best met by the fair tracing, if possible on Astrafoil or, failing this, on a good 'oily' vegetable tracing paper.

The fair tracing should ideally bear the following:

- (a) General locality and name of site.
- (b) Unit and datum of soundings and connection of datum to stable near bench marks and land survey datum; also to stable near temporary bench marks and tidal regime.
- (c) Date of survey.
- (d) Names of surveyor and assistants.
- (e) Natural scale. Fraction and graduated working scale.
- (f) The geographical position of one of the main trigonometrical stations shown on the chart or plan.
- (g) A true meridian drawn through one of the trigonometrical stations and extending over at least three-quarters of the length of the chart. It is not essential in harbour or engineering surveys that plan side borders should be oriented in the meridian. An inferior prominence can, if useful, be given to a magnetic meridian, using a half arrowhead on the side away from the true meridian, but, if this is done, the variation for a stated year should be given together with the amount of secular change.
- (h) The tracing should be gridded at about 6-inch intervals in relation to local origins, grid lines being finely drawn.
- (j) List of objects conspicuous to the navigator with heights given above M.H.W.S. where possible. Also all leading and clearing transits should be clearly drawn, showing true bearings and soundings taken on the line.
- (k) Outline of tidal regime at maximum and minimum ranges.
- (l) Concise memoir giving fundamental triangulation data in relation to local origin. Positions of local tide gauges and maximum rates and direction of set of tidal stream and currents from observations.
- (m) The general nature of the sea bed or bottom and any useful and relevant geological information.

ELECTRONIC CONTROL OF POSITION

At the present time (1952) the method for measuring distances by the reflection of radio waves as recorded by cathode ray techniques has revolutionized coastal navigation by providing accuracy

to within about 30 yards of the actual position under ideal circumstances and optimum instrumental adjustment.

The author is not concerned to describe these techniques in any detail, apart from saying that the use of the speed of radio waves as a fundamental of linear measurement will probably increase in accuracy as research progresses. The two methods at present used are :

(i) Radar.

(ii) Hyperbolic navigation.

(i) Radar proper produces a direct plot by reflection of all salient coastline oriented by gyro compass. This enables a ship's position to be plotted either by simultaneous distances of salient points or alternatively by imposing the P.P.I. (Plot Position Indicator) plot on an accurate chart and by comparison deducing the position. This method does not lend itself at the moment to anything like the survey precision required for large harbour scales, although techniques may yet be devised which will overcome the various sources of error to the surveyor's satisfaction.

(ii) Hyperbolic navigation is nothing more than plotting by interpolation within two intersecting lattice systems of hyperbolae depending upon three suitably placed fixed transmitting stations, two of which are 'slaves' to the main station. The receiving instrument measures the differences in time intervals between the reception of the signals and indicates the two respective readings which give the intersecting point by plotting on the lattice chart. It is obvious that the rate of linear change and angle of intersection of the hyperbolae will indicate the strength of the fix. The accuracy of this system under ideal conditions is ± 30 yards, and it is consequently not acceptable for survey work at scales larger than about 1/80,000 and then only after careful comparison with measured distances (Fig. 91).

For harbour and estuarine work the surveyor will find sextant graph techniques vastly superior in every respect to anything electronic research yet has to offer, but he will, of course, hopefully apply the usual standard linear tests to any new methods which present themselves in an age when research is proceeding at a stupendous pace. The value of any method which will give the position of a survey ship to a proved accuracy of say ± 5 feet in fog, or seriously reduced visibility, is obvious, although even here the surveyor will probably still prefer the precision now available in

the clear-weather intervals he utilizes for echo-sounding to such advantage. On the other hand, in localities where the incidence of fog throughout the year is high, the development of a local chain

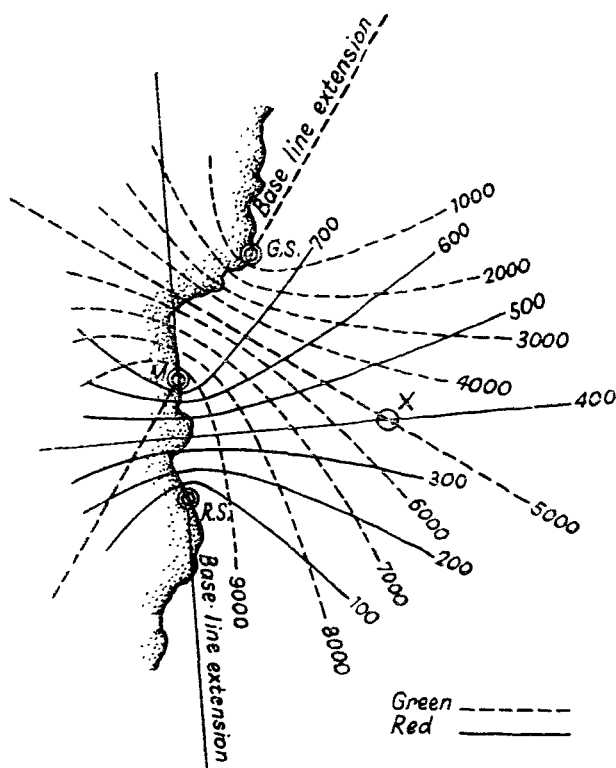


FIG. 91

of stations giving good hyperbolae cover on large scales to this accuracy would be attractive to a surveyor responsible for an important waterway carrying deep draft modern ships. The method by which he determines his position at fixes will not, of course, affect the routine and basic principles set forth in the preceding pages.

CHAPTER 9

ELEMENTARY PHOTOGRAMMETRY

THE CONCEPTION of using photographs for purposes of measurement appears to have originated with the experiments of Laussedat who in 1851 produced the first measuring camera. Nearly half a century elapsed however before the most important of all developments took place in the science of photogrammetry by the introduction of double image or stereoscopic projection. The instruments and techniques of plotting from stereoscopic pairs of photographs are now so highly developed that in nearly every case they are to be preferred to single-picture methods both on account of accuracy and speed of working. Nevertheless, there are occasions, particularly in exploratory surveys, when circumstances still make the single photograph a practical method of mapping. For this reason, and as a necessary commencement to the study of photogrammetry in general, we will start with the simplest case of the single photograph taken with the axis of the camera directed horizontally and the plane of the photograph truly vertical. This is the normal type of picture taken with a photo-theodolite.

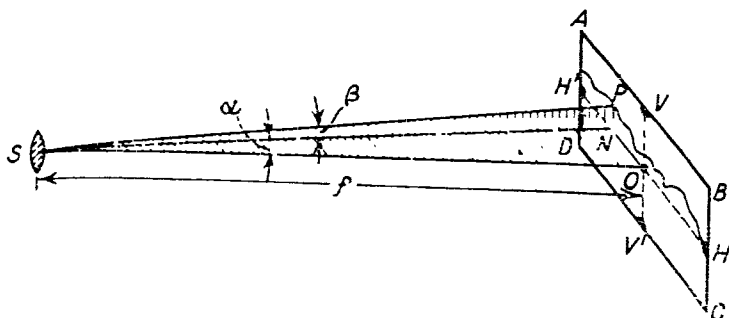


FIG. 92

GEOMETRY OF THE PHOTOGRAPHIC IMAGE

BASIC DEFINITIONS AND RELATIONSHIPS. In Fig. 92 ABCD represents the photographic plate (considered here as a positive and not a

negative). The optical centre O of the plate is located by the intersection of the two axes joining the registration ticks V, V^1 and H, H^1 (usually known as *collimating* or *fiducial* marks) which are engraved on the plate carrier and thus automatically recorded on the negative. The focus of the camera is set at infinity, and the focal length or *principal distance* of the lens is equal to f and is set normal to the plane of the plate.

SVV^1 is known as the principal plane.

P is any image point recorded on the plate.

N is the foot of the perpendicular from P to HH^1 .

$PN = y$, $NO = x$ are the plate co-ordinates referred to the optical centre or principal point of the plate.

On the assumption that we know the height of the station from which the photograph was taken and that the camera has been correctly set and levelled, then:

β , the vertical angle to P from S , is given by $\tan \beta = \frac{y}{SN}$

and α , the angle between the principal line and the direction of

the ray to P , by $\sin \alpha = \frac{x}{SN}$ or $\tan \alpha = x/f$

The graphical method of plotting is to prepare a template on tracing paper bearing a cross marked off with the principal distance f along the tail (f being the focal length of the camera multiplied by the enlargement factor of the print used). By placing the template in position over the photographic print, with the principal line over VV^1 and the arms of the cross along HH^1 , it is possible to mark off graphically ON and thus measure SN on the template, x and y being measured on the print. In this way the photograph can be treated in the same manner as a combined sight-rule and clinometer would be in the hands of the plane-table, with the difference that all plotting of the map is relegated to the drawing office, and the field work on each station is reduced to exposing the photographs and to taking a few angles on the theodolite sufficient to resect the station itself and establish its height.

Plate I shows the TAL photo-theodolite developed by Messrs. Zeiss Aerotopograph before the war. This instrument with its stand and accessories weighs only 17 lb. and is ideal for small-scale topographical mapping in rugged terrain.

We may next consider the much more complex case of the oblique photograph taken either on the ground or in the air. There are two classes of obliques—*high* or *horizon* and *low* or *non-horizon*. In the former the axis of the camera is tilted to include the visible horizon, while in the latter type the depression of the axis is too great for the horizon to show. Fig. 93 shows the conditions of perspective when the plane of the negative ABCD is inclined to the horizontal (ground) datum by an angle δ .

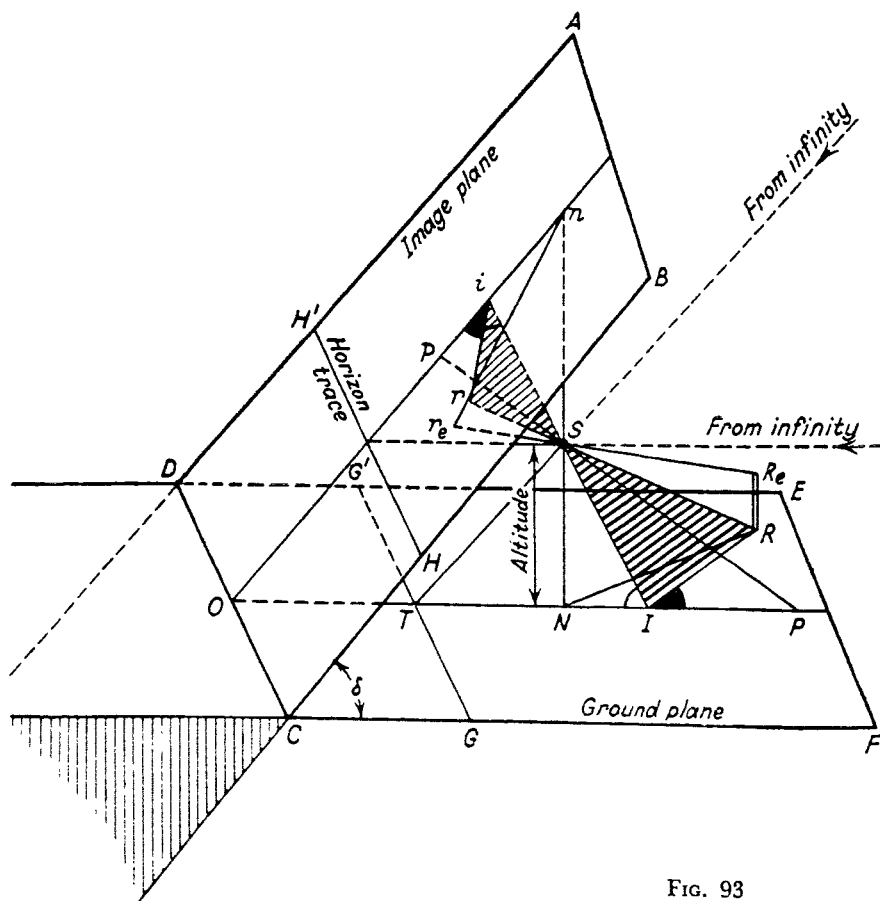


FIG. 93

S is the perspective centre of the lens.

N is the *plumb* or *nadir* point where the plumb line from S meets the ground plane, which is assumed to be flat and horizontal.

n is the image point on the negative of N .

p is the principal point of the negative, being the foot of the perpendicular from the lens S on to the photographic plate.

P is the point on the ground plane corresponding to image point p on the negative. $npoNPS$ all lie on the *principal plane*, which is at right angles to both the plane of the negative $ABCD$ and the ground plane $CFED$. HH^1 is the trace on the negative of the horizontal plane containing S the perspective centre and is known as the *horizon trace*. It is clear that the images of all infinitely distant points on the ground plane would be represented by the horizon trace on the negative. Similarly, if ST is drawn parallel to the plane of the negatives, then GTG' is the ground trace of all infinitely distant points on the image plane.

The bisectors of the angles NSP and nSp meet the negative and ground planes in points I and i respectively. These are known as *isocentres* or *metapoles*.

In Fig. 93 consider $\triangle s$ ISN and iSp .

$\angle N$ and $\angle p$ are both right angles

$\angle pSi = \angle NSI$ by definition

$\therefore \triangle ISN$ is similar to $\triangle iSp$.

$\therefore \angle SIN = \angle Sip$ and thus the line Ii intersects the ground and image planes at the same angle.

Let R be any ground point and r its image point on the negative.

Provided R and r remain on the same planes as I and i respectively, then the spatial triangles SRI and Sri will be symmetrical and thus $\angle RIP$ will be equal to $\angle rip$.

In other words angles subtended at I by points on the ground are equal to the angles subtended at i by the corresponding image points on the plate.

Now examine what happens with points lying above or below the datum plane such as R_e . For convenience we may say that R_e is vertically above R , in which case R is the orthographic projection of R_e on the datum plane. Let r_e on the plate be the corresponding image point of R_e , the ground point. It is clear that the plane containing SR_eRN is vertical and must necessarily also contain n , r and r_e . *Therefore the displacement of r_e from r due to height is radial not from the isocentre i , but from the plumb point n . Thus when there exists both ground relief and tilt, there is no single point having angle-true properties.*

Although the majority of air photographs used for survey purposes are referred to as *verticals*, as yet no apparatus has been

invented which will entirely eliminate tilt. Normally we have no means in a tilted photograph of determining the position of the plumb point and consequently also the isocentre. The principal point is, however, accurately located in the camera calibration by reference to the intersection of the lines joining the fiducial marks, which latter are engraved on the pressure glass and appear on every exposure made.

Suppose in Fig. 94 that n , i , p , are respectively the positions of plumb point, isocentre and principle point on the image plane, and that r_0 is the image which a ground point R would occupy on an untilted photo.

Considering tilt displacement alone, the error made in angular measurement to the point r by using p instead of i would be equal to $\angle r_0 p r = \alpha$. For very small tilts this can be accepted as negligible. Directly, however, there is ground relief r moves to r_e , and the difference between the true and false angular measurement at p is given by $\angle r_0 p r_e = \alpha + \beta$ (Fig. 94). Since the height error is usually much greater than the tilt error, the effect of both together cannot be tolerated except within narrow limits.

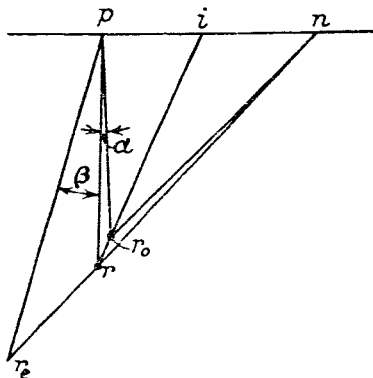


FIG. 94

In the method of 'radial line' graphical plotting a generally accepted 'rule of thumb' guide is that tilts should not be greater than 4° and the ground relief should not exceed 10 per cent of the mean altitude of the aircraft above the terrain. Within these limits the principal point can be used in place of the plumb point and isocentre without introducing serious error.

ANHARMONIC RATIOS. Fig. 95a shows four ground points VWRQ whose image points are $vwrg$. Related pairs of points such as Vv , Ww , Rr , Qq are known as *homologues*. Any two such homologous points (say VQ and vg) determine a plane with the perspective centre S . (This should be clear from a study of Fig. 95a.)

$VQSqv$ is one plane. Similarly $RQSrq$ is another plane and so on.

These planes cut the image and ground planes in the lines joining the homologous points.

The lines connecting any other point X with $QVWR$ represent a pencil of rays. Each such line when extended will meet the hinge of the two planes in T_1, T_2, T_3 and T_4 respectively. If x is the

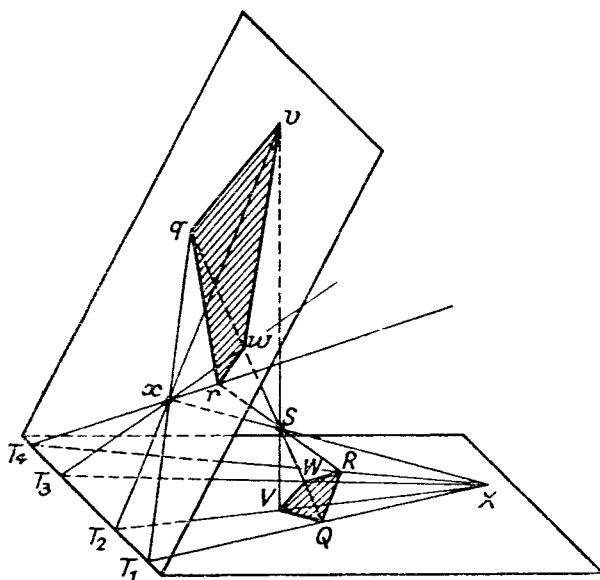


FIG. 95a

corresponding image point of X , then XQT_1xqS is one plane. Similarly XVT_2xvS is another plane and XWT_3xwS is a third plane and so on. It follows that any pencil of rays on the ground plane will intersect with the pencil of rays produced by the homologous points on the image plane.

Imagine the two planes of Fig. 95a folded back along their axis of intersection so that they both lie in the plane of the paper (Fig. 95b).

Let $LMNR$ be any transversal. Then it can be shown that:

$$\frac{a'}{b'} : \frac{c'}{d'} = \frac{a}{b} : \frac{c}{d} = K$$

These are known as anharmonic ratios and are used as the basis for the 'four-point' method of graphical plotting as follows.

intersection with B gives the location of L on the map. This method is sometimes used for superimposing a map grid on a photograph (see Fig. 97).

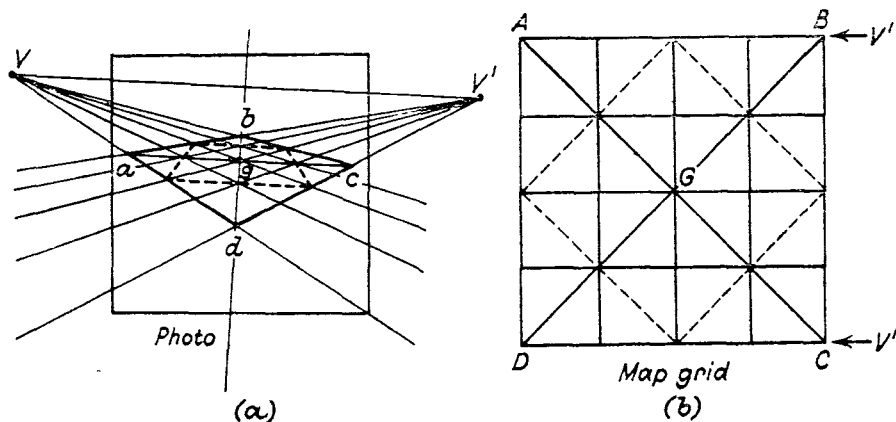
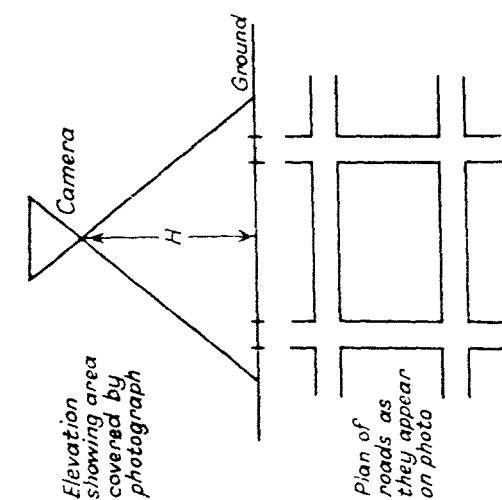


FIG. 97

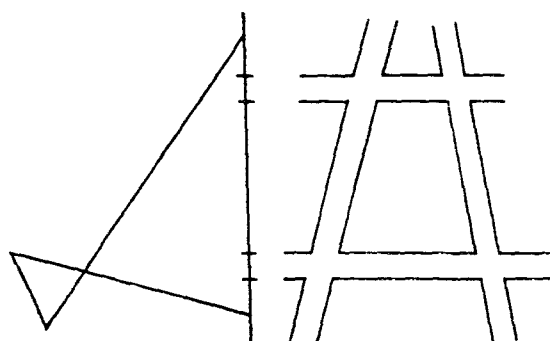
ABCD are the four corners of the map grid, which can be transferred to the photograph by the four-point method just described, assuming of course that the surveyed positions of at least four identified points on the map are known. If ab , cd are produced, they will meet at V^1 , the *vanishing point*. Similarly ad and bc produced will meet at V . From Fig. 93 it is self-evident that straight lines on the ground plane appear as straight lines on the image plane. Therefore the intersection of the diagonals ac and bd in Fig. 97a must correspond with that of the diagonals AC and BD on the map grid (see Fig. 97b).

Now agc represents the line of intersection of two infinitely distant pencils of rays which on the map are parallel and equidistant grid lines. If therefore we divide ac equally into the same number of parts as AC on the map grid, then the intersections of all the rays from V and V^1 through these points will give the corresponding cuts of the map grid on the photograph.

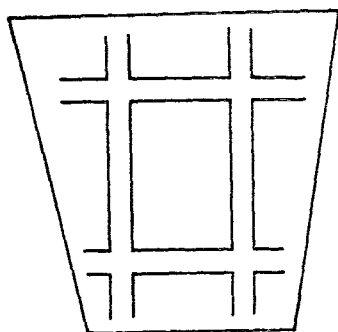
RECTIFICATION. The term *rectifier* is misleading, as none of these instruments will rectify the actual perspective of the photograph. They serve merely to compensate partially for some of the effect present in a tilted picture.



Case 1. Air camera vertical.



2. Air camera tilted.



3. Rectified print showing roads restored to conditions of untilted photo.

FIG. 98

Case 1. Fig 98 shows a rectangle of roads (in perfectly flat and level country) photographed from a truly vertical camera. There is here no distortion of the roads and on the photographic print they will appear as they would on a map.

Case 2. Shows the distorted appearance of the roads on a tilted photograph.

Case 3. Shows the appearance of the print (Case 2) after rectification. It will be seen that the roads are restored to their true shape, but the print is of course no longer square.

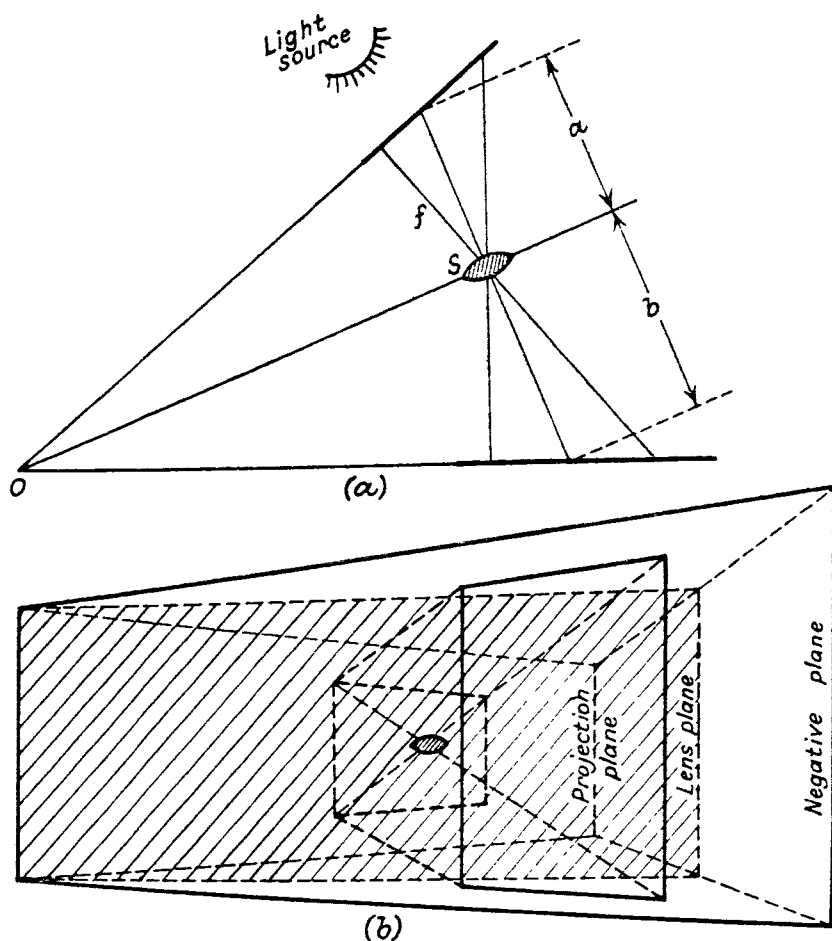


FIG. 99

In any optical projection it is a condition for sharpness of definition that the following two conditions are satisfied: (1) $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ where a and b are the distances of the image and lens planes respectively from the nodal centre of the lens S , and f is its focal length (see Fig. 99a). (2) The planes of the negative, the lens and the projected image intersect in a straight line. This is usually known as the Scheimpflug condition (see Fig. 99b).

All types of fully automatic rectifiers are fitted with mechanical systems (referred to as invertors) which maintain these two conditions. In the Wild E2 apparatus (Plate II) condition (1) is provided by a scissors invertor (Fig. 100) connecting the planes of the negative lens and table.

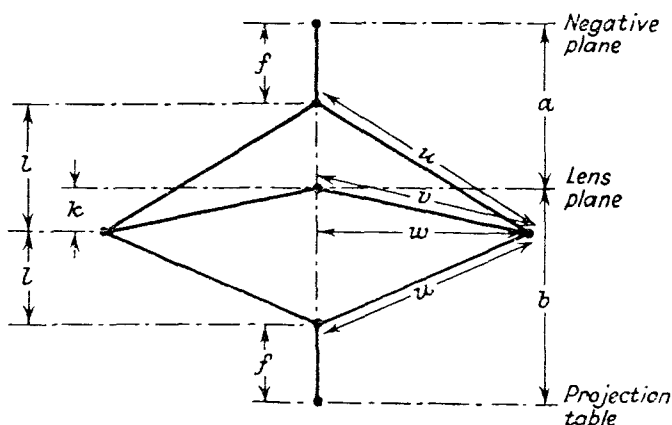


FIG. 100

By definition $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ or $(a + b)f = a \times b$

From the diagram $a = f + l - k$, $b = f + l + k$

$$\therefore a + b = 2(f + l)$$

$$a \times b = (f + l)^2 - k^2 = 2f(f + l)$$

$$\therefore f^2 = l^2 - k^2$$

But $l^2 = u^2 - w^2$

And $k^2 = v^2 - w^2$

$$\therefore l^2 - k^2 = u^2 - v^2 = f^2$$

Provided then that the scissors are dimensioned so that $u^2 - v^2 = f^2$, this condition of focus will be maintained. Fig. 101 shows how the Scheimpflug condition is complied with in the same apparatus.

Two levers NN_1 and PP_1 act at right angles to the negative and projection plane respectively and are connected by a long rod N_1P_1 which swivels about a point O at a fixed distance C from the lens plane. N_1 and P_1 slide on horizontal rails also at distance C from N and P respectively.

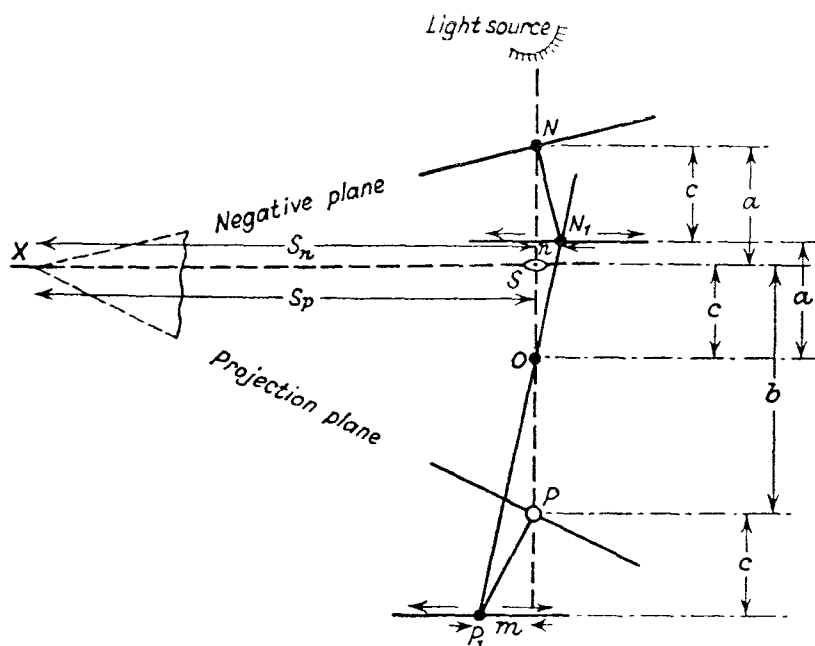


FIG. 101

From similar triangles:

$$\frac{S_n}{a} = \frac{c}{n} \quad \frac{a}{n} = \frac{b}{m} \quad \frac{S_p}{b} = \frac{c}{m}$$

$$\therefore S_n = \frac{ac}{n} = \frac{bc}{m} = S_p$$

Therefore SX the plane of the lens will always intersect both XN the negative plane, and XP the projection plane, and the Scheimpflug condition is fulfilled.

In general there are five independent elements necessary for rectification. These are:

1. Variation of the projection distance (change of scale).
2. Tilt of the plane of projection about a horizontal axis.
3. Rotation of the negative in its own plane (swing).
4. Displacement of the negative in its own plane vertical to tilt axis.
5. Displacement of negative in its own plane parallel to tilt axis.

These are shown diagrammatically in Fig. 102.

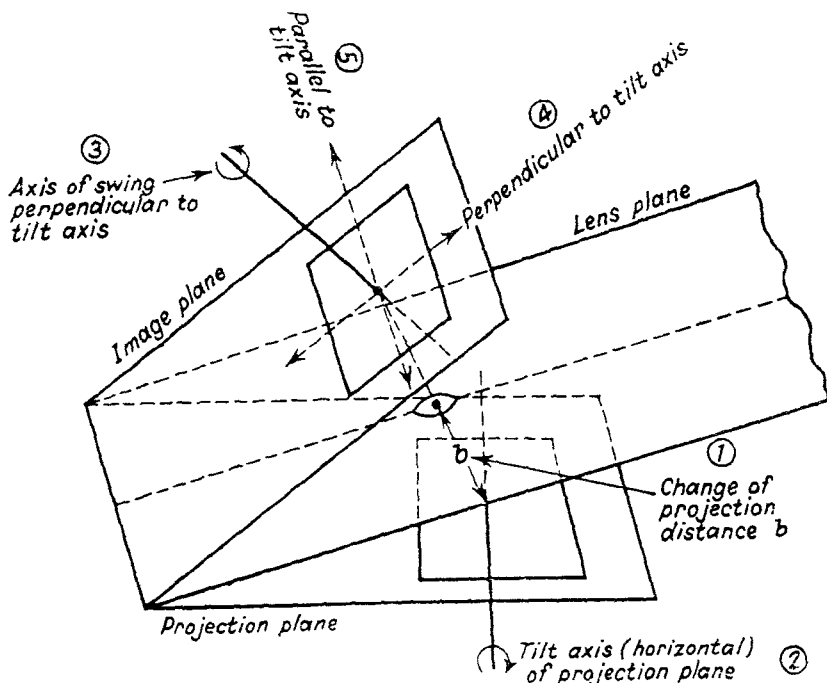


FIG. 102

In the case of photographs taken of hilly country there is no means of eliminating in one print the effects of tilt. This will be better understood from a study of Fig. 103 which shows what would occur if the same rectangle of roads illustrated in Fig. 98 were to lie in fact on the side of the hill. It is clear that the recorded position (*a*) on the photographic plate of a point A on the road crossing the hill will not be the same as that of B, the true orthogonal projection of A. B's corresponding image point (*b*) will be

displaced from (a) by an amount δp with the result that the road will no longer appear straight on the photograph but will be distorted in varying degree according to the height of the hill and the conditions of perspective. There is no means of rectifying for

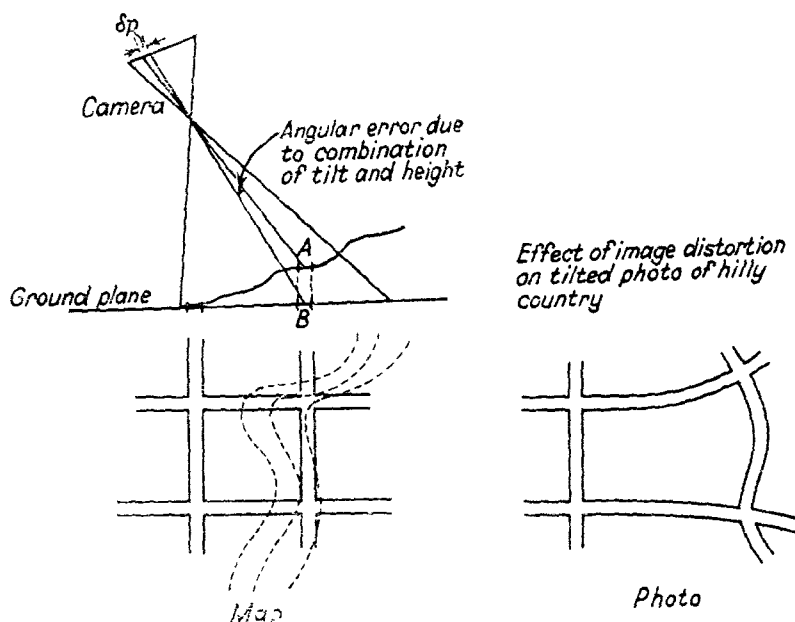


FIG. 103

such distortion except by dividing the area into a large number of different planes and rectifying separately for each. (The Brock process used in the United States works broadly on this principle.)

Rectified photographs are used for (1) the preparation of 'controlled' mosaics, (2) map revision and (3) original line maps of 'flat' areas. The photography is often carried out with a lens of long focal length in order to minimize as much as possible the effect of height displacement. For the same reason only the central portion of the enlargement is used when there is appreciable relief. Each enlargement is scaled in the rectifier to four control points. These may be supplied either (a) from an existing map, or (b) by normal methods of ground survey, or (c) by 'slotted template' control, a method of semi-graphic 'air-triangulation' which is now to be described.

PLOTting FROM PHOTOGRAPHS BY GRAPHICAL METHODS

RADIAL LINE TRIANGULATION. We have already stated that, provided tilts are kept below 4° and the relief of the terrain is not more than about 10 per cent of the mean altitude of the aircraft above ground, the principal point of the photograph (or some point near it) can be used in lieu of either the isocentre or the plumb point in so far as any rays drawn from it to points of detail on the photo will be true in angular relationship to the corresponding measurements made by theodolite or plane-table on the ground. Vertical air photographs for survey purposes are normally taken in strips in such a manner that successive exposures overlap in the direction of flight by 60 per cent (see Fig. 104). There is thus an overlap of 10 per cent (shown shaded in the figure) which is common to three photographs.

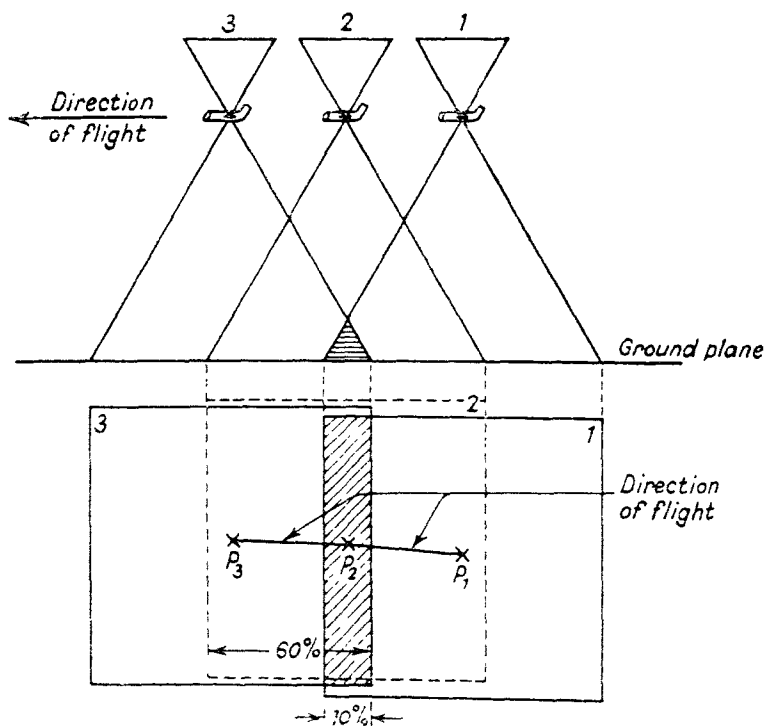


FIG. 104

Fig. 105 shows the same three photographs as Fig. 104 but separated. P_1 is the principal point of photograph No. 1 (or some image point very near the principal point). P_2 is the principal point of photograph No. 2 and p_2 the homologous positions of P_2 on photographs 1 and 3 respectively. Similarly p_3 is the corresponding position of the principal point P_3 (photograph No. 3) on photograph No. 2.

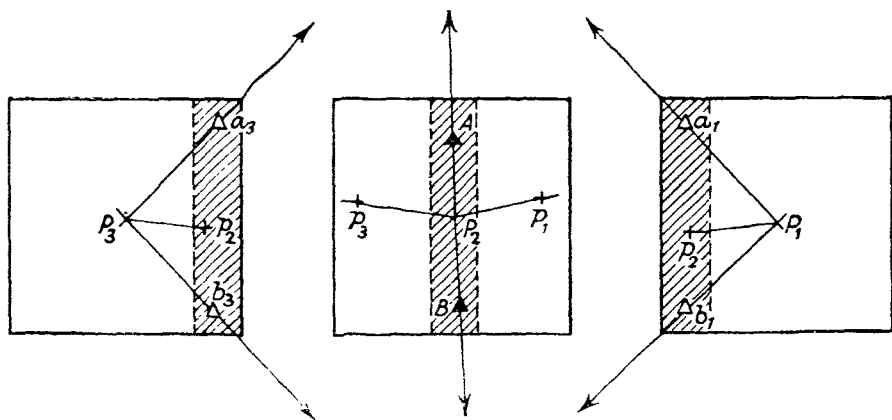


FIG. 105

P_1-p_2 gives the direction the aircraft flew between taking photographs 1 and 2 which necessarily has the same orientation as P_2-p_1 , so that, if we were to lay photograph No. 2 over photograph No. 1 along this common axis and photograph No. 3 over photograph No. 2, so that the trace of P_2-p_3 is aligned with that of P_3-p_2 , then the three photographs are related correctly in azimuth.

Suppose we now select on photograph No. 2 two points of detail A and B on the common overlap to the three photographs (shown shaded), and well spaced on either side of the principal point, then $a_1 a_3 b_1 b_3$ are the corresponding images of A and B on photographs 1 and 3 respectively. Since all angles drawn from $P_1 P_2 P_3$ are true, the intersection of the rays drawn from P_1 to a_1 and P_2 to A will give the true plan position of point A relative to the line of flight. In other words we have established the $\triangle P_1 A P_2$ to some scale which is fixed by the distance P_1-P_2 . (We can make this what we like, but it is usual to take the mean of the principal point bases for all photographs of the strip, thus establishing a mean photograph scale for the strip.)

tracing paper and mark off these distances along it. Then draw a semi-circle with centre Y and radius Yy . Draw Xt tangential to this. To scale any other point P on the strip lay the template so

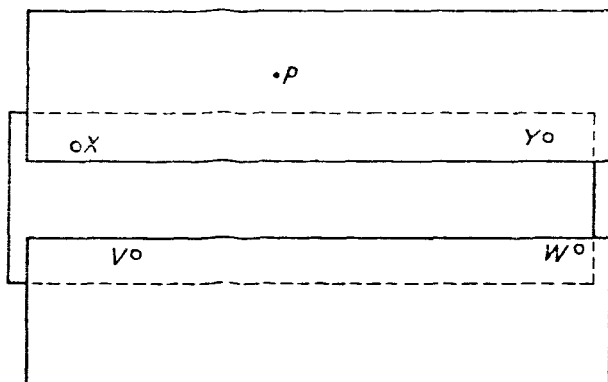


FIG. 107

that X (on the template) corresponds with x on the plot of the strip to be scaled and the line XY passes through P . When so positioned set a pair of dividers to the distance from P to the line Xt and strike an arc to cut XY in p (p always being on the same side of P as y is of Y). Then p marks the scaled position of P and may be pricked

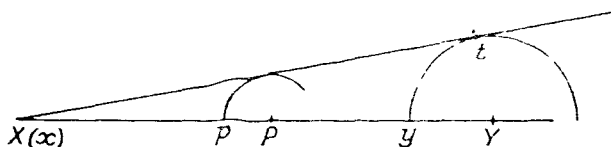


FIG. 108

through on the plot. The same method can be used for scaling between ground control points where these exist. It is best to begin with a strip on which there are two well-spaced ground points and to scale this strip first which establishes the scale of the plot; then bring all other strips to this standard scale.

Slotted template method. The satisfactory 'block' adjustment of graphical minor control plots is often difficult, particularly in areas where control is sparse and tilt error due to relief exists. Most, if not all, of these difficulties can be eliminated, and the best mean fit

automatically obtained between points of ground control, by the method known as 'slotted template' in which all rays from the principal points to minor and ground control points are replaced by slots cut in cardboard or acetate templates (Plate III and Fig. 109). One such template is prepared for each photograph, and its principal point and the photograph positions of all other control points are pricked through, circled and numbered on the template. With the aid of a special spring-loaded punch, a small circular hole is cut in the template centred exactly on the principal point. The diameter of this hole is such that it will fit snugly over a sliding metal stud in the cutting machine (Plate III). The cutting blade of the machine is designed so that it can be centred accurately over the photograph position of any point marked on the template, and when in position cuts a longitudinal slot of the same width as the metal stud and in a direction radial to its centre. Fig. 109 shows a completed template for one photograph containing in all nine slots.

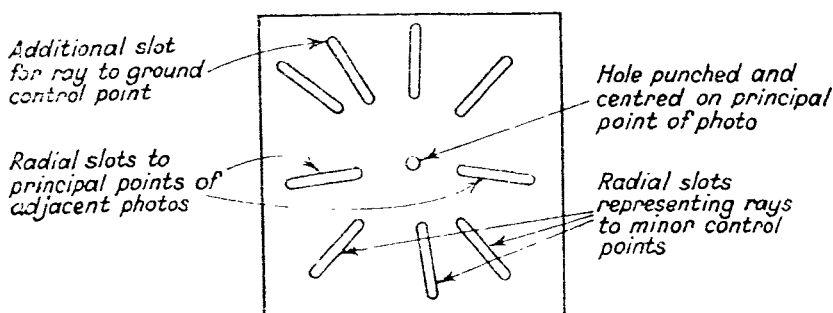


FIG. 109

Six of the slots represent radial directions of the minor control points, one slot a ground control point, and the remaining two represent the directions of the air bases. Each slot and the principal point hole is fitted with a stud which fits exactly both the central hole and the width of the slots. All studs are drilled centrally with a fine hole to accommodate a steel pin. In practice a specially prepared floor or dais is set aside for assembly of templates. Ground control points are accurately plotted on a map grid drawn on the floor and represented by fixed studs.

The assembled templates are adjusted until two or more slots belonging to each ground control point will fit (without buckling)

over the fixed stud belonging to that point. When this is achieved, all the 'free' studs representing minor control points and principal points will be automatically adjusted for scale and their positions transferred to the floor by gently hammering a pointed steel pin down the central hole of each stud. When the templates are removed, the position of all points can either be scaled off from the map grid or pricked through on to transparent sheets. They will then be used (1) as the framework for detail planimetric mapping or (2) for controlling rectified enlargements. Plate IV shows a completed assembly of templates covering an area representing some 2,500 square miles of country at a photograph scale of $1/30,000$. The black triangles (just visible in the photograph) denote the positions of ground control points.

Detail mapping. When the scaled minor control plot or slotted-template assembly is to be used as a basis for planimetric mapping additional points known as 'detail points' (see Fig. 106) are selected on the photograph and 'cut in' graphically on the plot.

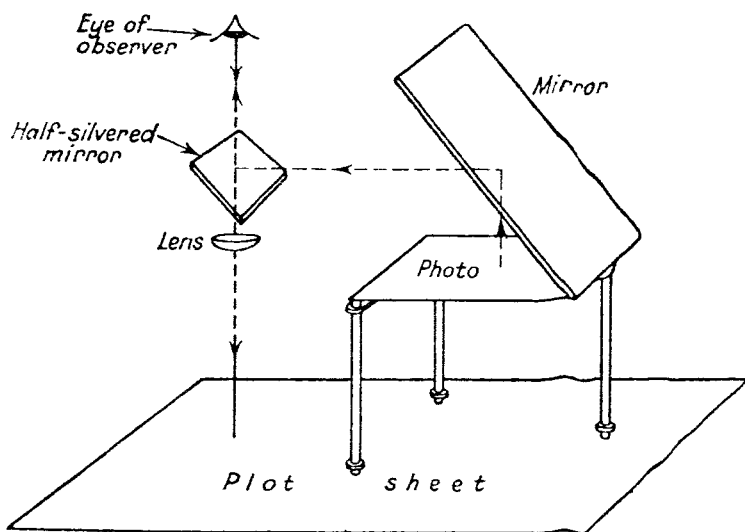


FIG. 110

The topographical detail is then filled in by tracing from the photographs, making any local adjustments for scale between the detail points; the latter are provided at close enough intervals to ensure that the degree of local adjustment or *equating* is very small.

When rectified prints are used, it is sometimes possible in flat country to dispense with detail points entirely and trace direct from the print between the minor control and principal points.

The Sketchmaster. Yet another method of supplying planimetry in flat country is by the use of a simple instrument giving a virtual image, such as the Sketchmaster, shown diagrammatically in Fig. 110. This consists of a tripod supported in a frame on which is carried the photographic print. The observer looks down with one eye through a half-silvered mirror and sees the image of the photograph superimposed on the map or plot sheet. Small corrections for tilt and scale adjustment between control points can be effected by adjusting the tripod legs. A selection of lenses is supplied with the instrument, one of which is fitted below the half-silvered mirror and assists in reducing parallax which is nevertheless nearly always present in some degree.

STEREOSCOPIC VISION. The impression of 'depth', which those of us who possess normal eyesight obtain, depends on the separation of the eyes. Each eye views an object from a slightly different position, and by a remarkable physiological process the two separate images combine together in the brain enabling us to see in three dimensions. Exactly the same effect can be obtained if a pair of photographs is taken of an object from two slightly different positions of the camera and is then viewed by an apparatus which ensures that the left eye sees only the left-hand picture and the right eye is directed to the right-hand picture. The two separate images will fuse together in the brain to provide the observer with a spatial impression of the subject of the photograph. This is known as *stereoscopic fusion*.



FIG. 111

In normal binocular vision the apparent movement of a point viewed first with one eye and then the other is known as *parallax*, and the angle subtended at the point by the eye-base (the normal distance between the eyes is about $2\frac{1}{2}$ inches) is known as the *parallactic angle*.

Fig. 111 shows two pairs of dots at different separations. Place a piece of cardboard midway between $a\ b$ and $a_1\ b_1$ at right angles to the plane of the paper so that the right-hand dots $a_1\ b_1$ are screened from the left eye and the left-hand dots $a\ b$ from the right eye. Then stare hard and you will observe that a and a_1 fuse together to form a single dot which appears *closer* than the fused image of b and b_1 . This apparent difference in level is known as stereoscopic depth, and depends on the spacing between the dots or the *parallax difference* as it is called.

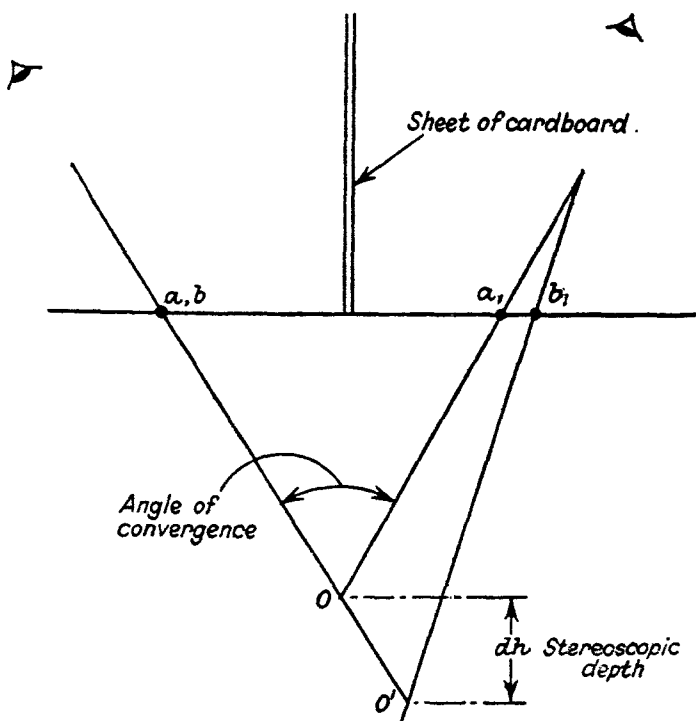


FIG. 112

Fig. 112 shows the corresponding diagram in elevation. It will be seen that the axes of the eyes are directed to points O, O' 'behind' the plane of the paper, whereas the eyes must be focussed for the plane of the paper if the dots are to remain sharply defined. In more specific terms—the *convergence* of the eyes is not in sympathy with their *accommodation*.

In natural vision this condition is automatically fulfilled since the eye is equivalent to a lens of variable focus. As the distance a_1 (or b_1) gets wider however, so the strain on the eyes in obtaining fusion becomes greater. With experience the eyes become more and more 'elastic', so that it is even possible for some observers to retain fusion when the axes of the eyes diverge. The geometry of Fig. 112 should not therefore be accepted too rigidly.

Stereoscopes are designed for two purposes:

- (1) To assist in presenting to the eyes the images of a pair of photographs so that the relationship between convergence and accommodation is relatively the same as it would be in natural vision.
- (2) To magnify the perception of depth.

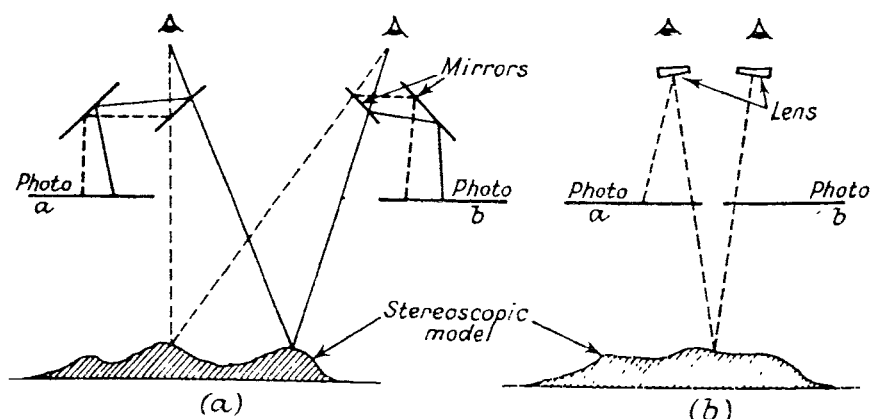


FIG. 113

Fig. 113 shows diagrammatically two of the most commonly used types of simple stereoscope. The mirror type stereoscope (Fig. 113a) allows the photographs to be widely separated, and the whole area of the overlap can be viewed at one time with little eyestrain but with some slight diminution of scale. Fig. 113b illustrates the pocket lens type of stereoscope which gives a $2\times$ magnification that is useful for interpretation. The lens stereoscope is, however, apt to cause eyestrain as accommodation is not in sympathy with convergence and the axes of the eyes are forced out of their normal conditions of vision.

We have already shown that the relative difference in spacing of the dots is a measure of the parallax difference. In

photogrammetric survey differential parallax is measured in units of length on the photographs. *Absolute parallax* may be defined as the linear measurement on one photograph of a stereoscopic pair between the image of a point and that to an infinitely distant point.

Fig. 114 shows two truly vertical photographs taken from the same height H above mean sea level. It is to be noted that the photographs are represented as positives in this diagram.

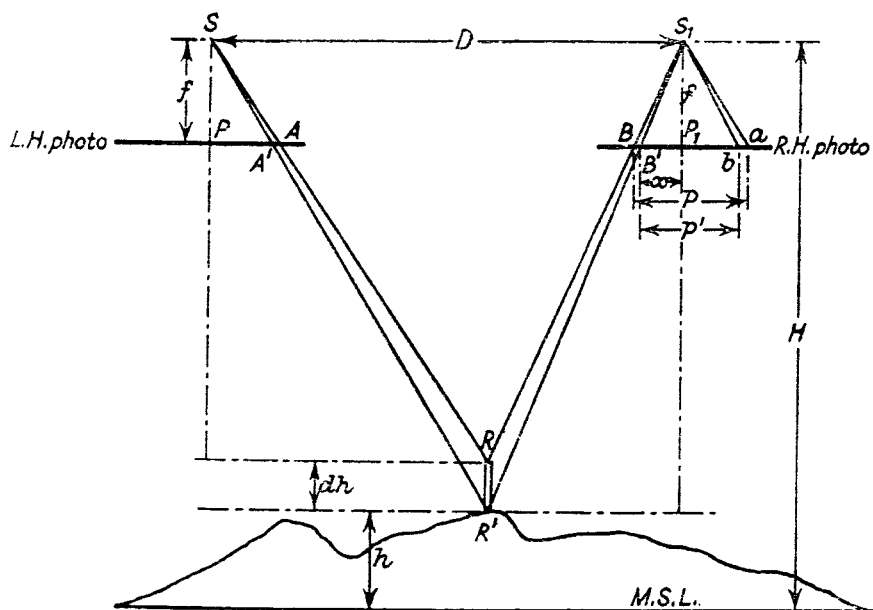


FIG. 114

The absolute parallax (p) of R , the top of the chimney, is given by Ba , where S_1a is parallel to SAR .

Similarly p_1 , the absolute parallax of $R^1 = B^1b$.

The height of the chimney (dh) can be calculated from the difference in absolute parallax.

If f is the focal length of the camera, then by similar triangles

$$\frac{p}{f} = \frac{D}{H - (h + dh)}$$

$$\therefore p = \frac{Df}{H - (h + dh)}$$

Similarly $p^1 = \frac{Df}{H-h}$

dp = difference of parallax between R and R¹

$$= p - p^1 = Df \left(\frac{1}{H - (h + dh)} - \frac{1}{H - h} \right)$$

$$= \frac{Df \, dh}{(H - h)^2 - dh(H - h)}$$

$$\therefore dp = \frac{p^1 dh}{H - h - dh} \quad \dots \quad (1)$$

or in another form $dh = \frac{dp(H - h)}{p^1 + dp} \quad \dots \quad (2)$

These are the fundamental parallax equations.

In cases where h and dh are small compared with H equation (1) reduces to

$$Hdp = p^1 dh \quad \dots \quad (1A)$$

and equation (2) to $dh = \frac{Hdp}{p^1 + dp} = \frac{Hdp}{p} \quad \dots \quad (2A)$

Measurement of parallax. From the foregoing it is clear that in a pair of truly vertical photographs all points of the same height have the same absolute parallax. Differences in height produce differences in parallax. There are various types of instruments for measuring parallax differences, the simplest being the *parallax bar* illustrated in Plate V. This consists of a metal rod carrying two sleeves which hold small glass or perspex slides each bearing an engraved mark at the centre. The left-hand sleeve can be locked in position by means of a clamp screw, while the movement of the right-hand sleeve parallel to the bar is controlled by a micrometer screw which can be read off in hundredths of a millimetre.

Plate V shows a pair of vertical photographs correctly set up under a mirror stereoscope with base line parallel to eye-base and the distance apart of the photographs such that fusion is obtained with least strain. The parallax bar must also be placed with its axis parallel to the base line and the distance between the measuring marks adjusted to suit the separation of the photographs. This is done by placing the micrometer approximately in the centre of its run and the right-hand mark over the principal point of the right-hand picture. The left-hand mark is then moved until over the

corresponding image of this point on the left-hand picture and clamped permanently in this position. Seen through the stereoscope, the two marks will fuse together as a single dot which appears to float in the three-dimensional model of the terrain. As the marks are brought closer together so the dot appears to rise relative to the ground. When the separation of the marks is increased the fused dot will appear to drop until it first touches the ground and then sinks below. When the latter occurs, the fusion of the separate marks is 'broken' by the stronger impression of the ground image and the dot is seen to 'split' into two. Fig. 115 shows diagrammatically what happens.

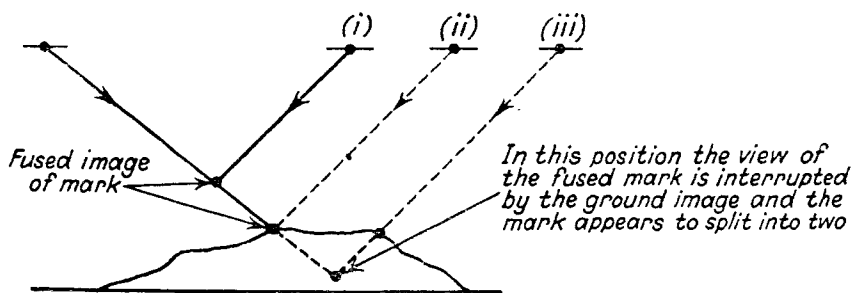


FIG. 115

It should be clearly understood that the separation of the photographs has no bearing whatever on the measurement of parallax, and is purely a matter of convenience to suit the type of stereoscope used. Thus a single measurement between the measuring marks of the bar, corresponding to the distance apart of two corresponding images on the photograph, has no meaning, but the *difference* in the measurements taken between two pairs of corresponding points will give the difference in parallax and thus enable the height of one point to be deduced from the height of the other.

From equation (2) above we have:

$$h_2 = h_1 \pm \left[\frac{dp(H - h_1)}{p_1 + dp} \right]$$

where:

h_2 is the height of point No. 2

h_1 is the known height of point No. 1

dp is the difference in parallax measured in mm. on the parallax bar

$(H - h_1)$ is the height of the aircraft above point No. 1 which can be deduced from the altimeter reading above mean sea-level corrected for the height of point 1

p_1 is the absolute parallax of the datum point $= \frac{fD}{H - h_1}$, D being deduced from the mean value of the principal point base measured from the two photographs.

When a measurement is being made, the left-hand mark is placed over the photograph point and by moving the right-hand micrometer the fused dot is made to 'float' down until it just touches the ground. Any lack of parallelism of the bar with the base line will introduce y parallax in a direction at right angles to the base line and can be removed by a very small swing of the bar in the plane of the photograph with the left-hand mark still held over the point. When the fused dot appears sharp and clear and just lying on the ground, a reading is taken on the micrometer and recorded. It is usual to take the mean of several readings for dp . With an experienced observer the error due to reading observed parallax differences should not be greater than $\cdot 01$ mm. The above remarks apply to untilted photographs. Tilt however is almost invariably present to some degree in the photographs, and the effect of this is to introduce false parallax which may be divided into two components, parallel, and at right angles to the base. The latter is known as *want of correspondence* and has no effect on the stereoscopic impression of depth.

Normally tilts should not exceed 2° in good survey flying, but even in the case of a 2° lateral tilt, the height error introduced between two points lying in a line at right angles to the air base and at a ground distance apart of 2,500 feet (3 inches on a photograph at 1/10,000 scale) will be approximately 87 feet. Thus in tilted photographs direct parallax heights are of very little value except for points that have identical plan position (i.e. the top and bottom of a tree) or which are situated in close proximity. It is, nevertheless, still possible to use parallax measurements for breaking down further a close network of spot heights provided on the ground. The most satisfactory method of doing this is to select two ground heights A and B lying on or near a line at right angles to the air base (Fig. 116a).

Since the heights of A and B are known, we can work out what the parallax difference should be to give the true difference in

height between the two points; call this dP . We can also obtain with the bar the measured difference in parallax between these two points; call this dp . Then $dP - dp$ will be the false parallax due to

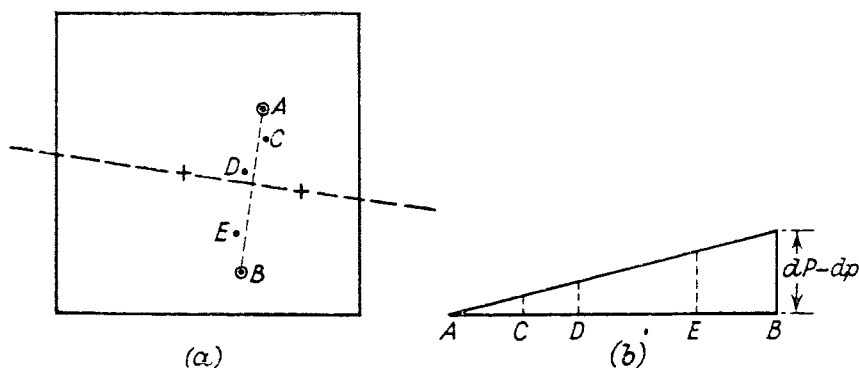


FIG. 116

tilt of the photographs or inclination of the air base, or other cause. By simple graphical proportion (Fig. 116b) we can obtain the corrections to apply at other points on the line joining A and B, the distances AC, CD, DE, etc., being transferred from photo to plot. The reason for working only along lines normal to the base line is that the horizontal scale tends to remain constant along such lines.

Contouring by stereoscope. Having provided a close grid of spot heights by ground methods and supplemented these again by parallax heights, the overlap in question can then be contoured under an open stereoscope by sketching by eye in a manner similar to that employed by the plane-tableer between a close net of clinometer heights. Such contouring is carried out on one or other of the contact prints forming the stereo-pair or alternatively on an overlay trace fastened with sellotape over the top of one print; in the latter case the material used must be very transparent so that it does not mask the detail of the photograph when viewed through the stereoscope. The contours on the print or trace have finally to be compiled into the map by 'equating' between detail points as already described for the planimetry.

CHAPTER 10

MODERN

STEREO-PHOTOGRAMMETRIC

INSTRUMENTS AND THEIR

APPLICATION

AIR SURVEY—INSTRUMENTS AND METHODS

THE LIMITATIONS of the radial line method and the parallax bar have been pointed out in the previous chapter. In order to overcome the effect of displacements due to tilt and height distortion, and to provide a means of accurate contouring, it is necessary to employ a much more elaborate type of instrument in which the spatial relationship of a pair of photographs at the instant of exposure is reconstructed.

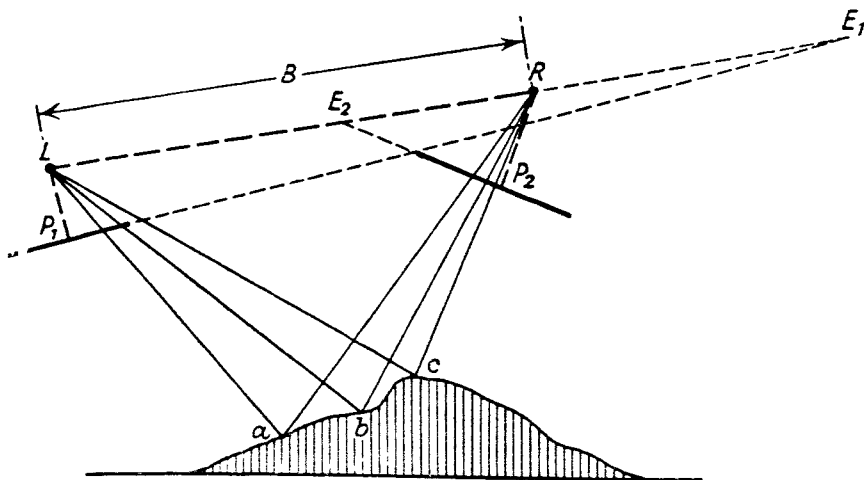


FIG. 117

EPIPOLAR PLANES. In Fig. 117 P_1 , P_2 are diapositives of two 'vertical' air photographs taken from camera stations L and R respectively. LR is the line joining the perspective centres from

which the exposures were taken and is known as the *air base*. E_1 and E_2 are points where the air base (or its extension) cut the planes of the photographs. These are known as *epipoles*. Clearly in the case of a pair of truly vertical photographs the epipoles will be at infinite distance from P_1 and P_2 , the principal points. If we are to reconstruct a true model of the land form, then pairs of rays passing through the corresponding image points in the plates must intersect in space. If, when projected, the rays La and Ra are not on the same plane, there is said to be *want of correspondence*. This can only be removed by reorientation of one or both of the two pictures P_1 and P_2 , so that they take up the same relative conditions as occurred at exposure. Each point a, b, c (Fig. 117) in a true stereoscopic reconstruction has its own epipolar or basal plane which contains the air base, the ground point and the corresponding photographic images of the point on the negative (or positive as the case may be). In this way a relief model of the terrain may be said to be built up of an infinite number of intersecting pairs of rays contained by epipolar planes which hinge about the air base like the leaves of a book from its binding.

Modern plotting machines are designed on the foregoing principle of epipolar planes. In general they fall into two distinct classes which may be termed the *optical* and the *mechanical*. In the former type the photographs are projected back through a lens system of identical characteristics to that of the survey camera, so that the pencil of rays emerging from the projection lens is simply a reversal of that which entered the camera lens at exposure; this is usually referred to as the Porro-Koppe method. The alternative system entails replacing the light rays by means of mechanical rods which can move in any direction about a universal joint corresponding to the nodal centre of the camera lens. Both systems have their advantages and disadvantages. The best known instruments belonging to the Porro class are the Zeiss Stereoplanigraph, the Poivilliers, the Multiplex and the Kelsh, while those of the mechanical group are the Wild A5 and A6 Autographs* and Santoni plotter.†

THE MULTIPLEX PLOTTER. The Multiplex (Plate VI), though it is not a true 'Porro' instrument, for the reason that the projector

* Now superseded by models A7 and A8 respectively.

† A new British precision plotting instrument known as the Thompson-Watts, based on the Porro principle, will shortly be in production. This is described fully in the Photogrammetric Record, Vol. 1, No. 3, April 1954.

lenses are not exact replicas of those used in the taking camera, nevertheless belongs to the projection class and is probably the most widely used of any type of plotting machine; it is also the simplest to understand. In essence the Multiplex consists of a series of projectors strung along a horizontal bar. In order to bring the size of the instrument within workable dimensions, the original 9" \times 9" air negative is, by means of a special reduction printer, reduced to a glass diapositive of size 4 cm. \times 4 cm. The reducing lens of the printer is designed also to compensate for the distortion of the camera lens, so that the resultant diapositive is in theory distortion-free. In practice these wide angle objectives tend to vary considerably in their distortion characteristics, and the reduction printer can only compensate for the average. Fig. 118 shows the optical train of the Multiplex system. Even though the projector and camera lens in this case are different, the Porro principle is still maintained, provided the rays emerge from the projector lens at exactly the same angle as they entered the camera objective. It follows that the whole system from camera to projection table must be in sympathy if accurate results are to be obtained. It would be wrong, therefore, to employ a reduction printer and projection system designed for use with one type of lens for mapping with photographs taken with another type of lens, when the calibration characteristics of the two types of lenses are different.

Every projector unit is fitted with a filter either red or blue alternately along the series (see Fig. 119) and the operator wears spectacles of identical colours (red one eye, blue the other), separation of the images being effected in the manner of the anaglyph. When the projectors are used farther apart, the rays intersect in a lower projection plane, and the effect is to increase the scale of the model. Each projector is also free to move in five other ways:

- (1) Swing about a vertical axis (κ)
- (2) Fore and aft tilt (ϕ)
- (3) Lateral (differential) tilt (ω)
- (4) bz , vertically
- (5) by , horizontally at right angles to bar.

By this means it is possible to set up a strip of photographs along the bar 'in correspondence', whereby the model produced is a scaled presentation of the land form in three dimensions. As a means of tracing the map-detail and contours there is provided a movable projection table (Plate VII) which is adjustable in height.

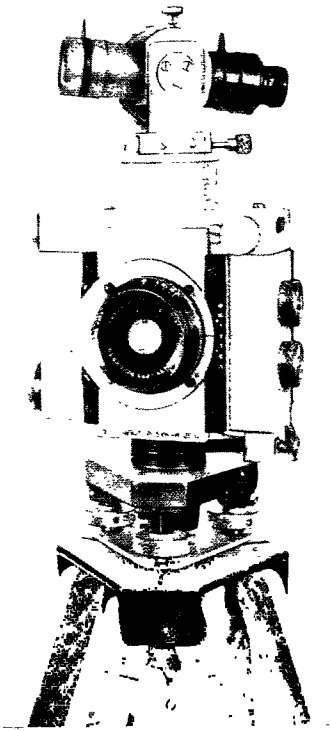


PLATE I. Zeiss TAL photo-theodolite



PLATE II. Wild E2 rectifying enlarger



PLATE III. Cutting a slotted template



PLATE IV. A slotted template assembly
(the black triangles denote fixed control points)

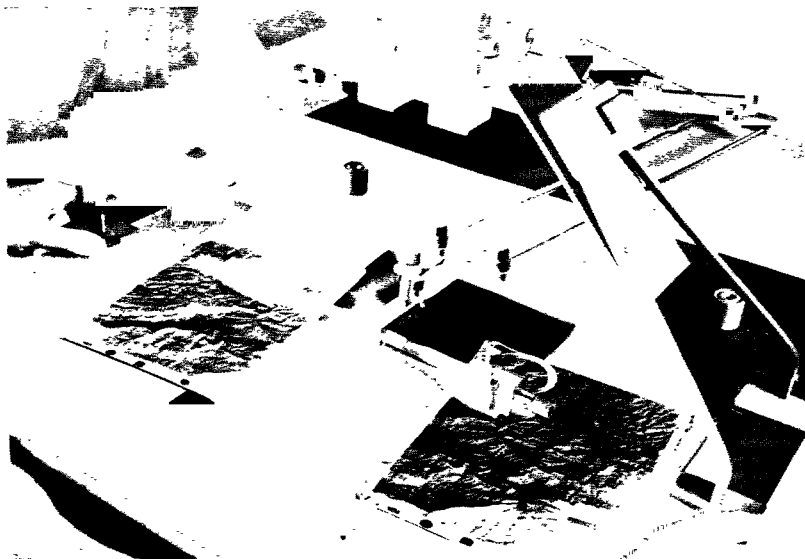


PLATE V. Mirror stereoscope with parallax bar

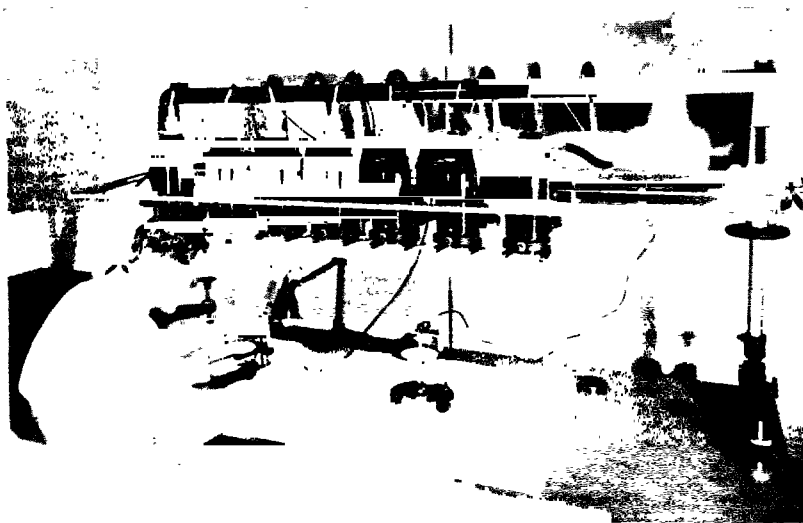


PLATE VI. Williamson multiplex equipment

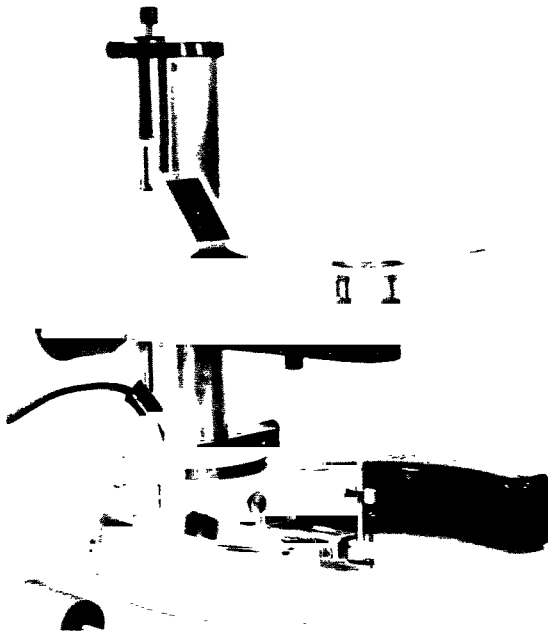


PLATE VII. Multiplex tracing table

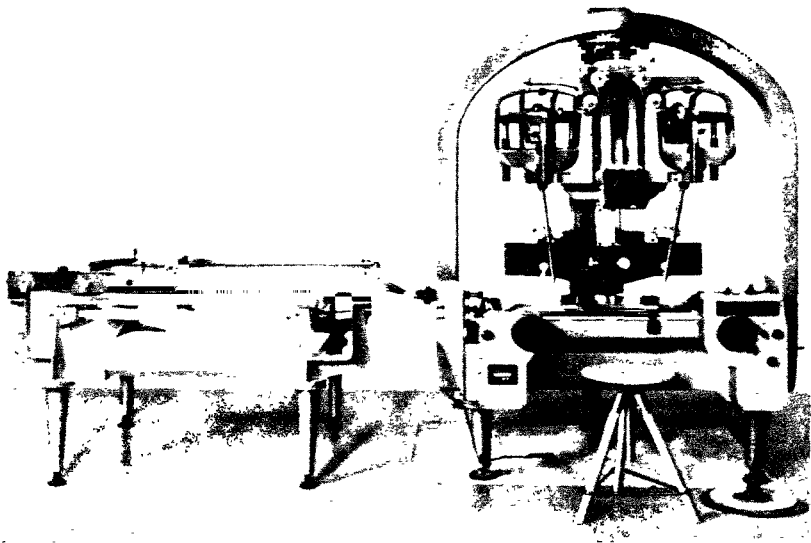


PLATE VIII. Wild Autograph A7 stereo-plotter

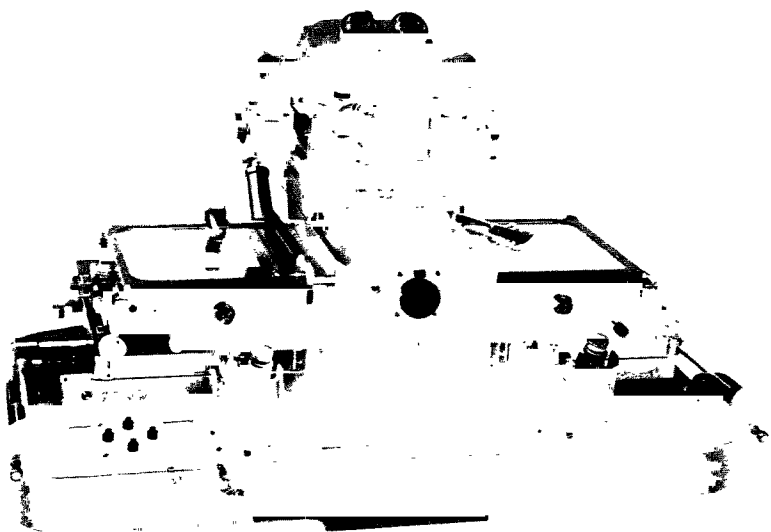


PLATE IX. Cambridge stereo comparator

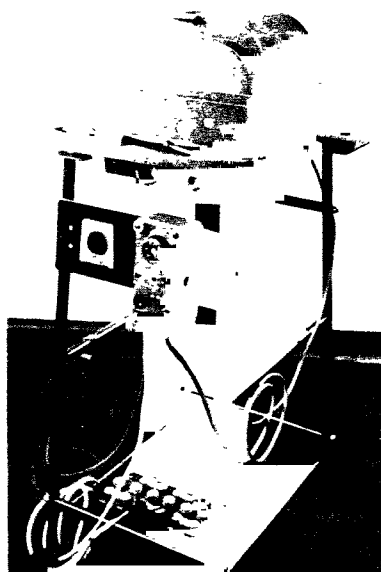


PLATE X. Willhens in Eagle IX survey camera and overlap control panel

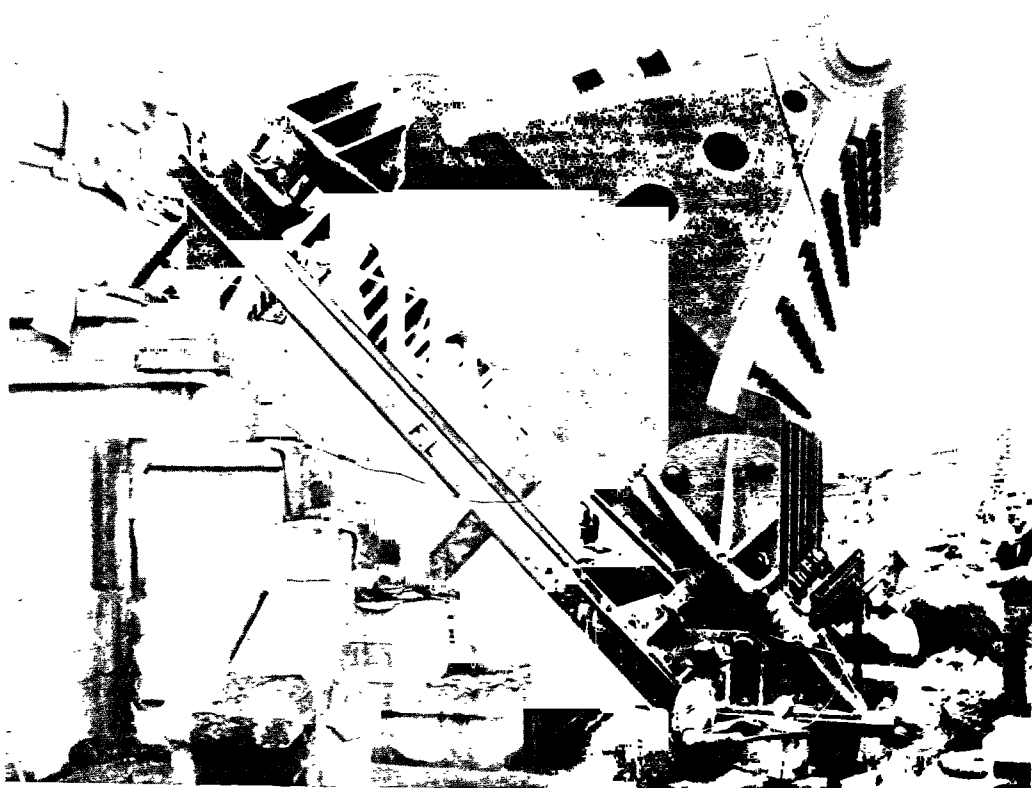
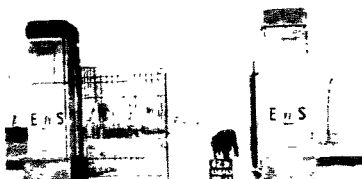


PLATE XI. Sydney Harbour Bridge. Positioning of one of the main pins

R. P. Description

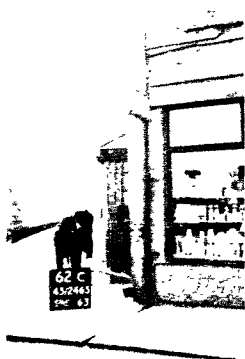
OS 425



12. View of corner of East side road of
1944 Survey from 1944 Survey point



13. Street front of building from
1944 Survey point



14. View of building from
1944 Survey point

Checked by

1944

from sketch by

Par No

PLATE XII. Ordnance Survey revision points (see Fig. 14)

A pin-hole in the centre of the table is illuminated from below and appears as a point of light which 'floats' in the model projected on the table. When the height of the table is adjusted to the plane of a point in the model, the mark then appears to touch the ground.

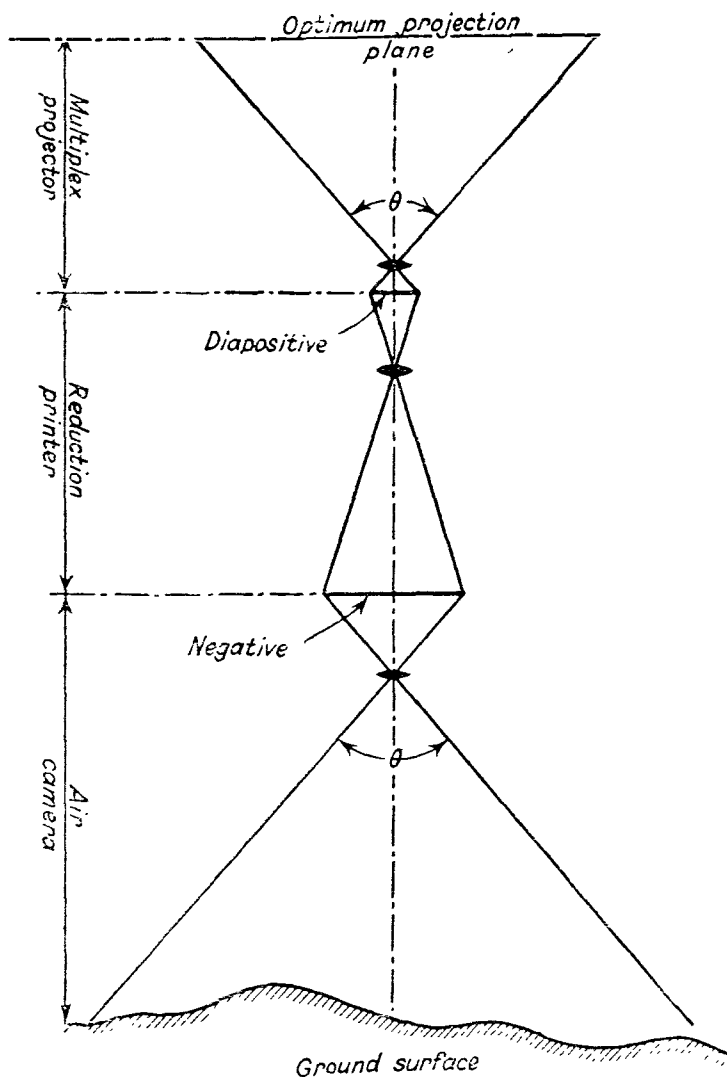


FIG. 118

Thus, in order to draw a contour, the table is set to the desired height (recorded in mm. on an illuminated scale) and moved over the plot-sheet so that the mark is in continual contact with the

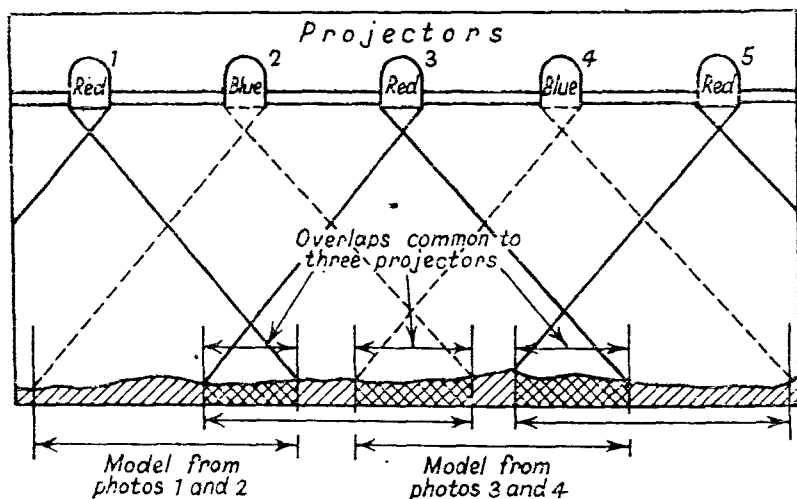


FIG. 119

ground. A pencil situated vertically below the mark on the table traces out the line of the contour. If the plane of the tracing table is set too high, the mark will appear to float above the ground, and, if it is too low, the mark will become blurred and eventually 'split'. When plotting plan detail, the floating mark must also maintain contact with the ground, or it will not be following the true orthographic projection of points in the model.

THE WILD A5 PLOTTER. In the Wild A5 Autograph* (Figs. 120 and Plate VIII) the light rays are replaced by telescopic rods which are free to rotate about a universal joint corresponding to the perspective centre of each skeleton camera. In the construction of the instrument a difficulty arises in that the base distance when scaled down is too small to allow for the operation of the cameras. The solution of this problem is affected by means of a guiding mechanism usually referred to as the 'Zeiss Parallelogram'.

* This has now been replaced by model A7, but the basic design is substantially the same.

In Fig. 121 C_1 and C_2 represent two cameras separated by the air base b , which is shown inclined to the horizontal datum. P is any point in the spatial model, and is fixed by the two rays (or rods

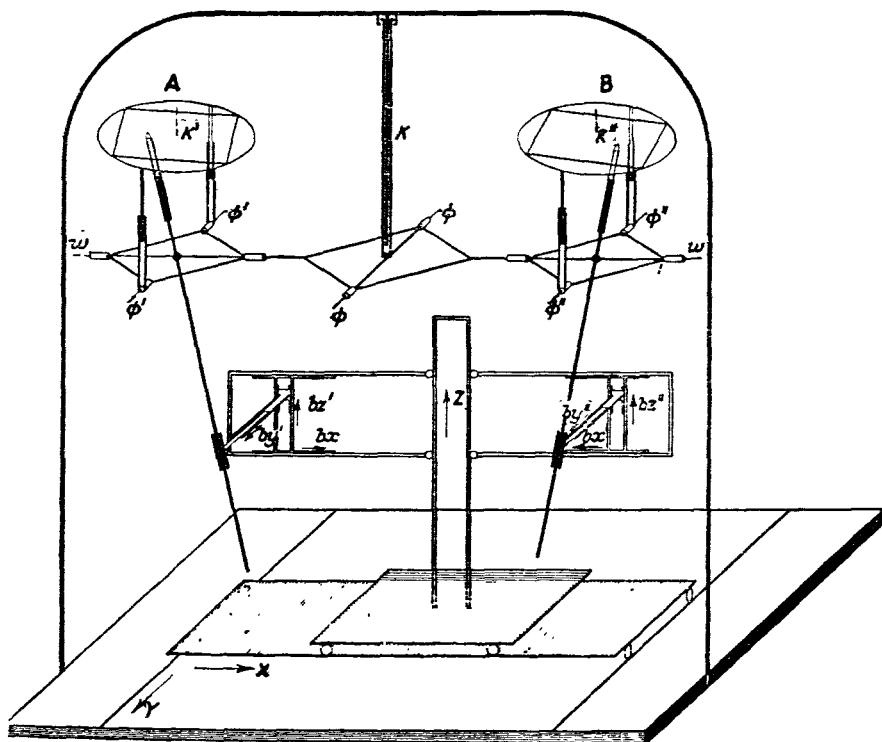


FIG. 120

in the case of the A5) C_1P and C_2P . Suppose we translate C_2P to a new position $C_2^1P^1$ where C_2^1 lies along the X axis of the instrument and $C_2C_2^1$ is parallel to PP^1 , then by symmetry P^1R will be equal to $C_1C_2 = b$, the air base. Provided $C_1C_2^1$ and PP^1 remain rigorously connected, the *Zeiss parallelogram* remains unchanged, and the relationship of P_1 to C_2^1 will be exactly the same as P to C_2 . In the Wild A5 the two sleeves P and P^1 are fixed to a movable carriage known as the 'base carriage', which is carried on the centre perpendicular column of the instrument. Each sleeve can be set relative to the carriage so that the base components by , bx , bz are correctly established.

Other principles in the design of the A₅ may be followed by referring to Fig. 120 and Plate VIII. The photographs in the form of glass diapositives are carried in two camera bodies A and B. One

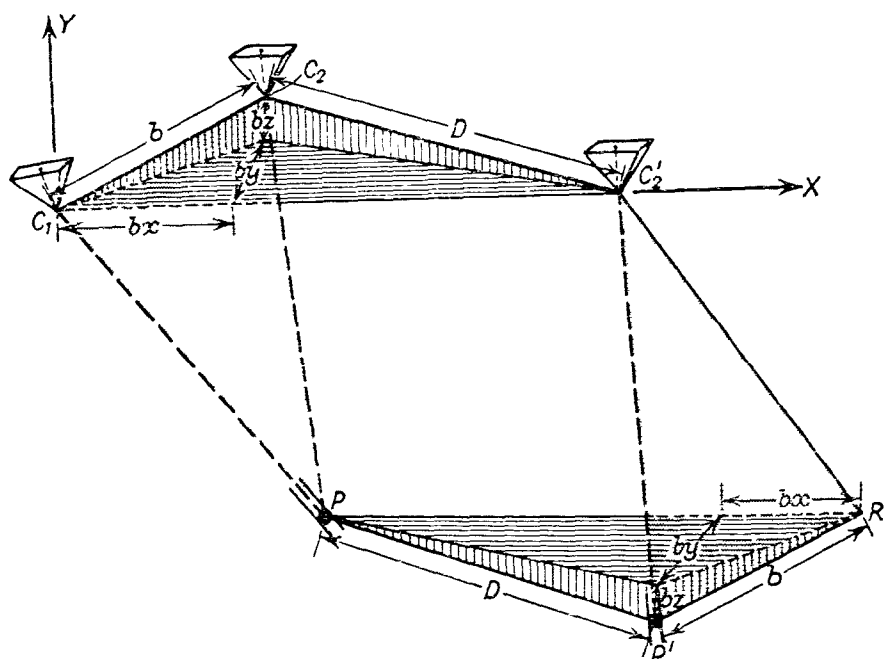


FIG. 121

of the advantages of the instrument over the 'Porro' type is that the principal distance of the cameras can be varied between 100 and 215 mm. and can thus accommodate photographs taken with lenses of varying focal length. In order to overcome the effects of inherent distortion of the survey lens, each camera is fitted with a special glass plate which is shaped to compensate for the distortion previously determined by the lens calibration; alternatively a compensating plate may be used in the diapositive printer. The recent development of both normal and wide angle lenses of negligible distortion should in future dispense with the necessity for compensating plates.

The diapositives are set in the plate carriers by making the collimating marks on the photographs coincide with those of the carrier. The principal distance of the lens concerned is then set by

means of a crank which raises or lowers the plate carrier relative to the perspective centre of the camera. These two settings, namely (1) the setting of the principal point on the camera axis and (2) the adjustment of principal distance to that of the taking camera, are referred to as the *internal* setting.

Each skeleton camera is capable of rotation singly in three mutually perpendicular axes about the perspective centre, namely swing (κ), fore and aft tilt (ϕ) and lateral tilt (ω). These rotations are sufficient in themselves for the complete orientation of the cameras, but in the A5 provision is made whereby both cameras can be given combined κ , ϕ and ω movements. (In the A7 the combined movements of the two cameras have been eliminated since they are seldom used and introduce an unnecessary complication.)

The base carriage carrying the two guide sleeves for the space rods is capable of movement only up and down the vertical column, and is operated by a foot-disc. The rise and fall of this carriage alters the separation of the rods, and thus follows the variations in height of the model corresponding to the raising and lowering of the tracing table in the Multiplex. For any particular horizontal plane in the model, the tracing of the plan detail is affected by two hand wheels which operate two mutually perpendicular trolleys moving on rails set in the x and y co-ordinate axes of the instrument. The y rails are fastened rigidly to the frame of the instrument, the x rails are fixed to the y trolley which in turn carries the vertical column on which the base carriage moves. The movements of the two hand wheels linked with the x and y trolleys are transmitted through a gear box to a pencil which is carried on the cross-arm of a co-ordinatograph plotting table. By combination of different gears the scale between instrument and plotting table can be varied over a wide range. In enlargement, however, it is not advisable to exceed a ratio of 1:3.

The optical system of the A5 (Fig. 122) exists merely to provide the operator with a stereoscopic model in which he can trace the intersection of the space rods. Each space rod is connected at its upper end to a small prism carriage (prism 1 in Fig. 122) carried on a swinging girder which moves only in the plane of the photograph. In this way the point on the photograph to which the end of the space rod is directed, is focussed on to the plane of an engraved mark M. Each eye is directed to one picture, and the observing system provides a magnification of $\times 10$. Seen through the

binoculars, the two reference marks fuse together and appear as a small dot which floats in the three-dimensional model similar to the illuminated mark on the Multiplex tracing table. It is important

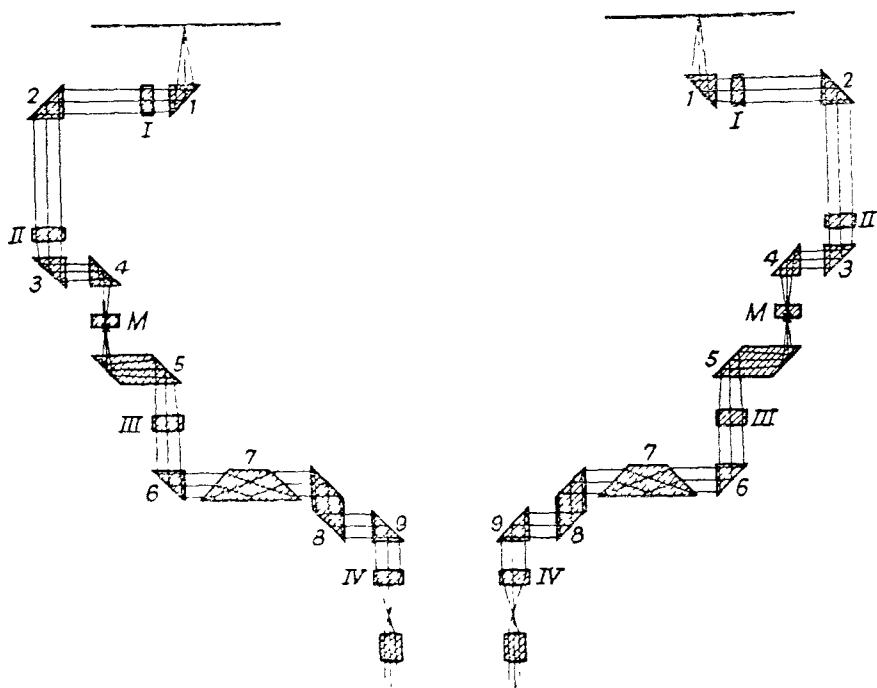


FIG. 122

to note that the accuracy of the observing system depends on the short path between the mark and the photograph plane. Small maladjustments in the remainder of the optical system may cause discomfort to the observer and apparent 'jumping' of the mark, but do not affect the accuracy at all.

MODEL ORIENTATION. Apart from the internal setting already described, the reconstruction of the conditions at exposure involves (a) **RELATIVE** or correspondence setting and (b) the **ABSOLUTE** setting or external orientation. Relative setting involves the mutual orientation of one camera to the other so that all corresponding rays lie on an epipolar plane. There are five movements involved:

- (i) rotation of each camera about the axis of swing—two movements κ_1, κ_2 ;
- (ii) fore and aft tilt of each camera—two movements ϕ_1, ϕ_2 ;
- (iii) rotation of one camera relative to the other about the air base, i.e. lateral or differential tilt—one movement ω .

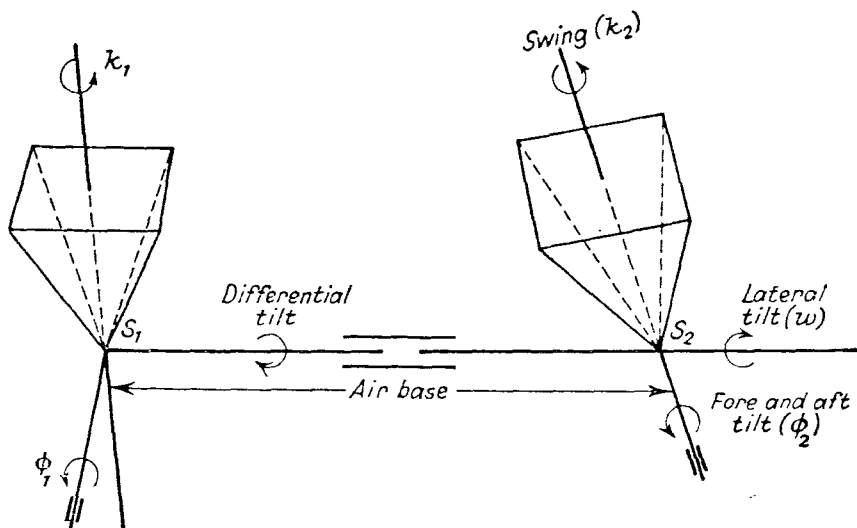


FIG. 123

These five movements are illustrated diagrammatically in Fig. 123. Relative or 'correspondence' setting is made without reference to

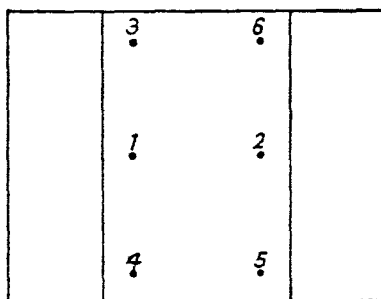


FIG. 124

any ground control and can be carried out by a systematic method of trial and error. There are many ways of doing this. The following

method is recommended by Dr. M. Zeller for the Wild A5.* Referring to Fig. 124, we remove the want of correspondence for each of six points in turn:

In 1 with ω_1 or by_1 .

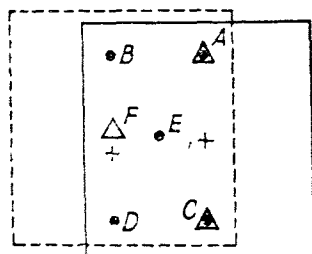
In 2 with κ_1 .

In 3 with bz_1 .

In 4, remove half the want of correspondence with bz_1 and then overcorrect by n^\dagger times the remaining want of correspondence with ω_1 . Finally remove the want of correspondence caused by the overcorrection with by_1 .

In 5 and 6 with ϕ_1 .

Instead of observing in the sequence given, the points can, of course, be taken in the order 1, 2, 4, 3, 6, 5. This type of setting only involves movements of camera 1. When required, camera 2 can be moved and camera 1 maintained constant. Once the overlap has been set in correspondence, the resultant model is true within itself but still has no scale nor azimuth and is unrelated to a horizontal datum. The *absolute* orientation of the model cannot be completed without some form of control provided on the ground. In order to scale the model and establish an azimuth, we require to know the co-ordinates of two identifiable points spaced sufficiently far apart to provide a satisfactory base. For the external levelling of the model we require a minimum of three spot heights distributed in such a manner as to ensure accuracy in levelling along and about the air base; a fourth point is usually provided as a check. The ideal distribution of this control is shown in Fig. 125. Points E and F are not essential but ensure the maximum precision when setting a model.



\triangle Co-ordinated points.
 \bullet Heighted points.

Fig. 125

Absolute setting of a model entails seven unknowns, namely:

- (a) Scale.
- (b) Displacement of the model in plan in two directions at right angles—two movements.

* Zeller, M. *Textbook of Photogrammetry*. H. K. Lewis, 1952.

$\dagger n$ is given by $1 - \frac{f^2}{y^2}$, where f is the focal length of the lens and y the distance of the point from the air base.

- (c) Adjustment of the height of the model.
- (d) Orientation of the model in azimuth.
- (e) Tilting about two horizontal axes—two movements.

The solution of these unknowns is carried out as follows:

(a) The scale is determined by making two given fixed points in the model correspond to their true distance apart on the ground. We must have, therefore, a minimum of at least *two co-ordinated ground-control points* spaced well apart in the forward overlap. (Normally a third point is provided as a check.) In the instrument alteration of scale is provided by a combination of base-setting and change of gearing between machine and plotting table.

(b) Displacement of the model in plan is carried out by placing the floating mark on one of the given fixed points of detail, and translating the paper under the drawing pencil until the pencil is over the plotted position of the point.

(c) Adjustment of the model for height is effected by placing the measuring mark on one of the ground control points, the height of which is known, and making the height column give the correct reading.

(d) For azimuth the drawing sheet is attached to the fixed point, the position of which was made to coincide with the corresponding position of the pencil (see (b) above). The reference mark is then set on the second point, and the drawing sheet rotated with the first point as centre until the pencil lies on the line joining the two fixed points.

(e) There now remains only the tilting of the model about two horizontal axes. For this it is necessary to know the heights of at least three ground points. Since the model can be made to fit on any three points, a fourth point is always required as a check to ensure that there has been no error by the ground surveyor either in identifying his ground points with the corresponding detail on the photograph or in his actual field observations.

In Fig. 126, A, B, C are three points in the model, the ground positions of which are known. If we set the height counter to read correctly on point A, and then in turn take readings on points B and C, we shall find discrepancies between the recorded and actual heights of these points of say $+15$ and -7 respectively. In order to compute the required longitudinal and lateral tilts to bring the model into sympathy with the ground height, draw CD parallel to OX and BE parallel to OY. The differences in level at E and D

along the line AC and AB respectively can be interpolated and thus the required tilts are given by:

$$\tan \phi = \frac{\text{numerical sum of interpolated error at E and B}}{EB} = \frac{18}{EB}$$

$$\tan \omega = \frac{\text{numerical sum of interpolated error at D and C}}{DC} = \frac{10}{DC}$$

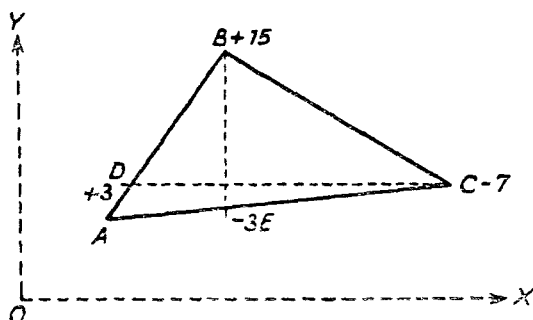


FIG. 126

AERIAL TRIANGULATION. So far it has been assumed that every overlap will be provided with four ground control points, and indeed this is necessary if the highest accuracy is to be obtained in the setting of the model. The amount of ground control required to do this is however considerable, and for topographical mapping, when the contour interval is not closer than say 10 metres, it is possible on most three-dimensional instruments to carry through a system of spatial triangulation across several models with little or no control between. A beginning is made with the first pair of photographs which are fully controlled; the third picture is added to the second by relative orientation only, and by using only the settings of the third camera. Similarly the fourth picture is added to the third, only the settings of the fourth camera being used, and so on along the strip until the next overlap with control is reached. Owing to inaccuracies in setting one picture to another and to other sources of error arising from lens distortion, changes of altitude of the aircraft, curvature of the earth, etc., as we proceed along the strip we accumulate errors both in scale and height. Scale errors are generally small and can be accepted as proportional along the strip, but the effect on tilt in the fore and aft direction due to the accumulation

of errors in strip setting may be considerable and will normally produce a curve of error of parabolic form. In other words the combined model along the strip, instead of running along the level as one would like it to do, bends upwards or downwards (see

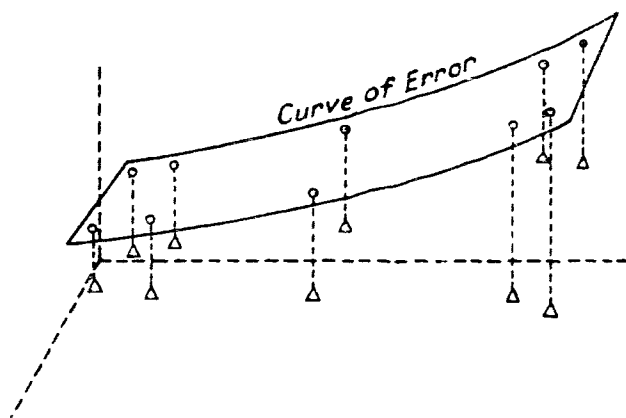


FIG. 127

Fig. 127). There is further a tendency of the strip to be subject to a torque effect which introduces a twist laterally, so that points near the lateral overlap read too high one side and too low on the other. To counteract the effects of this tendency for lateral torque and at the same time to enable us to plot the curve of error in the longitudinal direction of the strip, we require two spot heights on the ground placed roughly halfway along the strip and situated one either side in the lateral overlap.

The adjustment of an aero-triangulation strip may be carried out analytically, but the calculation is time consuming and tiresome. Fortunately a simple graphical solution of the problem has been developed which is in general use.*

Analytical method of aero-triangulation. Another method of carrying out aero-triangulation, chiefly favoured by the Ordnance Survey, is analytical. The plane photographic co-ordinates of corresponding points are measured by means of a stereo-comparator, and the triangulation is subsequently computed.

The Cambridge stereo-comparator (Plate IX), which was designed by E. H. Thompson and is manufactured by the Cambridge

* See 'Graphical Interpolation-Adjustment', by Jerry Zarsycki, *Photogrammetric Engineering*, December, 1949.

Instrument Co. Ltd., consists of two horizontal tables A and B capable of rotation about vertical axes mounted on carriages which are movable along co-planar tracks parallel to the eye base of the operator. The photographs are viewed stereoscopically through an optical unit, which in essentials consists of prism binoculars with magnification $\times 5$ and two gratitudes which provide the floating (or measuring) mark. The viewing unit is mounted on a carriage which is movable in the Y direction perpendicular to the eye base (X axis). The instrument is provided with two X scales and one Y scale to measure the displacements of the respective carriages in these directions. All the scales are read by means of micrometers.

The need for a relative Y movement of the two tables (to measure and eliminate the want of correspondence when using aerial photographs) is avoided by attaching two 'parallel plate micrometers' to the binoculars; each parallel plate micrometer comprises a glass plate which rotates on a micrometer head, thereby introducing a shift of the image in the Y direction. The photographic co-ordinates measured by the stereo-comparator must be corrected for the distortions of the lens and the film before commencing computation of the triangulation.

An improved method for reducing the effects of film distortion has been developed by the Ordnance Survey,* and is applicable to photographs taken with cameras that have a register glass on which is engraved a calibrated 'reseau' of small crosses at intervals of one centimetre. A contact print of this reseau appears on each photograph taken with the camera, and provides a large number of origins of known relative positions, to which co-ordinates may be referred. Any distortions of the lens are allowed for by corresponding adjustments to the calibrated co-ordinates of the reseau crosses. Thus the use of the reseau obviates the need to correct each measured co-ordinate for film and lens distortion. It also compensates automatically for lack of flatness of film, provided this is not so large as to cause lack of definition in the reseau crosses. Any blurring of the reseau will serve as an indication that film and register glass are not in contact.

It has, however, been shown[†] that the improvement resulting

* Shewell, H. A. L. Proceedings of the Commonwealth Survey Officers Conference, London, March, 1951.

Thompson, E. H. *Journal of the R.I.C.S.*, Vol. XXX, No. 81, July, 1951.

† Wassef, A. M. *Empire Survey Review*, Vol. XI, No. 84, April, 1952.

from the use of the reseau has been over-estimated. Thus, users of cameras which have no register glass to carry the reseau can still make use of the analytical method, provided they introduce corrections to the co-ordinates for lens and film distortion, the latter being measured by means of the collimating marks (perhaps a larger number of them).

The main advantages of the analytical method of aero-triangulation as compared with the use of plotting machines are: (*a*) that corrections for all the distortions and other determinable errors can be applied to the observational data before these are used to compute the triangulation, and (*b*) that the instrument used, namely a stereo-comparator, is simpler and, therefore, has fewer sources of instrumental error. In any practical case these advantages must be weighed against the lengthy computations involved. It should be noted that the analytical approach is still in the course of development. A method for the automatic recording of the observations has been suggested* and an effective reduction of computing time is being sought for either by the use of electronic computers or by developing a modified type of electric desk calculating machine.

Ground control. There is no method yet devised whereby aerial mapping can be completely divorced from work on the ground, nor, despite the assistance of radar and electronic aids, is this ever likely to happen. In photogrammetric mapping the extent and disposition of the control framework varies considerably according to the accuracy and type of map required and the method used in plotting.

The factors which influence the control requirement are generally speaking governed by (*a*) the extent of terrain relief, (*b*) whether the map is to be contoured, (*c*) the nature of the photography, (*d*) the scale and precision of the map.

For planimetric mapping at topographic scales in areas of moderate relief, i.e. where the radial line system of aerial triangulation is applicable, the normal requirement is a framework of co-ordinated (but not heighted) points, each of which is 'pin-pointed' on the photographs. Control points of this kind should be selected so that they fall in the lateral overlap of two adjacent strips and are sufficiently removed from the flight lines to provide a satisfactory 'cut' when rayed in from the principal points. They should also be selected at ground level and so far as possible at an average height

* Wassef, A. M. Photogrammetria, VIII. No. 4. 1951-52.

above sea-level in order to minimize tilt errors and to overcome the effect of sudden local changes in scale resulting from abrupt change in relief. In deciding on the desired spacing between control for slotted templates, the following empirical formula is due to Dr. L. G. Trorey.*

$$c = t \left(\frac{0.16}{e} \right)^2$$

Where e is the arithmetic mean error in millimetres (usually taken to be 0.5 mm. at scale of publication)

t = the number of photographs or templates

c = „ „ „ control points

Assuming a block of 5,000 square miles photographed at a negative scale 1/30,000 involving, say, 1,000 photographs required for mapping at 1/50,000: then

$$e = 0.5 \times \frac{50,000}{30,000} = \frac{2.5}{3}$$

$$\therefore c = 1000 \frac{(0.16 \times 3)^2}{2.5^2} = 37$$

or approximately one point every 10–15 miles.

In aero-triangulation by Multiplex or Wild, level control in addition to plan control is required whether the map is to be contoured or not. As already explained, the disposition of the spot heights must conform to the lie of the overlaps and the strip, and for this reason *the control must be carried out after photography is complete*. The extent of 'bridging' between control depends on a number of factors including the type of camera and lens used, the photographic material and the plotting instrument. A considerable amount of research has recently been carried out with a view to 'bridging' long distances. Dr. Zeller, employing a glass plate camera in conjunction with a recording statoscope to provide the variations in height between successive camera stations, claims to have bridged 100 km. with a mean square error in position of ± 13 metres and of height ± 4.7 metres.† Such results would, however, rarely if ever be possible under normal working conditions, and with negatives made on film. A practical limit for Multiplex bridging

* *Handbook of Aerial Mapping and Photogrammetry*, by L. G. Trorey. Cambridge University Press.

† Zeller, M. *Textbook of Photogrammetry*. H. K. Lewis & Co., 1952.

is six overlaps with two spot heights at the centre of the bridge and two at the end (see Fig. 128). With wide angle photography at $1/40,000$ scale, the distance between control of any kind (i.e. half

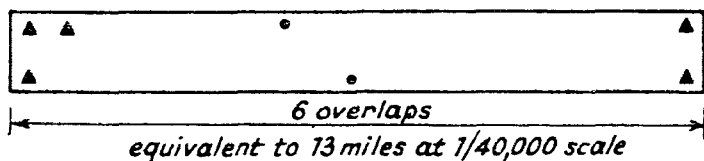
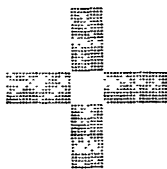


FIG. 128

the bridge) would then be equal to 5-7 miles and the distance between co-ordinated points 10-14 miles along the strip. For satisfactory working every alternate strip should be controlled in this manner. As the observable distance on the ground for secondary triangulation seldom exceeds this distance, the extension of longer bridges can only apply to geodetic work for which a much higher accuracy is required than that achievable by photogrammetric means. The argument for aero-triangulation over very long distances seems therefore to be largely one of academic interest only.

The selection of control points and their correct identification on the photographs require as much skill by the ground surveyor as in the methods he may use for surveying them. For the reasons given above the pre-marking of points before photography is largely a waste of time. The two most common exceptions to this rule are (1) in areas of practically featureless terrain where control is required for slotted template mapping only and (2) in very large scale cadastral or engineering surveys of built-up areas. A convenient method of control marks in the desert is to lay out the arms of a cross by spreading a mixture of three-parts lime to one of water. This will usually last for a month or two under normal conditions. The arms of the cross are laid out in four rectangles thus:



The size of each rectangle is roughly 20 ft. \times 40 ft. for photography at a scale of $1/30,000$ and proportionately larger or smaller for other scales. Where photogrammetric methods are employed for the

production of plans at extremely large scales (i.e. 1/500), there will always be a proportion of detail (namely between 10 and 30 per cent) which is either screened from the air or is unlikely to register on the photographs without some form of artificial marking. By judicious use of white paint or special markers before photography, the photogrammetrist can both increase the amount of detail which can be plotted instrumentally and at the same time provide convenient reference points for completion of detail, such as bridge abutments, station platforms, overgrown fences, etc., which the vertical air view will not show.

In all cases where control is supplied *after* photography, a set of contact prints is marked up by the photogrammetrist with large circles about an inch in diameter showing the zone in which each control point is required. These prints are then passed to the field parties, whose job it is to establish a suitable point within the zone and to identify it precisely. The mere pricking of the point on the photograph is not sufficient, as the pin-prick will occupy a large area on the ground. On the reverse of the photograph, or on a separate form, the surveyor must make a careful sketch showing the exact location of the point relative to features in its immediate vicinity and which can be seen on the overlap when viewed with a hand stereoscope. In forested country and terrain of little variation the problem of identification is often of great complexity and requires the utmost skill and patience on the part of the surveyor, but with the necessary experience there are few places where it cannot be done. Even in desert areas there are usually small clumps of camel thorn, sink holes or occasional patches of outcropping rock that form an identifiable pattern. As a general guide the following list of 'dos and don'ts' may be of value to those engaged on control for machine mapping.

Co-ordinated points:

Do not select points:

- (i) Where they are obscured by vegetation, shadow, halation or overhanging roofs of buildings.
- (ii) Where the junction of two paths forms an acute point.

Do

- (i) Take offset measurements to other objects near by which can also be identified on the photographs, particularly bushes and rocks which usually form some kind of pattern.

(ii) Draw a neat sketch on the back of the photograph showing the exact position of the point related to other objects *appearing on the photograph* with measurements.

(iii) Endeavour to choose objects that are sharp, such as the corner of a roof, the pointed peak of a hill, the intersection of two tracks cutting at right angles. If it is necessary to select a tree or bush or any type of 'woolly' object, always co-ordinate the centre.

Level points:

Do NOT select points:

(i) Where there is no texture on the photograph, i.e. on a wide road or large patch of bare ground that shows white on the photograph.

(ii) On a track running parallel with the line joining the centre points of the photographs.

(iii) At the bottom of a cutting or clearing in the forest where the angle of the line of sight at the ground point to the top of the surrounding features is more than 45° .

(iv) On a slope.

(v) In deep shadow.

Do select points:

(i) In an area of level ground *or* on the top of a clearly defined object such as a chimney or pointed mountain top.

(ii) Where the image of the ground on the photograph has texture.

(iii) That fall in the common overlap to three photographs and are not nearer than $\frac{3}{4}$ inch from the edge of the print. Points should also be chosen no closer to the air base (or line joining the centres of the photographs of a strip) than the distance between the principal points of two consecutive photographs.

(iv) On tracks running at right angles to the air base.

(v) In areas not overshadowed by surrounding objects. (See point (iii) of the 'Do NOTs'.)

PRECISION AIR CAMERAS AND LENSES. The survey camera is just as much a precision instrument as the theodolite and the plotting machine, and all three are equally important links in the chain of a photogrammetric survey. The modern survey camera is divided into three main component parts—the mounting or suspension

unit, the optical unit and the magazine. Plate X shows the Williamson Eagle IX camera. The mounting is designed on a gymbal system and has adjustments for drift and tilt; it is insulated against aircraft vibration. The optical unit incorporates (a) the driving mechanism, (b) a panel of recording instruments including a veeder counter, watch and altimeter, (c) the lens, (d) shutter and shutter operating gear and (e) the register glass on which are engraved the collimating marks. The magazine which accommodates sufficient film for 250 exposures is in itself a complete unit and contains the mechanism for winding on the film and for raising and lowering a pressure pad which keeps the film pressed flat against the register glass during exposure. In the Wild RC5 camera there is no register glass, but, instead, the film is flattened by vacuum against a spring-loaded suction pad which is pressed in contact with a metal frame bearing the calibration marks.

The interval between exposures is controlled by means of an intervalometer which may or may not form an integral part of the camera. In the Wild RC5 and RC7 cameras there is a built-in overlap regulator with a vertical sight containing a moving graticule. When the overlap setting is made on the camera, the correct shutter speed can be set by a knob controlling a rheostat until the image of the ground is synchronized with the lines of the moving graticule.

Focal plane shutters are no longer used in precision air cameras since they introduce serious distortion resulting from the relative movements of the aircraft and slit as the latter passes across the field of the negative. Some British cameras employ a louvre shutter on the principle of the venetian blind, but the majority of modern cameras are fitted with a rotating disc or leaf shutter which is built-in between the integral parts of the lens.

Air survey lenses may be divided into three general classes, *wide angle*, *normal angle* and *narrow angle*. It is common practice to talk rather loosely about lenses in terms of their focal length, which may easily give rise to misunderstanding for the reason that the effective area of ground included in a single photograph taken from a given height depends on the angle of cover embraced by the lens, i.e. on the combination of focal length *and* negative size. As an example, the Williamson Eagle IX camera, fitted with a lens of 6-in. focal length and having a negative format $9'' \times 9''$, has almost identical coverage at any given flying height to the

Wild RC₅ camera using a lens of $4\frac{1}{2}$ -in. focal length on a $7'' \times 7''$ negative.

The following formulae are worth remembering:

$$(1) S = \frac{12H}{f} \quad (2) B = \frac{fS}{30} = \frac{2fH}{5f} \quad (3) A = \frac{L \times l \times H^2}{f^2 \times 5280^2}$$

where H = flying height above mean ground level in *feet*,

f = focal length of camera in *inches*,

S = scale factor of negative,

l = side of negative in direction of flight in inches,

L = side of negative perpendicular to direction of flight in inches,

B = distance on the ground between exposures in direction of flight in feet,

A = area on ground covered by a single photograph in square miles.

The term *wide-angle* is applied to that class of lens giving an angular coverage of between 90 and 95 degrees measured across the diagonals; from any given flying height the wide-angle picture will give the maximum possible coverage. The ratio of air base to height above ground (*base-height* ratio) is at its maximum with wide-angle photography. It follows that the parallaxic angle subtended by corresponding rays will be comparably large, and thus the perception of depth when a pair of wide-angle photographs are examined under a stereoscope will appear strongest. Wide-angle lenses have in the past suffered from a serious falling off in illumination at the edges and variable distortion characteristics. The recently designed Wild 'aviogon' wide-angle lens has succeeded in combining a remarkably even intensity of illumination over the whole surface of the negative with almost complete freedom from distortion. Because of the great obliquity near the edges, wide-angle photographs are not suitable for mapping of built-up areas. For the same reason in high mountain country wide-angle photography should be undertaken from an altitude at least three times that of the maximum terrain in order to avoid abrupt changes of perspective between successive exposures.

The *normal-angle* lens has an angular field of 60° across the diagonals and can be used for all types of mapping even up to scales of $1/500$. The Ross 12-in. on a $9'' \times 9''$ format and Wild Aviotor 210-mm. on an 18 cm. \times 18 cm. are examples of the latest type of aerial objective

in this class. The displacement of the image with this type of lens should be below $1/200$ mm. even at the extreme edge of the picture.

Examples of narrow-angle lenses are the Ross 20" and 25" on a 9" \times 9" format. Narrow-angle photographs come nearest to providing a true plan picture of the ground and for this reason are particularly valuable in the production of mosaics and in the revision by graphical methods of town plans in areas of little relief. The appreciation of relief in narrow-angle photographs is poor, and they are therefore of no value for contouring. Photographs taken with a narrow-angle lens cannot be accommodated in existing types of three-dimensional plotting instruments.

Filters. In all air cameras it is standard practice to use coloured filters of optical stained glass. With panchromatic emulsion some form of yellow filter is used to reduce halation and atmospheric haze. A red filter can sometimes be of value in accentuating drainage in areas of low relief and in delineating the water-line along indeterminate coasts. It is not, however, possible to interchange filters absorbing light of different wavelength without affecting the calibration of the camera.

Negative material. The precision of the negative is, of course, dependent not only on the camera and lens but also on the dimensional stability of the photographic base. A great deal of research has been carried out towards producing a dimensionally stable film by the Eastman Kodak Company, who have produced an aerographic emulsion on what is termed *topographic* base. Even *topobase* film, however, will distort unless maintained under standard conditions of humidity and temperature during its storage and subsequent use. Glass plates provide the only material of certain dimensional stability but tend to be cumbersome to handle and are only normally employed on very high precision surveys of limited areas. Film intended for photogrammetric mapping should, whenever possible, be transferred to glass by making diapositives between one and four weeks after development.

THE ACCURACY AND SCOPE OF AIR SURVEY. The days when an air survey was considered 'good enough for a rough map' have passed. Survey photography and stereo-photogrammetric mapping have now developed into a highly precise science covering almost every type of scale in land surveying, from topographical maps to cadastral and engineering surveys.

In considering the application of photogrammetry to any particular survey problem, we are concerned with three things, accuracy, speed and economy. To the questions 'How accurate?' or 'How rapid?' or 'How cheap?' is an air survey there can be no direct answer until the nature of the problem and the requirements are known; neither is it possible to generalize in making a comparison between aerial and ground methods, since no two survey problems are ever alike and the factors involved are infinitely variable. Table 22, taken from recent practice, may, however, serve to give the student some idea of the capabilities of modern air survey and to indicate the conditions applying to photogrammetric mapping at varying scales and contour intervals.

It should be understood that the 'map accuracies' given in the last two columns of this table do not represent the *ideal* attainable by any particular combination of equipment and circumstances, but are representative of what may be expected under *average* conditions. 'Map accuracy' is at best a doubtful quantity, since two people seldom agree on a satisfactory interpretation. The following definitions of plan and contour accuracy, on which the figures given in the table are based, provide, in the writer's opinion, a practical and simple formula, though they may not appeal to those who prefer a more mathematical solution.

(1) Planimetric error.

'The accuracy of the map shall be such that the measured co-ordinates of any defined point of detail will show no error at natural scale greater in amount than . . . feet when referred to the nearest ground control point or map grid or graticule.'

(2) Contour error.

'85 per cent of all elevations located on the map contours will show no single error in excess of . . . feet when checked by spirit levelling from the nearest standard bench mark.'

The figures left blank in the definitions refer to those stated in columns 8 and 9 of Table 22.

Figs. 129 and 130 show two divergent examples of photogrammetric mapping. The map (Fig. 129) covers a strip of virgin country required for the initial routing of a new railway. In this case the scale of photography was at $1/36,000$ taken with a 6-inch lens from a flying height of 18,000 ft. above mean ground height. Aero-triangulation and plotting were carried out in the Williamson Multiplex. This map demonstrates well the value of

photogrammetric contours in depicting complicated topographic features.

Fig. 130 shows part of a survey for a local authority at a scale of $1/500$ contoured at 2 feet intervals. The flying height in this case

TABLE 22 SOME TYPICAL EXAMPLES OF AIR

Scale of Map	$1/50,000$	$1/50,000$	$1/20,000$	$1/10,000$
Contour interval in feet	50	—	50	—
Type of country	Undulating mixed forest & cultivation	Flat, open, little detail	Mountainous with scrub forest	Rolling with open forest
Camera & focal length of lens	Eagle IX 6-inch	Eagle IX 6-inch	Eagle IX 6-inch	Eagle IX 6-inch
Flying height above mean ground level and negative scale	20,000 ft. $1/40,000$	15,000 ft. $1/30,000$	20,000 ft. $1/40,000$	10,000 ft. $1/20,000$
Average distance between ground points in miles	5	15	5	5
Method	Aero-triangulation by Multiplex. Plotting of detail and contours by A6 Autograph	Slotted template for minor control. Radial line or rectified enlargements for map detail	Aero-triangulation by Multiplex. Plotting of detail and contours by A6 Autograph or Multiplex	Slotted template for minor control. Ray plotting and Sketch-master for map detail
Map error at natural scale of map plan detail in feet	± 100	± 150	± 50	± 40
Map error of contour in feet	± 25	—	± 25	—

was 2,200 feet with a lens (Wild Aviatar) of focal length $8\frac{1}{4}$ inches. A small proportion of the detail (not more than 10 per cent) was supplied on the ground. The contours are shown chain-dotted to distinguish them from the plan detail.

SURVEY WORK AND EXPECTED ACCURACIES

$1/10,000$	$1/5,000$	$1/2,500$	$1/1,250$	$1/500$	$1/500$	Col. 1
20	10	10	5	2	—	2
Rolling with low grass	Semi-urban parkland	Urban hilly	Reservoir site, open hilly	Undulating broken with some cultivation	Heavily built up industrial areas	3
Eagle IX 6-inch	Eagle IX 6-inch	RC7 plate camera $6\frac{1}{2}$ -inch on $5'' \times 5''$ neg.	RC5 film camera $8\frac{1}{2}$ -inch on $7'' \times 7''$	Eagle IX 6-inch	RC5 $8\frac{1}{2}$ -inch	4
10,000 ft. $1/20,000$	8,000 ft. $1/16,000$	8,000 ft. $1/12,500$	4,500 ft. $1/6,500$	2,000 ft. $1/4,000$	2,200 ft. $1/3,200$	5
2	2000 yds.	1000 yds.	700 yds.	500 yds.	300 yds.	6
Aero-triangulation by Wild A5. Plotting of detail and contours by A6 or Multiplex	A5 Auto-graph	A5 Auto-graph	A5 Auto-graph	A5 Auto-graph	Combina-tion of A5 and ground work	7
± 20	± 8	± 4	± 2	± 1	$\pm 6-8$ ins.	8
± 10	± 5	± 5	$\pm 2\frac{1}{2}$	$\pm 1\frac{1}{2}$	—	9



FIG. 129

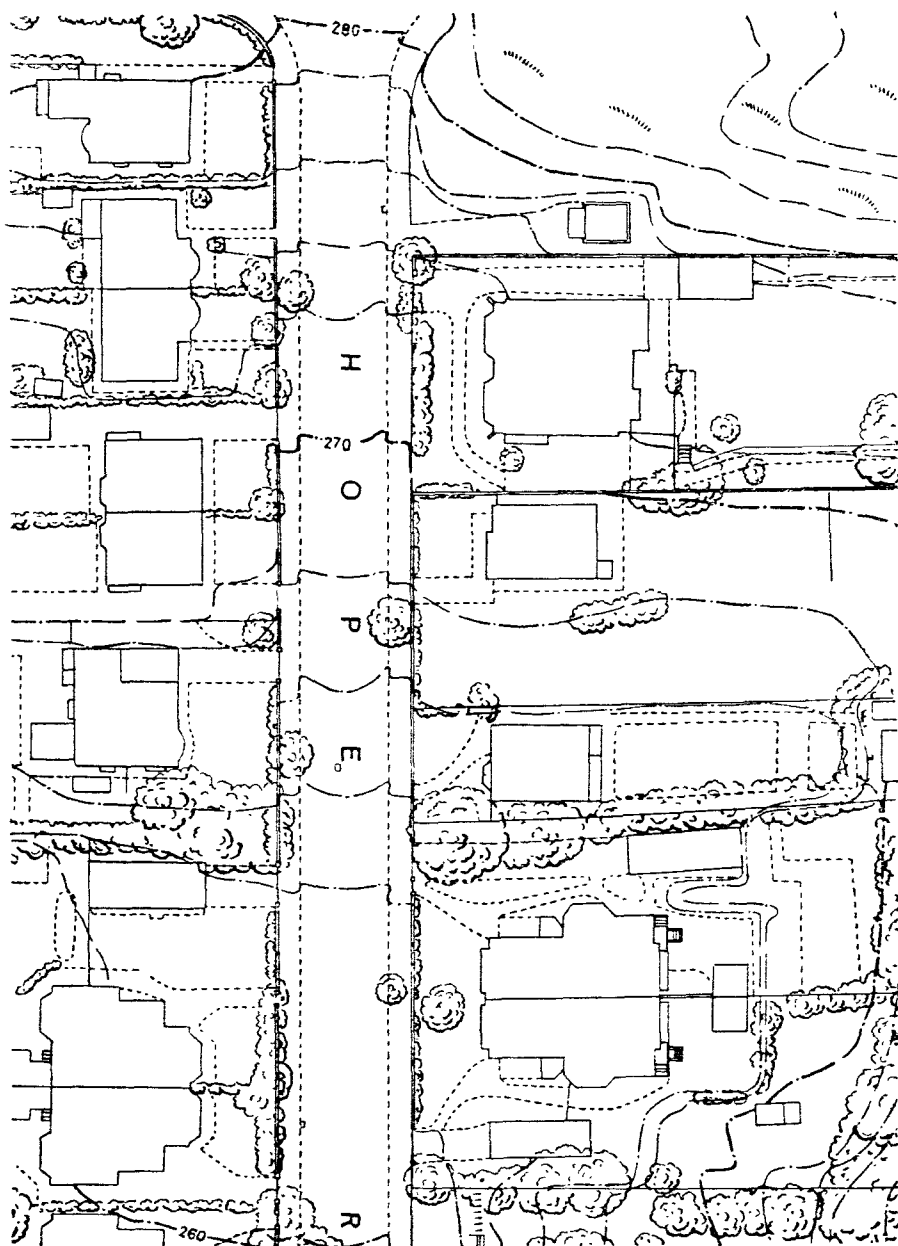


FIG. 130

CHAPTER 11

RAPID AND RECONNAISSANCE SURVEYS

THIS CHAPTER attempts to describe methods of surveying in which standards of accuracy are relaxed in the interests of economy, and the comprehensive title is not intended to mean that speed is the main object of such relaxation. It is thus concerned with reconnaissance surveys as a preliminary to more detailed work and with exploratory and sketch surveys of regions previously unmapped.

The importance of consistency in balancing the standards of accuracy between the various measurements, angular and linear, in a survey has already been stressed, and attention must be paid to this when the standards are being relaxed. But this search for consistency must be directed with discretion; e.g. it is not essential that the standard of accuracy of the vertical control should agree with that of the horizontal control, or again, to take an extreme case, in a survey controlled entirely by triangulation, the accuracy of the angular measurement need not be unduly relaxed because the original base line is measured by pacing, since only an error of scale is involved which may be rectified at some later date.

Many and diverse are the methods of controlling surveys and of surveying the detail, and considerable experience is required for selecting the most suitable for any particular requirement. The principal factors which will influence the choice of method will be the accuracy required, the nature of the country, the means available, both in experienced personnel and instruments, possibly the time factor, and certainly the ultimate cost; the question of accuracy is closely connected with that of scale and, particularly when the detail is being surveyed, plottable error will be the criterion. For most of the work discussed here the scale will be small and in the region of 1,250,000 ($\frac{1}{4}$ miles to 1 inch), but some classes of work demand a much larger scale even in the preliminary stages.

HORIZONTAL CONTROL

The following methods are available:

Triangulation		
Primary (Major)	}	For regular surveys of large areas (see Chap. 3)
Secondary (Minor)		
Tertiary		
Primary traverse		
Rapid theodolite triangulation		
Plane-table triangulation		
Secondary theodolite traverse		
Compass traverse		
Sextant subtense traverse		
Latitude and azimuth		
Astronomical positions and route traverse		
Tacheometry		

RAPID TRIANGULATION. When the nature of the country permits triangulation, this is undoubtedly the most satisfactory method of controlling any survey (except possibly the estate survey extending over one square mile or less). Very flat country, especially if thickly wooded, is unsuitable, but in all others triangulation should be practicable except when the reduction of visibility to three miles or less by atmospheric conditions is of frequent occurrence.

Rapid triangulation, although governed by the same principles as high order triangulation, will normally be carried out with lighter and less precise instruments; time will be saved by less refinement in measurement and by accepting less well proportioned triangles, and—possibly the biggest time-saver—unvisited stations will regularly be included in the triangulation.

The actual method of field work adopted must vary greatly according to the strength of the party and the speed required. Ideally, the triangulation will be carried out by a party working ahead of the detail surveyors, but when the total strength is small it may be more efficient for the detail to be surveyed with the triangulation.

Base measurement. Whenever possible the rapid triangulation should be connected with the local survey system, and, if two triangulated points of that system are conveniently placed, it may be possible to dispense with any form of base measurement, either by including

the two points as one side of the rapid triangulation or by making use of an 'inaccessible base'. When no connection can be made with an existing system, or if connection can be made only at the two ends of the rapid survey, it will be necessary to measure a base. When possible, the base should be at least half a mile long and extended on to a side of the triangulation in the normal manner, though it will probably be necessary to include poorer conditioned triangles in the extension than is really desirable.

Time will seldom allow a prolonged search for a good site, nor does the refinement of the measurement require it; time and labour may be saved in clearing the site if the measurement is made with a steel tape in catenary using locally cut stakes, with strips of tin on their tops as the terminals of the tape lengths. Constant tension should be maintained and temperature and slope allowed for; the measurement should be made once in each direction.

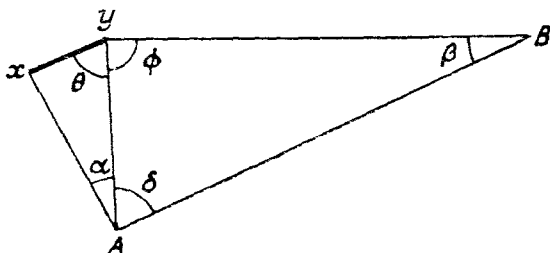


FIG. 131

When the lie of the land allows it, excellent results can be obtained from subtense bases. In Fig. 131 xy is the subtense base and AB a side of the triangulation. The subtense angles α and β should be measured by repetition, but θ , ϕ , and δ need be measured only to the refinement being used for the rest of the triangulation. xy should be one tape length, preferably a 300-ft. tape, and the signals at x and y used for the first extension should be the actual terminal marks used for the base, either measuring heads on tripods or large nails driven in stakes. The signals at A and y for the second extension must be centred with the greatest care, but in many cases Ay will be regarded as the base and will be extended by normal base extension methods. 'For an xy of 300 feet and α and β of 6° each, AB is about 5.2 miles'. In thickly wooded country four lanes of clearing would be necessary for the simple scheme shown in

Fig. 131, and such a scheme would not be chosen unless all stations were elevated. xy , the subtense base, must be corrected for slope and temperature before Ay is computed, but (except as far as they are required for height control) the other slopes are not required. Rougher forms of base line may be measured by ordinary surface taping with slopes measured or by horizontal taping (stepping); or by a short tacheometric traverse (say one of four legs, i.e. c. 3000 ft.).

If connection can be made with one point of an existing system at the beginning of a rapid triangulation and with another at the end, the survey can be adjusted between these two points and no base measurement is needed as far as the triangulation is concerned. In practice, of course, a rough measurement will be required in order that the survey of detail may be started before the completion of the triangulation.

Angular measurement. Many excellent light-weight theodolites are now on the market, and some of the optical micrometer type when fitted in metal cases weigh less than 8 lb. (tripod 12–14 lb.). Good work can also be done with a $4\frac{1}{2}$ -inch vernier theodolite weighing (in its box) about 14 lb. Whatever type is used it must have a compass.

Base extension angles should be observed on four rounds, either F.L. and F.R. on two zeros or with a change of face and zero on each round; but two rounds, with a change of face and zero between them, will suffice for the other angles. Vertical angles should be observed once on each face. Stations in the base extension scheme should be beaconed, but elsewhere most of the observations will be to natural objects. However, some form of signal should always be left at stations which have been occupied, and occasionally it will be essential to mark in advance the next forward station.

Field routine. Selection and identification of points. The azimuth of the base and the latitude of one end should be observed, and a longitude, based on radio time signals, is also desirable (note, the longitude and azimuth observations may be combined), though these observations may be dispensed with if direct connection can be made with the local survey system. In addition, the magnetic bearing of the R.O. should be read at this and every other occupied station.

From the base-extension stations all prominent natural marks must be observed, both those that will probably be suitable for

future triangulation stations and those that will be useful for the detail surveyor as intersected points. In addition, sketches should be made of the immediate surroundings of each point observed to assist in future identification. The selection of future stations is fraught with great difficulty; in many cases only those on the direct route will be visited, and frequently this direct route is not known in advance. Further, on descending from a station, the surveyor will probably lose sight of all the points he has just observed. If this is not so, a careful watch should be kept on the changing appearance of the landscape, particularly of the points observed, as he moves forward.

The identification of points previously observed is often exceedingly difficult, and any step which can assist in this matter should be taken. Sides of triangles should be kept short, usually 6 to 12 miles; they should seldom exceed 20 miles. Observations should be made only to the highest points of hills, and, if the hill-top is flattish and indefinite, to those conspicuous natural marks which are close to it. The back bearing to the previously occupied station should guard against gross errors of position. A plane-table plot of the triangulation kept by the triangulator may prove invaluable, not only as an aid to identifying points but also as a ready means of judging the value of intersections and for planning future operations. Apart from duplicating the theodolite observations at each triangulation station, the table should be set up at intervals along the route whenever a position can be resected from previously fixed points. From these subsidiary set-ups it will be possible to make preliminary location by intersection of points to which, so far, only one theodolite ray has been observed and also to watch the changing appearance of points which have already been located but to which further rays will be required. In addition, new points can be intersected which may be of great assistance when resecting the plane-table set-ups. On the site of a station, identification of the actual spot must be based on the sketches made at previously occupied stations. Astronomical observations for azimuth should be observed at every occupied station and for latitude (and longitude if radio signals can be heard) at intervals of about 50 miles.

Triangulation schemes. The normal scheme will usually be a comparatively narrow chain of triangulation, and frequently only those stations along the direct route will be occupied, stations on the

flanks being unvisited. It frequently happens that a station has to be made to which no forward rays have been observed. Such a station must be resected from three previous stations. If the fix is a good one the triangulation can proceed, but if the resection is at all shaky it may be necessary to erect a beacon and return to a back station to make the missing observation.

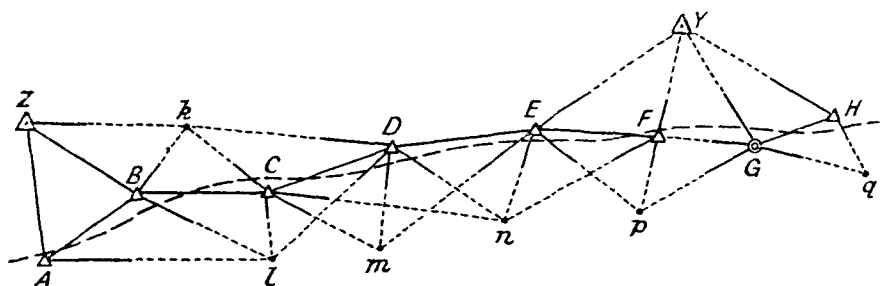


FIG. 132

Fig. 132 shows part of a rapid triangulation in which connection has been made with existing triangulated points Z and Y, from which scale and orientation may be determined. A, B, C, D, E and F are occupied stations close to the (pecked) line of route; k , l , m , n , p and q are unvisited stations. A rough base, extended to the side AB, enables BZ to be calculated; k and l are then fixed in the triangles ZBk and AB l , *et seq.*, endeavour being made to compute two independent values for each forward occupied station; e.g. C is computed via k and l , D, also via k and l . When station F is reached, the line ZABCDEFY is adjusted as a traverse between the fixed points Z and Y; revised co-ordinates for A, B, C, D, E and F are obtained and hence a revised scale and orientation. G represents a station to which no forward ray was observed, but, as it lies in a position favourable for satisfactory resection, the triangulation is carried on from there as though unbroken.

Breaks in triangulation. Complete breaks in the triangulation may occur in thickly wooded country. In such cases the sections of the triangulation must be treated separately, each with its own base line. The gap should be bridged by some form of rough traverse, and observations for latitude, and longitude if radio signals can be received, made each side of the gap.

PLANE-TABLE TRIANGULATION. The operations just described may be carried out in their entirety by graphic methods on a plane-table. When this is done the survey of the detail is usually made concurrently with the triangulation. Careful work should produce good results, but as a general rule this form of control is confined to much smaller areas than those envisaged above—e.g. a subsidiary chain run from station D of Fig. 132 northwards and eastwards round to Y.

LATITUDE AND AZIMUTH. In fairly open country and when the general direction of the route is north and south, a series of isolated control points may be established by 'latitude and azimuth'. The latitude is observed at every point occupied and also the azimuth to the adjacent stations, though the point to which the forward azimuth has been observed will often be impossible to identify on arrival at its site; so reliance will usually have to be placed on the azimuth of the back station at which some form of signal will have been left.

No great accuracy can be expected from the method. The probable error of an observed latitude due to uncertainty of the local deviation of the plumb-bob and observational errors will be at least 2", giving a probable error in the 'diff. lat.' between two stations of $2\sqrt{2}"$, say 300 feet; and this multiplied by the secant of the azimuth represents the probable error in the direct distance between the two. Because of this secant azimuth increase in error, the method should not be used when the azimuth departs more than 45° from the meridian; and, because the error is independent of the distance between stations, this distance should be as great as visibility will allow. (A standard of accuracy of 1/500 demands that stations should be apart 500×300 feet when north and south of each other and $500 \times 300 \times \sqrt{2}$ when the azimuth is 45° , say 30 and 40 miles.)

The 'mid-latitude' formula given on p. 76 (Chap. 3) should be used, the steps in the computation being, when both azimuths have been observed:

Compute the distance

$$S = \Delta\phi \rho \sin 1'' \sec \frac{1}{2} [\alpha_A + (\alpha_B + 180)]$$

where $\Delta\phi$ = diff. in lat. in seconds

α_A and α_B = the observed azimuths

and the value of ρ is for the mid-latitude, $\phi_M = \frac{1}{2} (\phi_A + \phi_B)$.

Then the diff. of long.

$$\Delta L = \frac{\nu \sin 1'' \sin \frac{1}{2} [\alpha_A + (\alpha_B + 180)]}{S \cos \phi_M}$$

ν being the value for ϕ_M .

When only one azimuth has been observed:

(a) compute approx. diff. long. from

$$\Delta' L = \Delta \phi \frac{\rho \tan \alpha_A}{\nu \cos \phi_M}$$

(b) compute $\frac{\Delta \alpha}{2}$ from

$$\tan \frac{\Delta \alpha}{2} = \tan \frac{\Delta' L}{2} \sin \phi_M \sec \Delta \phi$$

(c) compute $\frac{1}{2} [\alpha_A + (\alpha_B + 180)] = \alpha_A + \frac{\Delta \alpha}{2}$

(d) and (e) compute S and ΔL as given above.

ASTRONOMICAL POSITIONS AND ROUTE TRAVERSE. This is somewhat akin to the last method, but a closer analogy is the ordinary combination of dead reckoning and astronomical observations used in navigation. It is eminently suitable for rapid work in open but featureless country which is traversable nearly everywhere by motor vehicles, tracked or otherwise.

The method consists of a series of rough compass traverses adjusted between control points situated at distances of 20 to 50 miles apart. At each control point observations are made for position with a theodolite. Detail within the range of visibility of the route followed is plotted on a plane-table as the traverse proceeds, but is altered as necessary to fit the traverse when that has been adjusted to the control.

For the success of the method a radio capable of picking up time signals should be carried; the observations may be the usual ones for latitude and longitude, but it is preferable that simple altitudes should be observed and that these, corrected for refraction and parallax, should be worked out and plotted by the Marc St. Hilaire position-line method used in navigation.

It is a convenience for plotting a survey of this sort for the plane-table to be plotted on the Mercator projection with a graticule of latitude and longitude, and with an appropriate scale of land miles

and yards (or kilometres) for the mid-latitude of the projection. Best results for position will be obtained from observations to stars at altitudes between 20° and 50° , spaced at intervals of not less than 45° round the horizon; preferably, individual stars should be paired with others at about the same altitude on the opposite horizon. However, observations to sun and planets should not be neglected, and it should be noted that, provided the rough altitude and azimuth is set on the theodolite, it is possible to find and observe the planets Venus and Jupiter by day.*

SECONDARY THEODOLITE TRAVERSES. Rapid surveys may be controlled entirely by traverse in a manner similar to those described for precise traverse in Chap. 5. Secondary theodolite traverses or ordinary engineering traverses, even if the linear measurements are only made with a chain, are really more suitable for filling in detail between fixed stations not more than about 10 miles apart than for the rapid surveys being considered.

COMPASS TRAVERSES. For rapid surveys of comparatively small extent the compass traverse is very suitable, and reconnaissance surveys of areas of about 10 square miles, control and detail, may be carried out with satisfactory results by means of a 2-inch liquid compass and pacing. Larger areas may also be surveyed by compass traverse, and it is often the most convenient way of surveying the detail between main control points in thickly wooded country.

When large areas are to be surveyed by compass traverse, it is often convenient to subdivide the area by a network of precise compass traverses which follow the ground most favourable for linear measurement, the traverse legs being kept as long as possible. A large prismatic compass mounted on a tripod should be used for the bearings, which should be read to $5'$, and a steel tape under constant tension, but held horizontally, for the linear measurements. When possible, the traverses should close on fixed triangulation stations; otherwise the traverses should intersect each other to provide closed figures for adjustment. Usually the adjustment may be made graphically. The primary network is then broken down by secondary and tertiary compass traverse. (Frequently the secondary

* Note. Information concerning the planets is not given in the Star Almanac. The Abridged Nautical Almanac should therefore also be included in the equipment if planets are to be observed.

is omitted). In the former the bearings will be made with a prismatic compass mounted on a tripod or staff capable of reading to $\frac{1}{4}^{\circ}$ and the linear measurements by horizontal steel taping. In the latter a 2-inch liquid prismatic compass held in the hand read to the nearest degree is sufficiently accurate, and the linear measurements are made with a steel tape along the ground, though frequently a 300-ft. light steel wire rope is preferred.

In all compass traverses bearings should be taken from each end of the line so that if local attraction exists it may be discovered, but, even with this precaution, owing to difficulty in reading the compass and also to unknown diurnal variation, no bearing can be relied on to less than about 10 min. This corresponds to a lateral error of $1/344$, and it might be thought that there was inconsistency in making the linear measurements with the precision mentioned above; but it can be seen that, if the directions of the traverses in the net are comparatively straight and cut each other at right angles, they provide mutual checks on their directions which counterbalance, to some extent, the lack of accuracy in the angular measurement.

SEXTANT-SUBTENSE TRAVERSE. Rough traversing with all angular measurements made by sextant, and distances measured by subtense angles to a 10-foot pole, is extensively used by the hydrographic surveyor for surveying the detail of the coastline and its immediate topography. The method is convenient for inserting detail between two fixed positions, intervisible and not more than a couple of miles apart, but it can also be used for controlling rough rapid surveys, and for this purpose has one positive advantage in that the equipment required is very light.

A convenient subtense rod may be made from two canvas-covered rectangular wooden frames, each about 18 in. square, secured by pins and sockets to a light pole or bamboo. The canvas is painted white with a $2\frac{1}{2}$ -in. black stripe perpendicular to the pole, the distance between the centres of the black stripes, when the targets are secured to the pole, being exactly 10 feet (or any other desired distance, though anything greater than 10 feet becomes clumsy to handle). The angle subtended by the stripes is measured by bringing them into coincidence with the sextant. Normally the pole is held horizontally, but in cramped positions it may be held vertically. In either case it must be perpendicular to the line of

sight when the angle is read, and this perpendicularity may be achieved by fitting a sight vane to the centre of the pole or by a gentle swaying of the pole; in the latter case the maximum angle subtended is the reading required. The angle should be read both on and off the arc and the mean accepted.

Tables of distances may be easily computed, allowance being made for the separation between the index mirror and the line of collimation. Such a table, computed for a separation of $2\frac{1}{2}$ in., is included in the Admiralty Manual of Hydrographic Surveying (H.M.S.O.). It is best not to exceed distances of about 200 yards or, in terms of angles, the subtended angle should not be less than about 1° .

The measurement of direction presents difficulty and demands precautions if serious plottable error is to be avoided. (i) The traverse angle stations should be marked by a ranging pole of a height equal, at the least, to the height of eye, and, to avoid serious errors of centring, the angles must be observed with the eye held as closely as possible to this pole. Similarly the centre of the 10-foot pole must be held closely to the ranging pole when the subtense angles are being measured. (ii) As the sextant measures the true angle subtended at the instrument, which may differ considerably from the horizontal angle required, errors are incurred when the objects observed are at an elevation different from that of the observer. The horizontal angle may be calculated from the observed subtended angle by the formulae of spherical trigonometry if the angles of depression or elevation are also observed

$$\tan \frac{1}{2}\alpha = \sqrt{\operatorname{cosec} s \operatorname{cosec} (s - \theta) \sin (s - x) \sin (s - y)}$$

where α = horizontal angle required

θ = observed subtended angle

x and y = 90° - angle of elevation or

90° + angle of depression of the objects observed

and $s = \frac{1}{2}(\theta + x + y)$

but, since it is usually impracticable to measure these angles of depression or elevation, the best that can be done is to select points at eye-level immediately above or below the objects requiring to be observed by means of a plumb bob and then to observe the angle between these selected points. (iii) 120° is almost the largest angle that can be observed by most sextants with any accuracy. Consequently, if the traverse is roughly straight, the

main traverse angles must be observed in two parts by selecting an auxiliary point at right angles to the traverse line and observing separately back station to auxiliary point and fore station to auxiliary point. Preferably this auxiliary point should be distant and approximately at the same elevation as the observer; if the topography enforces the selection of a near object it is difficult to avoid inaccuracy. When the fixed station towards which the traverse is being run is in sight, it is best to measure all forward directions from this station, without any reference to the last traverse station visited except that, while the back fixed station remains in sight, the angle between it and the last traverse station should be measured as a check. An alternative is to measure the directions of the traverse legs with a prismatic compass.

A traverse of this nature should be plotted by protractor as the work proceeds and adjusted finally, when the terminal station is reached, by ordinary graphic methods.

TACHEOMETRIC SURVEYS. A complete survey, horizontal control, vertical control and detail, may be made by tacheometer and for some purposes, especially road or rail route surveying when the requirements are for a closely contoured map of a narrow strip of country, the method is ideal, but the survey when completed gives a much more detailed map than is envisaged in this chapter. As far as the horizontal control is concerned the method follows that of a theodolite traverse with distances measured tacheometrically.

VERTICAL CONTROL

The following methods are available:

Precise levelling	} For regular surveys of large areas (see Chap. 4)
Trigonometrical levelling by theodolite	
Ordinary levelling by engineer's level	
Ordinary levelling by hand level	
Trigonometrical levelling by clinometer	
Trigonometrical levelling by tacheometer	
Barometric levelling	
Hypsometer	

SPIRIT LEVELLING. Height control of a rapid survey by ordinary spirit levelling, whether carried out by an engineer's level or a

hand level, calls for little comment. No difference in principle is involved, though longer sights than normal may be permitted. A target levelling staff, which may be improvised by fitting a friction tight sliding sleeve over a normal staff, enables longer sights to be made, but its use demands a high standard of skill on the part of the staff-holder if time is to be saved and accuracy maintained. It is important with these long sights that the rule of equalizing fore- and back-sights should not be relaxed.

TRIGONOMETRICAL LEVELLING. If the horizontal control of the rapid survey is being carried out with a theodolite, whether by rapid triangulation or traverse, it will be convenient to establish the height control by trigonometrical levelling as described in Chapter 4, and vertical angles should be observed to all points concurrently with the horizontal angles.

In the type of rapid triangulation illustrated in Fig. 132 it will be possible to make but few reciprocal observations, but as nearly every point will be observed from at least three stations a check will be kept on the heights.

In the case of the theodolite traverse reciprocal observations will enable the height control to be carried forward from station to station. Should there be some convenient summit on the flank of the traverse which can be seen from several stations widely enough dispersed to enable its position to be intersected, vertical angles observed to it whenever possible should provide some check on the control. An absolute height may be obtained for any station from which the sea horizon is visible by observing the angle of depression to the horizon; this angle is the dip of the sea horizon (the combined angles due to curvature and refraction), and is related to the height of the observer by the approximate formula:

$$h \text{ (feet)} = \frac{R \sin^2 1'}{2 (1 - \mu)} \times \delta^2$$

where R = the mean radius of the earth in feet

μ = the coefficient of refraction

and δ = the dip *in minutes* of arc

For rays passing (of necessity) over land and sea, a good average value for μ is $\cdot 075$. The formula then becomes

$$h \text{ (feet)} = 1\cdot 04 \delta^2 \text{ (}\delta \text{ in minutes)}$$

A correction (similar to that for the height of beacon observed) must be made for the height of the tide at the moment of observation, i.e. for the height of the horizon above or below survey datum. Because of the uncertainty of the value of μ , the impossibility of a reciprocal observation, the uncertainty of height of tide and the difficulty in distinguishing the true horizon, the result of the observation may be very approximate; nevertheless at times the method may be of great use, especially if no great elevations are involved. (From a height of 1000 feet the distance of the sea horizon is about 42 miles (see Chap. 4, p. 99), and unless the atmosphere is very clear and unclouded it can be very difficult to distinguish).

Trigonometrical levelling by clinometer. In the case of a rapid survey conducted solely by plane-table all height information, control and detail will be supplied by clinometer. The ordinary 'Indian' clinometer should not be used for sights of over 3 miles, and indeed it is difficult to achieve this range; if longer sights are required a telescopic alidade is a necessity. Most clinometers are graduated in tangents as well as degrees, but for the purpose of control it is advisable to record the angle rather than the tangent. Heights determined by rays longer than one mile should be corrected for curvature and refraction by the table given on p. 48, Chap. 3, or by the simple rule

$$\text{correction in feet} = \frac{4}{3} D (\text{miles})^2$$

TACHEOMETRY. As explained on p. 329, the tacheometric method of stadia measurement is not entirely suitable for the types of survey being considered, but, if a closely contoured route survey is being carried out by tacheometer, it is often convenient to control the heights as well as fill in the contours entirely by tacheometry. No opportunity, however, should be neglected of making connection with known heights, either by trigonometrical or tacheometric levelling, should they be visible or accessible from the route being followed.

BAROMETRIC LEVELLING. Because of the portability of the instruments used, and because each observation (after certain corrections have been made) gives a determination of height which is virtually independent of the distance between stations, this is probably the most popular method of supplying the height control of reconnaissance surveys. However, it should be stated at the outset that

the method is really more suited for filling in the detail work between control points whose heights have been determined by more rigorous methods.

The barometric pressure recorded by a barometer may be considered as a measurement of the depth of the instrument below the upper surface of the atmosphere, and consequently a series of simultaneous readings of barometers at different positions in the survey area is similar to a series of simultaneous soundings taken during a hydrographic survey. The analogy may be carried a step farther for both series of measurements are made from a moving zero, and, since a multitude of simultaneous observations is impracticable, the movement of the zero must be observed at a base camp and used to correct the other measurements made in the field. For the most accurate work mercurial barometers should be used, but these are too fragile and unportable for rapid survey, and it is only the aneroid barometer that will be considered in detail.

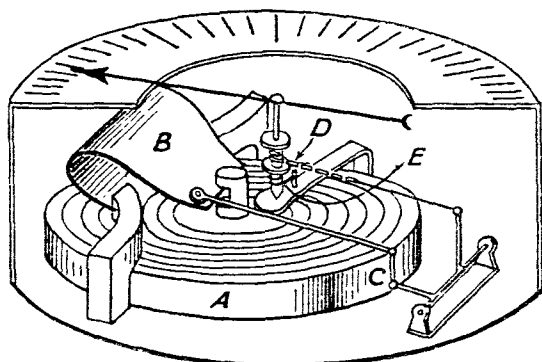


FIG. 133

The aneroid barometer (Fig. 133) consists essentially of a thin metal drum, usually of german silver (A), which is hermetically sealed and from which the air is exhausted. The instrument is described in more detail in Vol. I, but it should be noted that the method of graduation is by no means standardized: most instruments are graduated with a pressure scale and many have a height (elevation) scale in metres or feet as well; the latter may be either fixed or movable so that adjustment for the local barometric conditions can be made. The pressure scale may be graduated in inches

or centimetres of mercury or in millibars. The linear form of pressure scale is not satisfactory and so, at the conference of Commonwealth Survey Officers held in 1951, a resolution was passed that 'this conference considers that all surveying aneroids should carry a scale engraved in millibars . . .'

To avoid errors of parallax, the aneroid should always be held in the same position when reading, either vertically at eye level or horizontally. To free the recording mechanism (C,D,E) it should always be tapped lightly before reading. The instrument takes a short time to adjust itself to new height conditions and on arrival at a station it should be given about five minutes to settle down before being read.

Accurate graduation and compensation of aneroids is difficult, and the readings at different points of the scale and under different conditions of temperature should be compared with a good mercurial barometer and the corrections noted. Because of changes in the elasticity of the vacuum drum and its supporting spring (B), and wear on the mechanism generally, the corrections will not remain constant, and frequent comparisons are necessary. In particular, a new comparison should be made after prolonged stays in areas of low pressure. The fineness of reading depends on the range, which varies considerably in different instruments; 0-6,000 feet, 0-10,000 and even 0-16,000 feet are ranges covered. Refined reading is impossible, and the instruments should not be used near the limit of their range. If the work includes extreme variations in elevation, instruments covering different ranges, e.g. 0-6,000 and 5,000-10,000, should be carried. Aneroids should be treated with care and not given unnecessary jolting. A satisfactory arrangement is to use a 'battery' of about three instruments transported in a padded theodolite carrying case.

Observations. The atmosphere is never in static equilibrium, and surfaces of equal atmospheric pressure are never constantly level, so that two perfectly constructed aneroids at the same elevation above M.S.L. but 20 miles apart would seldom indicate the same pressure (or the same reading on the height scale), the difference between the readings being the barometric gradient between the two stations. The pressures recorded by the instruments are a combination of that due to the elevation and that due to the constantly changing atmospheric pressure, and it can be seen that, without

knowledge of the latter, a solitary reading of the aneroid is useless for determining the absolute elevation of the station above M.S.L. The method adopted, then, must be to determine difference of elevation between two stations preferably by several simultaneous observations at the two. The farther apart the stations and the greater the separation in time between the observations, the greater will be the uncertainty of the determination since undetected fluctuations of temperature and humidity affecting the pressure are more likely to occur. Two methods of observing may be adopted: (i) single observations, (ii) simultaneous observations.

Single observations. This is the less accurate method but is the more economical in labour and equipment. The aneroid and thermometer are read at the reference station and then carried from point to point and read wherever elevations are required. All elevations are then computed as differences of elevation between the reference station and the station being read. If no allowance is made for the changes in atmospheric pressure occurring during the day, the computed elevations may be much in error, but allowance can be made if a suitable routine of observation is adopted, such as returning to the reference station, usually the base camp, once or twice during the day to read the pressure there. This will enable the trend of the barometer to be ascertained and allowed for on a time basis, under the assumption that the same variations observed at the base exist throughout the area covered during the day. Should two or more stations of known heights lie in the area, readings taken at them and computed will also indicate changes of pressure occurring during the day. Fig. 134 shows an idealized scheme.

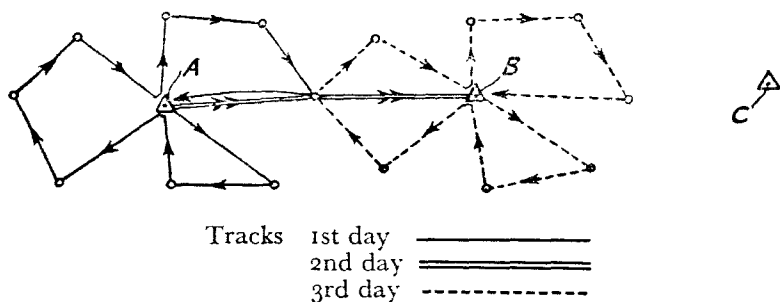


FIG. 134

A third method is, at intervals during the day, say before leaving camp, at 10 a.m., 2.0 p.m. and on arrival at new camp, to remain at a point (not necessarily a station) for half an hour and by two readings obtain the trend of the pressure gradient. In all these operations the atmospheric temperature must be observed when the aneroid is read, and, as stated before, a battery of these instruments is better than a solitary instrument.

Simultaneous observations. In this method two instruments, or better, two batteries of instruments, are employed. One kept at the base station is read at regular intervals, and at the same time the atmospheric temperature is recorded. Both are plotted on squared paper, and pressures and temperatures read from the graphs at the appropriate times are the 'simultaneous' observations used with the field observations to compute differences in elevation. An essential part of the routine is to compare the field and station instruments before leaving and on return to base camp to determine the index error. The station battery may with advantage be replaced by a good self-recording micro-barograph.

Diurnal variation of pressure. In temperate and high latitudes the atmospheric pressure is subject to large and irregular changes, but in the tropics the pressure is subject to small *regular* fluctuations with a twelve-hourly period which repeat themselves day after day. The maxima, at 1000 and 2200, and the minima, at 0400 and 1600, occur at the same local time everywhere, and barograph records in the tropics resemble tidal curves with regular times of high and low water. Use may be made of this fact for the fourth method of correcting aneroid readings; at fairly frequent intervals the instruments are kept in camp for several successive days and read at half-hourly intervals. The readings of each instrument are plotted against time on squared paper and a mean daily curve drawn which may then be used to determine the corrections to be applied to readings according to the times of observation. This method is frequently used when obtaining intermediate heights between benchmarks not more than about 15 miles apart.

Computation of barometric elevations. Various formulae have been proposed since Laplace made the first theoretical investigation of the subject, and graphs and tables to simplify the computation are given in various publications.

(i) Laplace's formula for mercurial barometers is

Diff. of elevation = $60,345.5 (\log \beta_1 - \log \beta_2)$

$$\times \left(1 + \frac{t_1 + t_2 - 64}{900} \right) \quad . \quad . \quad (a)$$

$$\times [1 + .0026 \cos (\phi_1 + \phi_2)] \quad . \quad (b)$$

$$\times \left(1 + \frac{\Delta h + 52,252 + 2h_1}{R} \right) \quad . \quad (c)$$

where β_1 and β_2 are the barometer readings at the lower and upper stations reduced to a temperature of 32° F. by the formula reduced reading

$$= \text{actual reading} \times [1 - .00009 (\text{temp. of attached therm.} - 32)]$$

Note: With compensated aneroids this reduction to 32° F. is unnecessary and the term 52,252 is omitted from factor (c) of the formula.

(a) is a factor allowing for the mean temperature of the air column and an average amount of humidity, t_1 and t_2 being the air temperatures in $^\circ\text{F.}$ at the lower and upper stations.

(b) allows for the variation of gravity with latitude, ϕ_1 and ϕ_2 being the latitudes of the lower and upper stations.

(c) allows for the diminution of gravity with altitude, Δh being the calculated diff. of elevation before factor (c) is applied. h_1 is the elevation at the lower station and R = the radius of the earth in feet (say 20,890,000).

(ii) Bailey's modification of the formula is

Diff. of elev. in feet = $60,346 \{ \log \beta_1 - [\log \beta_2 + \log (d)] \}$

$$\times \left(1 + \frac{t_1 + t_2 - 64}{900} \right) \quad . \quad (a) \text{ as before}$$

$$\times (1 + .002695 \cos 2 \phi_m) \quad . \quad . \quad (b)$$

In this β_1 and β_2 are the *unreduced* barometer readings and (d), which is *not* required for compensated aneroids, is the factor for reducing β_2 to the temperature β_1 . $(d) = [1 + 0.0001 (T_1 - T_2)]$ where T_1 and T_2 are the attached thermometers at the lower and upper stations. (b) is the latitude correction, ϕ_m being the mean latitude of the two stations. Bailey's tables, which are printed in *Close's Text Book of Topographical Surveying*, tabulate

$$\log \left[60,346 \left(1 + \frac{t_1 + t_2 - 64}{900} \right) \right] \quad \text{as Table (a)}$$

$$\log [1 + 0.0001 (T_1 - T_2)] \quad \text{as Table (b) [not required for aneroids]}$$

$$\text{and } \log (1 + 0.002695 \cos 2\phi_m) \quad \text{as Table (c)}$$

If not available, such tables may be soon computed, arguments for Table (a) being $(t_1 + t_2)$ at 5°F. intervals and for Table (c) being ϕ_m at intervals convenient for the area being surveyed.

(iii) Loomis follows Laplace with only minor modifications. His tables as adapted by Galton are given in *Hints to Travellers* (Royal Geographical Society). They are possibly a little more accurate than Bailey's tables but are not so convenient to use.

(iv) For greater refinement the 'international' formula, which allows for the actual humidity of the air at the time of observation, may be used

$$\Delta h = 60370 (\log \beta_1 - \log \beta_2)$$

$$\times \left(1 + 0.00264 \cos 2\phi_m + \frac{h_1 + h_2}{R} + \frac{3P}{8\beta_m} \right)$$

$$\times [1 + 0.002036 (t_m - 32)]$$

β_1 and β_2 are the barometer readings at the lower and upper station
 β_m is the mean reading

h_1 and h_2 are the heights of the lower and upper station

t_m is the mean temperature of the air

$P = P_w - 0.00045 \beta_m (t_d - t_w)$ in which P_w = saturation pressure of water vapour and

t_d and t_w are the readings of dry and wet bulb thermometers.

(v) As the above formulae assume that the temperature of the air column is uniform at all heights and do not take into account the lapse-rate, i.e. the rate of fall in temperature with height (roughly 3.6°F. per 1000 feet), *Aneroid Tables Based on a Standard Atmosphere and a Standard Lapse-Rate* (H.M.S.O. 1935) were prepared by an inter-services committee which was appointed in 1930 to consider the most suitable formula on which to base the height scale for aneroids (and altimeters). These tables are probably the most accurate of any published.

(vi) An ingenious method of solving the problem is given by Salt in *A Simple Method of Surveying from Air Photographs* (Professional Papers of Air Survey Committee No. 8), in which a graph is

plotted of the ratio, difference of elevation/barometric inch for different temperatures and pressures. This graph is entered for β_m and t_m (notification as before), and the result (feet per barometric inch) is multiplied by $(\beta_1 - \beta_2)$, β being in inches.

Errors of barometric levelling. The number of formulae given above (necessitated by the variety of tables that may be accessible) is sufficient indication of the lack of precision that may be expected from the reduction of the observations, but these are negligible when compared with the errors due to (a) atmospheric causes, (b) instrumental errors and (c) observational errors.

(a) are due chiefly to the impossibility of correct estimation of the temperature lapse rate and humidity of the air column. The uncertainty increases the more widely the stations are separated and the more unsettled the weather; in fact, observations should be suspended in stormy weather.

(b) has already been discussed on page 333, but it should be noted that instrumental errors are virtually absent in a good mercurial barometer.

(c) observational errors are those of parallax, which may be reduced by repetition of readings; of sluggishness, which may be reduced by waiting a few minutes at the station before reading, especially after a quick change in elevation; and of incorrect estimation of temperature, to minimize which both aneroid and thermometer should be shielded from the sun. Clark gives 5 to 20 feet as the likely limit of accuracy for a single observation and with batteries of no fewer than three aneroids.

Hypsometer or boiling-point thermometer. The pressure of the atmosphere is directly reflected in the temperature of the boiling point of water, and use can be made of this fact to determine differences of elevation. The hypsometer is a portable apparatus by means of which a thermometer held in the steam of boiling rain-water is read when the mercury has stopped rising and remains stationary. At the same time the shade temperature of the surrounding air is observed.

Observations at the two stations should be simultaneous (though that is seldom practicable) and may be reduced by tables given in various publications such as *Hints to Travellers* or by the empirical formula:

$$\Delta h = \{[T_2 (521 + .75 T_2)] - [T_1 (521 + .75 T_1)]\} \times \left(1 + \frac{t_1 + t_2 - 64}{900}\right)$$

where T_1 and T_2 are °F. below 212° at lower and upper stations and t_1 and t_2 are air temperatures „ „ „ „ „

Since 0.1°, about the limit of accuracy to which the thermometer can be read, represents over 50 feet, it can be seen that only rough results are possible, and so this instrument is seldom used by British surveyors though it is still used by some Continental surveyors.

SURVEYING DETAIL

When the control has been established, various methods of surveying the detail are available:

Plane-tabling	}	Suitable for regular surveys of large areas
Theodolite traverse		
Chain surveying		
Tacheometry		
Photographic surveying		
Compass traverse		
Sextant „		Spirit levelling
Route „		Clinometer
Sketching		Barometric levelling

Most of these have been already discussed either in this chapter or in Volume I, and but little amplification is necessary for adapting the methods to rapid survey.

PLANE-TABLING. In open country the plane-table is undoubtedly the most convenient instrument for supplying the detail. The scale adopted is usually that intended for reproduction; in the case of rapid surveys it will generally be small, of the order of 1/250,000 (or 4 miles to the inch). It is convenient to plot the field sheets, after they have been mounted on the tables, with a graticule of latitude and longitude on which the control may be plotted by geographical co-ordinates. If larger scales are used, it may be more convenient to use a grid and rectangular co-ordinates.

Most of the field work in rapid surveys will be done from plane-table stations resected from the fixed points of the control. Detail is surveyed by sketching between intersected points. Much experience is needed for this sketching, and the beginner will need to intersect many more points than the experienced man who, on

really small scales, may dispense with intersection entirely. In this connection it should be noted that the tendency with beginners is always to exaggerate, whether it is the depth of a river meander, the smoothness of flat country or the ruggedness of hill country. Elevations will be determined trigonometrically by telescopic alidade or Indian clinometer, or barometrically by the aneroid, or probably by both, the trigonometrical method being used to determine the elevation of the plane-table stations and the aneroid for determining spot heights in their vicinity and on the route between stations. On the assumption that the height control will have been determined by trigonometrical levelling, it should be possible to obtain three values for the elevation of each plane-table resection and at least two values for each intersected position. The weighting of these values should be roughly in inverse proportion to the length of the ray observed; allowance must be made for curvature and refraction. The mean values will establish a secondary height control for the barometric spot heights. If the aneroid is fitted with a height scale it is convenient to make use of it for this rough work, by setting the scale at the correct reading and noting the time before leaving the first station or base camp. Spot heights are observed as necessary, the time being noted at each observation. The errors of the readings at the control stations are divided amongst the intervening observed spot heights proportionately to the elapsed time. See example in Table 23 opposite.

Form lines should be sketched in at the plane-table stations, and also when the surveyor is travelling between them, later, if there are a sufficiency of spot heights, they may be transformed into regular contours.

COMPASS TRAVERSE. In thickly wooded country compass traversing will probably be the most satisfactory way of surveying the detail. Careful planning is necessary to cover the ground efficiently, and when possible the work should be planned as a network criss-crossing between stations previously fixed by the control traverses or by triangulation. The procedure is outlined on page 326, and normally a 2-in. liquid prismatic compass is used. Distances are usually measured by steel taping, though light wire ropes, up to 300 feet long, are sometimes preferred. For very rough work, such as short traverses off to the side of the detail traverse being run, pacing will suffice. Much concentration is, however, necessary,

and, when a reliable chainman is available, the counting of the paces should be done by him, leaving the surveyor free to note the topography as he passes. Instruments may be obtained which count automatically; these are the *passometer* which records the number of paces and the *pedometer* which records in miles and quarter miles.

TABLE 23

METHOD OF BOOKING BAROMETRIC SPOT HEIGHTS

Station	Time	Interval (hrs.)	Aneroid No.B372	Error	Corrn.	R.L.	Remarks
P.T.14	0730		302	0	0	<u>302</u>	R.L. from \triangle^s D.E. & G. Aneroid set to 302
14.1	0815	0.75	365		-1.9	363	
14.2	0945	2.25	280	$\left(\frac{12}{4.75}\right)$	-5.9	274	
14.3	1055	3.4	350		-8.6	341	
P.T.15	1215	4.75	454	+12	-12	<u>442</u>	R.L. from \triangle^s E.F. & G.
	1345	0	456	+14	-14		
15.1	1440	0.9	575	$\left(\frac{5}{2.5}\right)$	-15.8	559	
15.2	1530	1.75	780		-17.5	762	
P.T.16	1615	2.5	835	+19	-19	<u>816</u>	R.L. from \triangle^s E.G & K.

Both are about the size and shape of a watch, and are carried upright in the pocket. The jolt of the pace actuates the mechanism. The pedometer can be adjusted to suit the pace of the user. Heights will usually be surveyed by aneroid controlled by the method given above. It is well, in addition, to carry an Abney level, so that heights may be checked trigonometrically when control points become visible through clearings.

SEXTANT TRAVERSING WITH 10-FOOT POLE. This is described on page 327, and, as mentioned there, the method is much more suitable for detail surveying than for control.

ROUTE TRAVERSING is a general name given to any method of traversing of a low grade of accuracy. Directions are nearly always

measured by compass, but in thick bush the bearings may have to be made towards the sound of a whistle blown by a chainman at the end of a 300-ft. rope. Such a method, known as *rope and sound* traverse, has been much used in thick forested areas. Distances are measured with light wire or hemp ropes 310 feet long, and the bearing is observed to the sound of the whistle at every third rope length, the distance then being taken arbitrarily as 900 feet. Astonishingly good results may be obtained, provided the control points are not more than five or six miles apart; it is particularly useful for plotting unimportant streams, forest paths, etc. Elevations are observed with the aneroid. Other methods of measuring distance are by the measuring wheel or perambulator, the usual type of which is similar to the front fork, wheel and handlebars of a bicycle fitted with a cyclometer. Estimation by time may also be used by noting the time spent on each leg of a traverse run between two control points.

In all route traversing every opportunity should be made of fixing detail on the flanks of the traverse by intersection of compass bearings and by bearing and distance.

Although plane-tableing is usually plotted on the scale intended for reproduction, it is better to plot compass and route traverses on a larger scale; after graphical adjustment they may be reduced to the required scale for inclusion on the 'master' field sheet.

SKETCHING. Two forms of sketching can be of great use in rapid survey, but considerable experience is required before either can be done with both speed and accuracy.

Field sketches made from control points can be of great assistance in unravelling awkward problems that may occur when the topography is being plotted later. The field sketch must be drawn as an engineering elevation rather than as an academy picture; in fact artistic talent can frequently be more of a hindrance than a help. Squared paper is useful for maintaining scale, and it is convenient to make the vertical scale double the horizontal (10° per inch is convenient for the horizontal scale). A prominent feature should be selected in the middle of the field of the sketch as a zero and a few salient points fixed and plotted by horizontal angle, observed by theodolite or compass, and vertical angle, by theodolite or Abney level; intervening features are inserted by eye, and finally

the value of the sketch may be enhanced by observing and noting on the sketch the horizontal angles (or bearings) and vertical angles of a few of the more prominent features sketched.

Route sketches are sketches of the planimetry of the country made while on a route traverse. Where possible, conspicuous features are fixed by intersection, either by plane-table methods using some improvised alidade or by compass bearings; but elsewhere the topography is sketched by eye. It is most important to preserve the scale and not to waste time sketching in detail which cannot be shown in the final production. Many gaps in the route sketch may be filled in if there is a climbable hill accessible to the route of the traverse.

PHOTOGRAPHIC SURVEYING is dealt with in Chapter 9. Suffice it to say here that it is usually a substitute for plane-tabling. The camera stations take the place of the plane-table stations, and the subsequent plotting is done by plane-table methods in the office. The method is of greatest use in mountainous country (and in fact was largely developed by the late Dr. Deville for the survey of the Rocky Mountains), particularly when advantage has to be taken of a very brief spell of fine weather.

NAME BOOK. Correct nomenclature is a difficult problem in any survey, and field sheets soon become undecipherable if names are written on them too frequently. It is therefore desirable to distinguish the topographical features by numbers and to keep a name book in which the names of the features may be inserted when they become known. When there is a conflict of opinion as to the correctness of a name, the most responsible local inhabitant should be asked to give a deciding vote.

SURVEYING FROM AIR PHOTOGRAPHS. This subject is dealt with in Chapters 9 and 10 and is only introduced here to emphasize that, when the necessary equipment is available and suitable weather prevails, much more reliable rapid survey work can be accomplished from the air than from many of the methods described above.

MAP REVISION

The inclusion of map revision in this chapter must not be taken to mean that rapid survey methods suffice, for few operations in

surveying need to be undertaken with more care and deliberation. A map on which such methods could be tolerated is probably in need of complete re-survey rather than revision.

But few 'rules' can be given, since the cause of revision may range from a new fence across a field to the building of a new satellite town. In fact the only part of the procedure common to all cases is the careful identification on the ground of the points of detail shown on the largest scale map from which the new work may be 'tied' to the old. Considerable skill in map-reading may be required, particularly if much development has taken place since the original survey. In the town, the corners of buildings, in the country, the intersections of field boundaries, are the most likely points to select, but road-widening, new building, etc., may frequently have obscured or replaced the actual points shown on the map, the alteration often being small enough to escape detection if care is not taken; e.g. the building of a new porch may have 'moved' the corner of a building by 10 feet or so since it was originally surveyed. Its new and unweathered appearance may be a guide to very recent development, but the only safe method is to check the points of detail finally selected by measurement from other apparently recognizable features shown on the map.

In highly developed countries routine map-revision will be undertaken by the Government survey department, but private users may need to bring their maps up to date before such routine revision is made. In such cases the survey department will usually furnish the private reviser with a descriptive list of all the co-ordinated points in the locality, which should be of great assistance; though it should be noted that careful identification of points is still necessary.

When the revision is small and there are many identifiable points in the vicinity, ordinary chain survey methods of direct measurement can be employed. Many useful hints as to how these measurements may be made are given in *An Outline of the History and Revision of 25-inch Ordnance Survey Plans* (H.M.S.O. 1932, reprinted 1950). A typical (and very simple) example is shown in Fig. 135, in which the old material is shown in light line and the new in heavy line. The encircled positions are identified points of detail whose positions have been tested by measurements to other mapped detail. Direct measurements along the wall (measurements checked from both ends) establish points A, B and C;

similar measurements along the opposite hedge establish D, E, F and G. Similarly X and W along one hedge and Y and Z along the opposite determine the sides of the new buildings. 1, 2 and 3

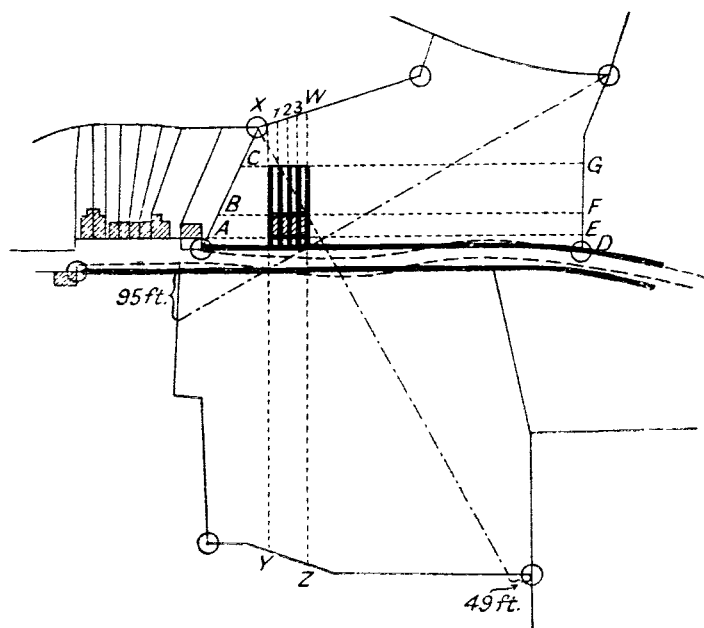


FIG. 135

and a measurement along the frontage AE determine the individual property boundaries. Finally check sighting lines are observed as shown by chain line. The chief thing to note about this simple procedure is the fulfilment of the first principle of surveying of working from the whole to the part. Should a minor error of measurement have been made in determining the position of E, for example, the effect of the error on the position of the house frontage would be much less than if the same error had been made in a measurement along the side of the house in line WZ.

Other types of problem would be the plotting of new work erected, say, in the middle of open heathland with:

(1) No identifiable points within a mile or so.

Such a case might be dealt with in the following way: From the survey department obtain details and co-ordinates of trig points

in the district. Say A and B (Fig. 136) are suitably placed. Establish local control points X, Y and Z so that AXZ and BYZ are in straight lines. Observe the following angles:

at X	1, 2 and 3
at Y	4, 5 and 6
at Z	7

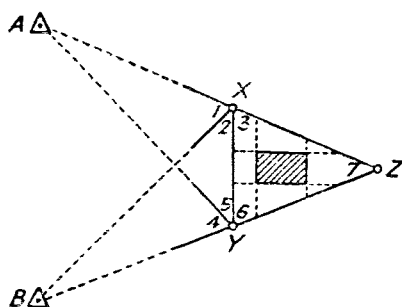


FIG. 136

In addition measure the sides XY, XZ and YZ, noting on the chain lines the cutting points of the boundaries of the new works.

The direction and length of XY may then be calculated from the co-ordinates of A and B by the *inaccessible base* calculation (p. 88) and thence XZ and YZ by ordinary triangulation. The chained measurements of the sides act as checks and provide the data for plotting the new work.

(2) One identifiable trig point within easy reach of the new work and another co-ordinated point visible.



FIG. 137

In this case (Fig. 137) a closed traverse AXYZWA (with XW also measured) would be run, the orientation being determined by the ray AB.

In all cases further checks are provided by chaining right round the works themselves.

MAP REVISION FROM AIR PHOTOGRAPHS. Air survey is discussed in detail in Chapters 9 and 10, and there is no need here to do more than mention, what must be obvious, that when the necessary equipment is available map revision may frequently be carried out very readily from air photographs.

When the new work is of comparatively small extent and of negligible elevation, provided it is surrounded by well-mapped detail, complete revision can often be carried out from a single photograph either vertical or oblique. The conditions necessary for such single photograph revision are detailed in Chapter 10. When complete coverage is available, map revision is carried out by the ordinary methods of air survey; identifiable points of detail supplying the control which would otherwise be supplied from triangulated points.

CHAPTER 12

ENGINEERING SURVEYS FOR LARGE SCHEMES

THE SUBJECT matter of this chapter is essentially the application of the craft of surveying to engineering problems. True craftsmanship has an infinitude of methods; it is therefore only possible to indicate certain principles and categories which engineering surveys tend to follow. Such detailed descriptions as are given are more for the purpose of illustrating these principles and categories than to provide an exhaustive description of the survey for any particular engineering project. In fact, surveys for engineering projects are essentially *ad hoc* problems; they do, however, fall into two broad categories, those for preliminary investigations and those for setting out the works for the contractors. These are usually referred to as the *location* and *construction* surveys.

LOCATION SURVEYS

Depending on the nature of the survey a preliminary investigation, which is little more than an inspection of the terrain, is followed by the true preliminary (or location) survey; sometimes the two are carried out together. The former is to limit the subsequent survey to routes or sites which alone are feasible, and the latter to obtain sufficient data (*a*) to decide which are the best routes or sites and (*b*) to enable adjustments to be made to the centre line of the route or the lay-out of the site to obtain the most economical construction.

Such surveys therefore demand an accuracy consistent with their purposes. Levels and location of contours must be accurate enough to determine changes in cut or fill for different trial centre lines. For some hydraulic works, especially if the lay-out is approximately defined, it pays to establish benchmarks to precise levels which are subsequently used for the construction survey.

In general the normal accuracy of ordinary levelling and theodolite and chain traverses is sufficient; sometimes in uninhabited

country stadia and theodolite traverses have sufficed for much of the survey. A rigid control is not required, the methods of plane survey being sufficient over lengths of many miles irrespective of geodetic control, though of course such trig stations as are encountered, if they exist at all, are tied in to the survey.

There is nearly always some controlling factor—depending on the type of construction for which the survey is required—which determines the feasibility or otherwise of the proposed scheme; and the experienced engineer will concentrate his party on this factor at the outset of his survey. For example, on surveys for low head hydro-electric schemes levels are predominant, and they in turn are controlled by certain physical features. It may be a watershed if storage is involved, and in this case survey above the level of the water-shed in any part of the scheme is obviously redundant. River cross-sections where the river is deep and sluggish in its flow are redundant in the first place, whereas in narrow swift-flowing sections they are of importance. For railways, roads and canals a narrow band of contours along the one or more possible lines is the aim of the survey, and grade is the controlling factor.

For transmission lines a knowledge of the mechanical design is of advantage as economical spans, minimum ground clearance, etc., have a bearing on the survey. Where the ground profile between supports is flat, or convex upwards, it must be accurately determined, especially for high spots, whereas if it is concave between supports the shape of the profile is of no importance.

Surveys for town planning or building estates are nearly always on land that has been already surveyed and will be dealt with later in this chapter.

It should now be clear that for preliminary surveys there are two categories, those over unsurveyed territory and those for which reliable surveys already exist. Furthermore, the engineer in charge should have a sound knowledge of the type of work for which he is surveying if much unnecessary labour is to be avoided; this is particularly the case in unsurveyed territory where the cost of maintaining a survey party is high. On him, too, depends not only the rejection or adoption of the scheme as a whole but also the selection of the detailed lay-out as chosen from the location survey. Once such a lay-out has been selected it is very difficult to change it after the many legal, social and technical processes that are involved have been set in motion.

A description of a particular preliminary survey for a particular project will now be given; but the reader should always bear in mind the gist of the opening paragraphs of this chapter—namely, that every preliminary survey has its own individual aim, with its own individual limiting factors and problems.

SURVEYS IN PREVIOUSLY UNSURVEYED TERRITORY. As an example of work in this category the following scheme for a water power project is typical of the procedure required. A river flows, as shown

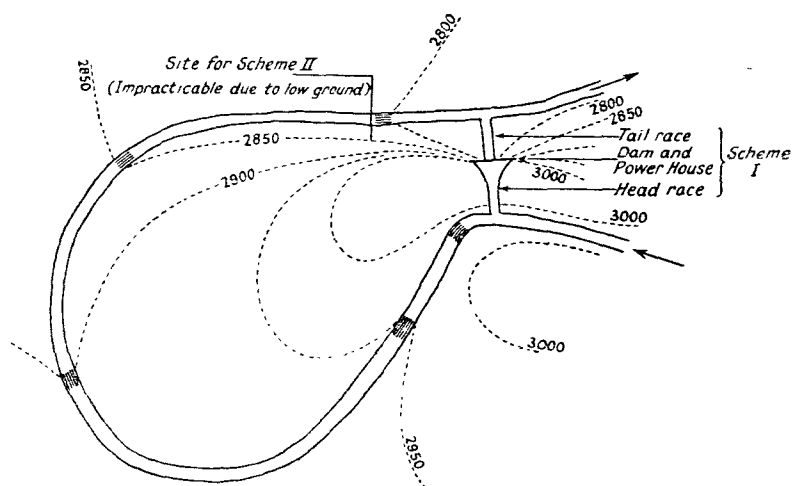


FIG. 138

in Fig. 138, over a series of falls which form a loop in plan. The water level at the top of the loop is fixed by considerations beyond the scope of the survey and is hereafter referred to as 'the head water level'.

The possible developments are:

- (i) a cut across the neck of the loop with a low diversion dam to turn the water into the cut, and a power house at the foot, below the lowest fall on the loop;
- (ii) a dam across the head of the lowest fall on the loop raising the water level to that of the *head water level* (which also may have the advantage of providing storage), the power house being sited below the fall;

(iii) two or more power sites along the loop with slack water between each, thus making full use of the entire fall, as do schemes (i) and (ii).

Such considerations as compensation water under scheme (i) or the flooding of land under scheme (ii) are not being considered. If the territory had been developed or the timber or mineral value was appreciable, there would exist a strong argument in favour of the third and probably the most costly scheme.

The first duties of the engineer in charge of the preliminary survey are then:

Scheme (i). A quick investigation with levels across the neck to ascertain that the elevation of the height of land is not too great to rule out the possibility of a cut (it is assumed that the volume of water is too great to consider the possibility of a tunnel) and that the centre line of the cut can follow the contours to the proposed site of the power house.

Scheme (ii). A cross-section above the lowest fall carrying the levels to the same elevation as the 'head water level'. From Fig. 138 it is obvious that flying levels perpendicular to the river on the left bank at the lowest fall would soon show that Scheme (ii) is impracticable, as the ground lies much below the 'head water level'. Hence any further survey here is redundant (except possibly for Scheme (iii)). If this level could have been reached, a quick investigation of the entire water-shed on each bank of the loop would then be made to insure that the water could be contained at the elevation of the head water level.

If Scheme (i) proves satisfactory a route survey for the proposed cut must be made. This consists in contouring a band on each side of the chosen centre line or centre lines if more than one distinct line appears possible. It should be apparent now that the engineer in charge should have a sound experience of water-power work, as he must make his bands of contours sufficiently wide to cover the modifications that the plot may show to be necessary in order to keep the cost to a minimum. This requires knowledge of the hydraulic design of the cut, without which the survey might have to be extended far beyond what in the end might prove necessary. He must have a knowledge of power-house lay-out in order to minimize the grid surveys required.

Traverse stations should be referenced wherever it is possible. In timbered country three trees are blazed and nailed near the

ground. Lengths from the nailhead to the station are then taken. It is seldom possible to take reference angles owing to the absence of clearly defined objects such as steeples, etc.; occasionally it is impossible to reference a station or make it permanent (e.g. in soft ground in open country). Permanent benchmarks must be established. The party should include a draughtsman, and each day's work be kept plotted up to date in order that the engineer in charge can appreciate the progress and direct the work. One minor but very important point for this type of survey should be noted. The river *must* be surveyed for some distance below the lowest fall to determine conditions of tail water and the possibility of lowering it by cuts.

The foregoing should indicate the general lines of such a survey, but to the imaginative reader many other possibilities will arise. For example it might be more economical to dam the second lowest fall to the head water level above the loop and lead from it by open channel to a power house at the lowest fall. All such possibilities must be considered by the engineer in charge, who must organize the work of his party in order to eliminate the work on the less likely projects.

The use of air surveys (Chapter 10) in unsurveyed country should always be borne in mind.

Sometimes, occupied with the technique of his work, the engineer is apt to overlook data which it is his duty to collect. For example, the type and size of the timber that may be flooded out is of great importance, besides the more obvious items such as the nature of the rock, soil, direction of streams crossing a traverse, etc.

LOCATION WORK WHERE RELIABLE SURVEYS ALREADY EXIST. In these circumstances the character of the work is very different. Much of the preliminary planning can be done in the office; but it should be very clearly understood that *nothing should deter the responsible engineer from the personal visit, preferably on foot*. It will be good for his health even if no unforeseen feature is revealed. The physical nature of the ground is no longer the only primary consideration; property rights, conflicting interests and amenities now are very much in the picture. Survey is still required, but often limited to areas where existing surveys are not sufficiently detailed for the requirements; this particularly applies to levelling. Unlike the cost in unsurveyed territory, the cost of maintaining the survey

party is here small, and features that have been omitted can easily be surveyed as required, and carried out by a junior engineer.

All boundaries must be carefully noted within the limits of the survey and the names of the property-owners where they can be ascertained. Local information is very useful but should be accepted with some caution. A young engineer was once surveying a river for flood levels for compensation payments to riparian owners, and he was using the local inhabitants to show him the highest point the water had reached and then taking the level of such points. After one such incident on a still summer's day he heard the following conversation from across the valley:

"What did yon yin want, Pat?"

"I'm thinking he would be one of them fellas surveying for the Compensation. Annyway, I showed him higher nor what sorra a spate ever reached in my life."

SURVEYS FOR TOWN PLANNING AND BUILDING ESTATES. Co-operation with the city engineer or architect is essential, and preliminary interviews will determine the nature of the controlling features, which are usually (apart from considerations of amenity and architecture) sewage, junction of new with existing roads and water supply.

Contours on existing surveys are rarely sufficient, even for a preliminary investigation, and grid levelling is required with interpolated contours. As for more detailed considerations, every engineer has his own methods of collecting the necessary data on survey, but the following may be of use for anyone who has not carried out this type of survey before. It particularly applies to building estates.

With a theodolite set out one or more large rectangles. Chain the sides, putting in poles preferably on both the long sides at the required intervals, say 100 ft., thus forming lines at 100-ft. intervals parallel to the short sides of the rectangle. Line in one pole about halfway across each line. The rodman now has no difficulty in keeping accurately 'on line' for each line, and with his chainman rapidly gets his position on plan for each ground-level, while the instrument man can pick up all the ground-levels for several lines with one set-up of the level if the fall of the ground permits.

Great care must be taken that the rodman calls out clearly the letter (it is rather better to refer the lines by consecutive letters of

the alphabet than to use numbers) of the line and the chainage for each ground level. This is particularly important where the ground falls too rapidly along the lines for one set-up of the level, and portions of a number of lines are taken from one set-up followed by another set-up to complete the lines. A missed or duplicated ground-level can vitiate a large portion of the work if the error is not observed at once. The fall of the ground may make it preferable to run the lines parallel to the long sides of the rectangle.

One reminder, obvious but sometimes overlooked: tie in to at least two clearly defined objects that are unmistakably shown on the architect's plan, in order to correlate the two plans.

AIR SURVEY. A brief reference was made above to the possibly great advantage of air survey under certain conditions. Two excellent examples are given in *Route Surveys* (Rubey, MacMillan and Co.), one for a project in a highly developed and surveyed area and one in a more or less unsurveyed and undeveloped area. Space forbids more than a brief note about these, but the following points are of interest:

For the first scheme by the combination of existing surveys with an aerial survey the entire project was laid out in the office, and all the subsidiary activities such as land-buying, negotiations with other authorities, specifications, contracts, etc., were carried out without a surveyor going into the field.

For the second scheme, covering an area of 100 miles by 25 miles, the field work was limited to 10 astro fixes for control and 110 miles of traverse and levelling on the final line selected from the aerial survey plans, which also yielded most of the engineering information for quantities, bridges, etc. The scheme was for a new road.

CONSTRUCTION SURVEYS

Here the ultimate object is to set out markers for both vertical and horizontal control from which the foreman can readily determine by very simple direct measurements or sights the position of the structure in plan and elevation.

To give the actual position of the structure is valueless as the markers will be disturbed at the very beginning of the construction work. For example, as anyone who has watched the building of a simple rectangular house will know, the plan is located by eight pegs set well outside the area of the house, two marking exactly the

line of each gable end and two marking exactly the line of each wall; four builders' lines stretched over each pair of pegs will then register the gables and walls in plan. Again, as anyone who has kept his eyes open when travelling our roads should know, driving a peg in the ground to register the level of the invert of a sewer or pipe at that position is useless, as it will be knocked out at the first run of the trenching machine. The method employed is to establish in plan the line of the sewer at two points beyond the limits of the immediate excavation. At these points a pair of four-by-two timbers are driven in and a cross batten fixed, the top being at exactly the same height (h) above the invert at each of the two points. These contraptions are called 'sight rails'.

A man standing behind one of these and sighting along the top of the cross battens will be sighting along a line exactly h ft. above the invert of the sewer; therefore any form of rod h ft. long, held with its top on this line of sight at any point between or beyond the sight rails, will have the lower end coincident with the invert, provided that the grade of the sewer is constant. This rod is usually a single light timber with a cross batten at the top, and is called a 'boning rod' in this country and a 'plat-board' in America. The operation can be seen on almost any highway where excavation is in progress.

These two examples, that for the rectangular building and that for the sewer invert, are the simplest cases of setting out and serve to illustrate the purpose and general method of all setting out. No measurements are required—a workman can obtain from them the information he requires—and there are no sighting points or lining points in the immediate vicinity of the actual work. Those are the two principles that have universal application on those sections of construction surveys dealing with the actual jobs—simplicity, and no pegs on the position of any work.

On works of any magnitude, however, there is bound to be a construction survey setting out the exact relative position of different parts of the structure. A high degree of precision is required, and the engineer must be highly skilled in the craft of surveying. For example, take a bridge crossing a wide river. The *exact* position of the abutments, as shown on the erection drawings in relation to each other, must be established. The centre line will have already been pegged, or can be established from stations of the location survey. In this case careful triangulation is required. Two base

lines preferably at right angles to the centre line, one on each side of the river, gives the best solution if the nature of the ground permits. It not only allows the exact position of the abutments to be fixed but allows any point on the bridge as shown on the erection

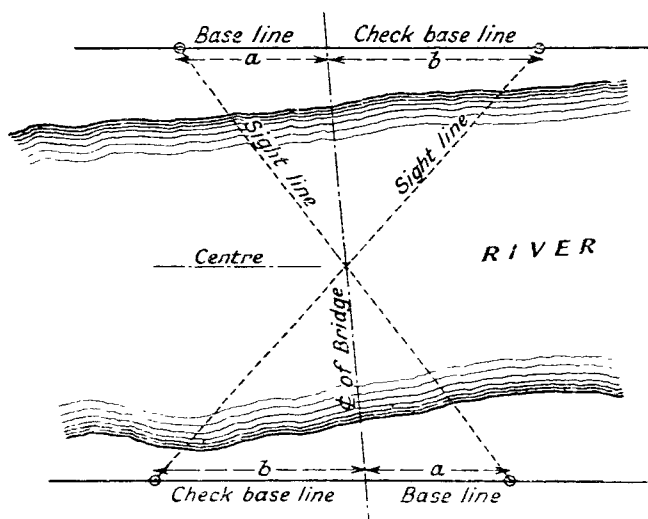


Fig. 139

drawing to be located by *simple sighting*. Fig. 139 shows the centre of the bridge fixed by sighting from a point on one base to another on the base on the other bank but on the other side of the bridge; the other pair of diagonally opposite base lines is used as a check. Similar sighting methods to fix other points on the bridge can easily be devised. If the nature of the ground prevents the base lines being at right angles to the centre line, or is such as to preclude accurate linear measurement, the base lines can be oblique to the centre line of the bridge but parallel to each other.

The procedure and calculations under such conditions, or any other conditions that may occur to the reader, can readily be devised by studying the methods of plane triangulation given in Chapter 3. Geodetic accuracy is of course not required, but the standard methods of accuracy in base line (Chapter 2), angular measurement, and adjustment of triangles should be employed where they are applicable. A study should be made of the typical examples given later in this chapter.

To recapitulate then, there are two aspects to be borne in mind when dealing with construction surveys: (1) setting out the immediate day-to-day work for the contractor and (2) the initial survey required to set out on the site the accurate positions of relative portions of the structure as detailed on the erection drawings. The latter (2) is really a test for the ingenuity of the engineer in applying the principles covered by this treatise. Experience counts a lot, but sound knowledge of basic principles, coupled with 'horse sense', counts more. Nevertheless experience is almost essential on works of any magnitude, as knowledge of the reactions of foremen and workmen to different methods is required; it is not advisable to use new methods, no matter how improved they are, when the men are conservative in their views about them. Terms used in construction have to be understood, and there is needed a general appreciation of the difficulties in, and time needed for, setting out work at the time and place required by the contractor's foreman.

DETAILED SETTING-OUT OVER SMALL AREAS. Once more it is emphasized that every case has its own problems; the examples now offered are more for the purpose of giving to the student with imagination a general idea of the type of work he will have to face on construction than to suggest any rigid method of solving the problems.

Bridge abutments. The centre line of the bridge (Fig. 140) and the peg X, marking the face of the abutment, having been established from the location survey, set up the theodolite at X and establish pegs A just clear of the limit of the construction. The wing walls are usually fixed on the erection drawings by the angle they make with the centre line. Establish peg B (the distance XB can easily be calculated) and over it set up the theodolite and turn off the angles to set pegs C. The lines of, and corners of, the wing walls can now be located at any time from pegs A, B, and C without linear measurements. If B is inaccessible, temporary pegs can be set at the corners by linear measurements; over them set up the theodolite and set duplicate the pegs C on the lines of each wing wall.

Admittedly the above assumes a favourable terrain, and frequently linear measurements from pegs A have to be used to establish the corners; but it should keep before the reader's mind the underlying principle of simplicity which should always be practised for

the actual location of the structure for the contractor. Difficult terrain will demand other methods which anyone with imagination, who has mastered the preceding chapters, can readily devise for himself. It is advisable to duplicate all pegs as far as possible.

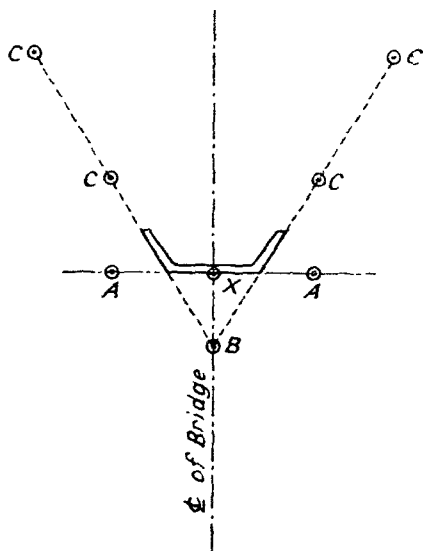


FIG. 140

Short tunnels. The traverse of the location survey will have located the centre line of the track at each end of the tunnel with a fair degree of accuracy. For a short tunnel a theodolite can be aligned in to the true line by trial if the profile of the ground permits. If not, a trial line is extended from one end, the bearing being based on the traverse of the location survey, and the offset to the line at the other end measured. The chainage being known, the correction to the bearing is calculated and the final line run, which should of course check accurately with the centre line of the track at the other end.

CONTROL OF SETTING-OUT BY PRECISE SURVEYS. Precise surveys are required for almost any large engineering works to establish the accurate relation of remote portions of the work before any detailed setting-out can be done. Again it is emphasized that in the following pages the purpose is to illustrate the general principles involved even though to do so it is necessary to descend to details.

For this purpose two cases will be dealt with: tunnels in general, and the setting out of the Sydney Harbour Bridge in particular.

Long tunnels. For long tunnels, where the intervening profile of the ground is such that no such line can be run as described at the end of the last section, triangulation should be employed. It should be carried through by well-conditioned triangles from a base line on the track at one end of the tunnel to another at the other end. Base lines should be measured with all the precautions as given in Chapter 2 (except reduction to sea-level which is unnecessary) and the triangulations should be balanced as in Chapter 6 for plane triangles. The pegs at each end of the tunnel should be thoroughly referenced, not only by linear measurements to near objects but also by angular measurements to distant ones. When extending the line through the tunnel every precaution should be observed to obtain an accurate result.

Except in short tunnels, stations must be established in the tunnel itself. These can be fixed in the floor or the roof. Either has its advantages and disadvantages, the choice depending on the conditions of traffic congestion, amount of water, etc.

For roof stations a horizontal steel angle or beam should be concreted into the walls at the height judged to be suitable for the conditions in the tunnel. This forms a solid base on which to erect the station. The standard self-centring type used on the Ordnance Survey is excellent, but a traversing scale and vernier is of great advantage when a station is to be set from a mean of several runs, the final position being the mean vernier reading. Where a ground station is preferred, it should be set in a box in a mass of concrete below rail-level with a steel cover plate. However, the work is much simplified, and with little loss of accuracy, if the traversing scale with vernier and sight is mounted on a tripod and the final reading on the vernier is transferred to the roof or ground station by plumb line.

When the tunnel is long, intermediate shafts or adits have to be run in order to give more working faces and use must be made of these to check the underground traverse. In the case of adits the procedure does not present any extra difficulties; reasonably long sights can be obtained, and the angles transferred to the main tunnel centre line by the usual methods, all precautions for accuracy being observed. But in the case of shafts there exists the

serious problem of transferring a horizontal line accurately from ground-level down a comparatively narrow shaft to the tunnel and there picking it up accurately. The principle is simple; the application very difficult. Two plumb lines are suspended, their bearing measured above ground, and the same bearing is taken from the plumb lines at the bottom of the shaft, and from it the bearing of the tunnel is set out. Such shafts are sunk just offset from the tunnel to avoid trouble from falling material and to facilitate traffic.

Above ground the theodolite is set by trial on the extended line of the plumb lines and a station established. This should be as close as the focussing of the telescope will permit. No advantage is gained by setting it at a longer distance, as the closer it is the better the plumb lines are observed. The bobs, which are heavy (20–50 lb.), are immersed in liquid. Some engineers prefer a liquid of high viscosity; some prefer the plumb bobs swinging free in air. Whichever is used, the plumb lines must be employed in conjunction with horizontal scales to note the amplitude of the swings. It is best to place the scales in front of the swinging lines and also to read on one edge of the plumb line; this has no effect on the bearing of the lines if they are of the same diameter. The main difficulty is the setting of the theodolite by trial on the same line as the plumb lines. This method has been largely superseded by the *Weisbach method*.

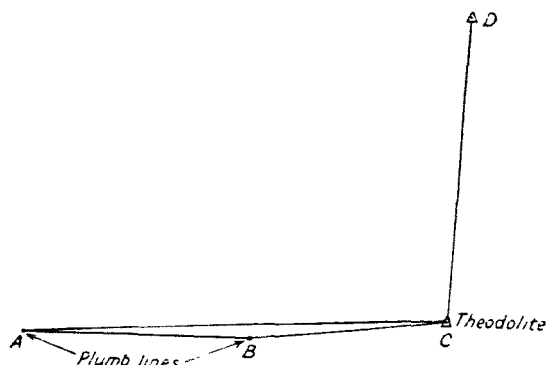


FIG. 141

Weisbach method. Stations C and D are in the tunnel but A and B are in the vertical shaft (Fig. 141). The theodolite is set at C approximately on the line of the plumb lines A and B. The angle BCA and

the lengths of the sides of the triangle are measured, whence the angle BAC is calculated and the bearing of AB obtained. As the relevant angles are very small, they are proportional to their relevant sides, and thus the calculation is very simple. The procedure below ground is exactly the same as above ground.

The procedure of reading is as follows:

The vertical hair of the theodolite follows one side of the swinging plumb line, and the reading (on the horizontal linear scale held just in front of the plumb line) at the end of the swing is observed. Similar observations are carried out at the other end of the swing and the operation repeated many times until a satisfactory mean is obtained. The actual angles (DCB or DCA) read to a whole number on the scale, and are then corrected to the mean value of the readings of the swings. Above ground it is usually possible to read the angles DCB or DCA direct to the appropriate side of the plumb lines, as the amplitude of the swing may here be neglected. The station D below ground should be in the same approximate position as that above ground.

LOCATION OF MAIN PINS OF SYDNEY HARBOUR BRIDGE. Fig. 142 shows the general lay-out of the base lines and the triangulation stations. (It might be mentioned here that since the date of this survey the tendency has been to use shorter base lines which are expanded to an extended base forming one side of a main triangle.)

The erection drawings specified distance of 1,650 ft. between the centres of the main pins, of which there are a pair at each end of the bridge, symmetrical about the centre line; therefore it was possible to erect stations on the intersection of the centre lines of the pins and the bridge, which would not interfere with the progress of the work. These stations consisted of towers built of large concrete slabs 4' 6" square. They are indicated by L and K in Fig. 142 and were set by precise linear measurements from the main triangulation stations C and D.

At stations A, B, C, D, E, F, K, J, and L metal plugs were set in concrete blocks covered with cast iron boxes. At stations G and H the metal plugs were set in solid rock. Each plug was marked with a centre punch and a transverse line. The metal plugs in the towers at L and K were very carefully referenced and these references were always checked before any readings were taken from these two stations in case of any movement of the towers. The angles

were read at 15° intervals which involved 24 readings to complete the circle. The error of the summation of the angles of each triangle was generally less than $\frac{1}{10}$ of a second. The base lines were measured with an invar band, using three different methods, and the results

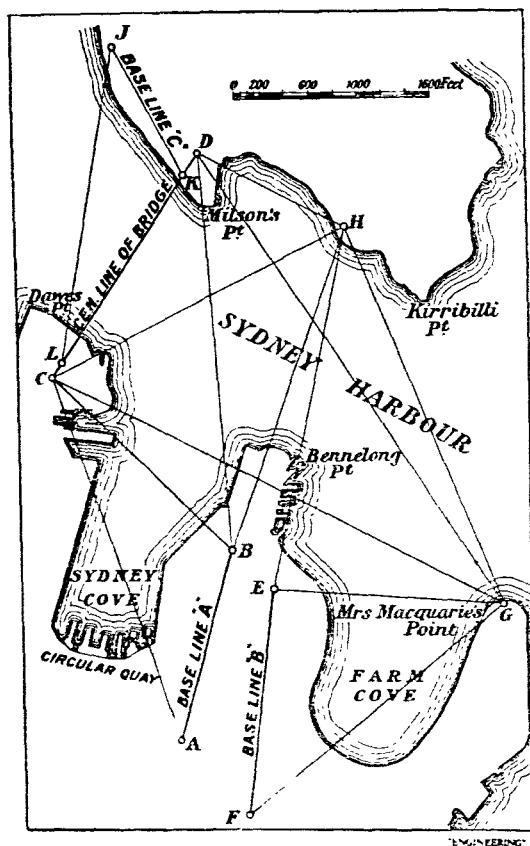


FIG. 142

obtained were as follows: Using base line A the calculated length of CD was 2268.441 ft. Using base line B the calculated length of CD was 2268.453 ft. Using base line C the calculated length of CD was 2268.456 ft. Notice the extreme care taken to obtain results that are beyond any criticism. This is an important point in the initial setting out of large engineering works from the erection drawings.

The foregoing is a *précis* of the initial part of a typical construction survey, e.g. the exact location of different parts of the structure in relation to each other. So far, what is demanded of the engineer is a sound knowledge of the principles of surveying with a good experience of taking the necessary measurements. There follows, however, the method of transferring this accuracy to the actual construction itself, that is to say, the accurate transfer of what was obtained at L and K to the actual pins themselves, and this accuracy must be maintained after the pins are erected. Here, then, is where a high degree of experience is required. This *précis* of the Sydney Harbour Bridge main pin erection will serve to illustrate how much this experience is needed.

Plate XI shows the final assembly of one of the four bearings, each of which weighed 230 tons. The steel stools on which the erection had been carried out had been removed after a system of hydraulic jacks took their place. These hydraulic jacks were so arranged that the assembly could be moved in any direction, and they were used, together with a system of fine adjustment, to manoeuvre the centre line of the main pins into the exact location required, as determined from the stations at L and K.

To appreciate the gear, time, and attention to detail that is required to obtain the desired result, it is worth while reading the description given on p. 559, Vol. 125, of *Engineering*; though the methods on other works might be entirely different, it will give the student an idea of the immense amount of attention to detail that is required to carry the survey to finality on works of such magnitude.

THE 1/1250 ORDNANCE SURVEY

No chapter on surveys for engineering structures would be complete without reference to the 1/1250 Ordnance Survey now being carried out principally in built-up areas. The detail is picked up from a breakdown of the tertiary triangulation of the country to minor trig points such as church steeples, etc., from which traverse stations are established. Precise traverses are then run between these stations to a limiting accuracy of 1/5000 (the actual accuracy is much higher than this). These stations are referenced to points (marked 'rp' on the sheets and called 'revision points'). The National Grid co-ordinates to the nearest centimetre of these points together with a very clear photograph of each can be obtained by application to the Director-General.

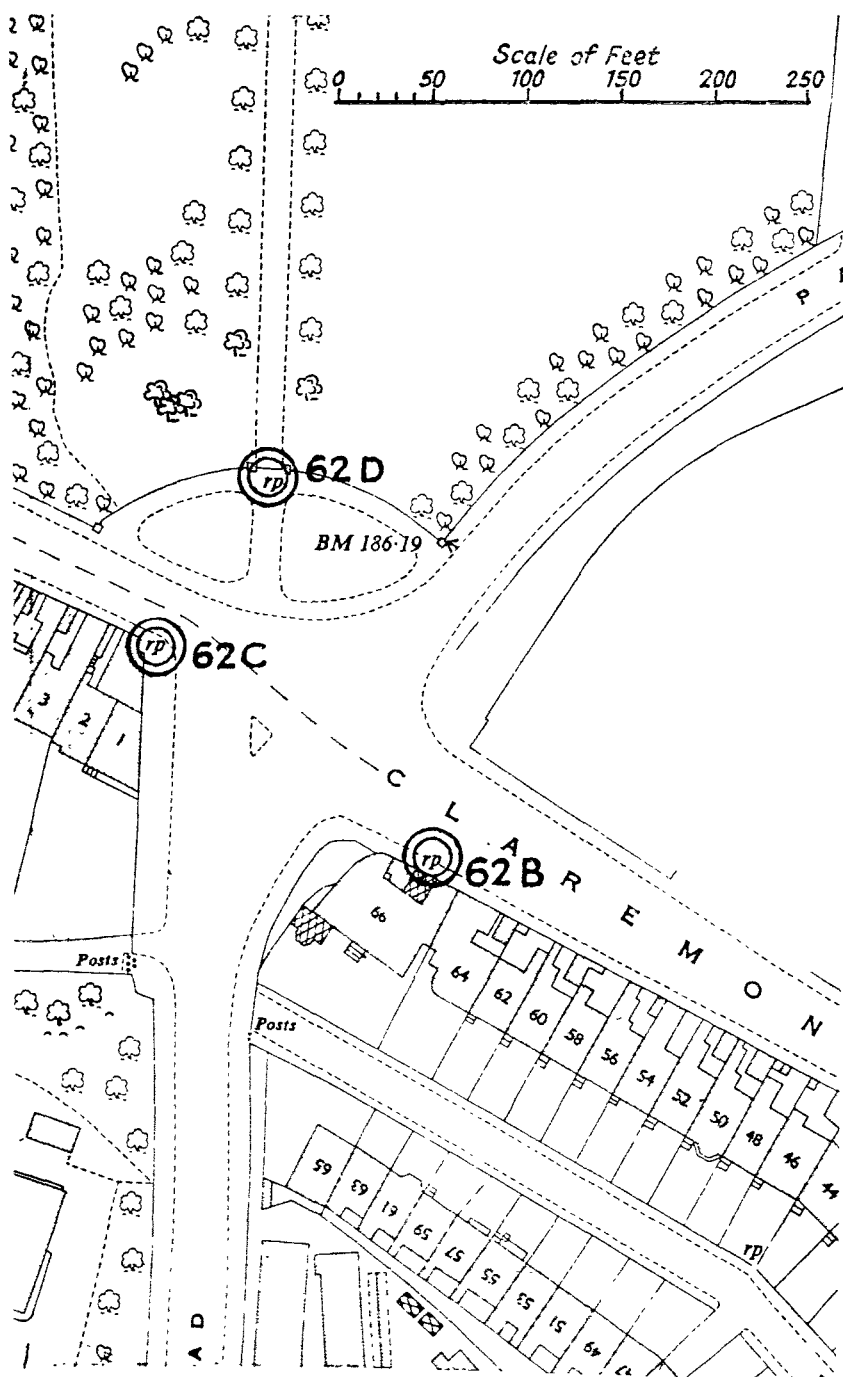


FIG. 143

Any engineer who has had to conduct the survey for large works in an area that is built over at the time of the survey, will readily appreciate the advantage in control that such points can afford. Fig. 143 shows a portion of one of the sheets containing the rp's, and Plate XII shows a typical photograph. It is of interest to add that the 6-inch and 25-inch sheets will be based on this survey.

CHAPTER 13

CADASTRAL SURVEYING

CADASTRAL SURVEYING is surveying carried out for the *Cadastre*, which may be defined, roughly, as the land register, comprising '(a) technical record of the parcellation of the land through any given territory, usually represented on plans of suitable scale with (b) authoritative documentary record, whether of a physical or proprietary nature or of the two combined, usually embodied in appropriate associated registers'.* Cadastral surveying therefore supplies the maps and plans by which parcels of land described in the landbook may be recognized on the ground, and is such an integral part of the land register that in many countries the land registry is administered by the survey department. But, just as the system of land registration differs from country to country, so does the type of surveying necessary to accompany it.

Cadastral surveying differs from topographical surveying in both scale and function, since its main purpose is not map-making but merely the formation of a plan on which property boundaries are accurately plotted which will form an indispensable ancillary of the cadastral record; e.g. the rectangular sheet lines of the topographical map 'which ignore natural boundaries and shear ruthlessly through all land parcels they encounter'* is an extremely inconvenient method of division. Therefore cadastral plans are frequently made of cadastral blocks, which, when possible, are bounded by recognizable natural features and never cut through parcel boundaries. Cadastral surveying also differs from engineering surveying in function. Both are usually large-scale plans, but, whereas an engineering plan may frequently be complete in itself, a cadastral plan must (nearly always) be connected to the main survey control of the country so that the beacons and marks delineating properties may be recovered on the ground.

It can be seen that cadastral survey is not easily defined, and it is not made easier by the fact that the term is sometimes applied

* *Land Registration*. By Sir Ernest Dowson, K.B.E., and V. L. O. Sheppard, C.B.E. (H.M.S.O.)

carelessly to mean any large-scale property survey in which all detail is plotted to scale and nothing is symbolized.

CADASTRAL SURVEYING IN UNITED KINGDOM. In this country 'general boundaries' (i.e. indefinite boundaries) only are required in the registration of title to land; but as these, particularly on agricultural land, are well established and usually follow the physical features of the countryside, all registrations are based on the Ordnance Survey maps and but few disputes about boundaries arise. In consequence, there is no cadastral surveying in England as the term is understood in other countries where, particularly when land is valuable, guaranteed or precise boundaries are required frequently in what may be quite featureless country.

DUTIES OF CADASTRAL SURVEYING. Dowson and Sheppard summarize the duties of cadastral surveying in any territory that is to be brought on the land register as the definition of the parcellation of the land (i) within the appropriate administrative and economic subdivisions of such territory, in a manner which (ii) will enable any parcel to be unambiguously located on the ground at need and, if need be, to the satisfaction of a court, (iii) will ensure that there are no gaps and no suppositious overlaps in such parcellation, and (iv) will embody all changes in its distribution and surface pattern as these occur. These duties, like those of the register, which they complement and serve, are continuous and unending. (For other definitions of the requirements of a cadastral map, see E.S.R., Vol. II, pp. 382 and 496, 'The Cadastral Map', Vol. IV, p. 338, 'Cadastral Mapping', and Vol. X, p. 2, 'Direct Use of Air Photos for Cadastral Purposes in Zanzibar'.)

RANGE OF PROBLEMS

The foregoing is sufficient to indicate that scales and methods adopted must vary greatly according to circumstances, but all have this in common that the fundamental rule of surveying of working from the whole to the part is of paramount importance; and as far as possible all work should be connected to the main triangulation network of the country. Standards of accuracy are frequently determined by the use to which the land is put; e.g. in Egypt a strip of land 2 metres wide will support four rows of cotton plants consequently ± 25 cm. is the accuracy required because no encroachment involving the loss of a row of cotton plants can be

tolerated. In Zanzibar, on the other hand, ± 3 feet could be accepted because the principal crop is clove trees planted 21 feet apart. The same sort of considerations determine the scale; e.g. in Switzerland, the scales laid down for cadastral work range from 1:250 for areas built up or to be used for building, through 1:1000 for well-cultivated closely parcelled land, to 1:5000 for Alpine slopes and pastures of intermediate extent.

The type of work involved may be divided broadly into three groups: (i) the survey of crown land, not yet alienated and largely undeveloped, (ii) the survey of cadastral blocks, fixing beacons and delineating boundaries of land already parcelled and (iii) the survey of individual properties on first alienation from the crown land or when ownership is being transferred. This last type of survey is, in many countries, allotted to private licensed surveyors.

CADASTRAL CONTROL ON UNDEVELOPED LAND. Under the first heading may be discussed the survey of land virtually unsurveyed and the establishment of beacons which will form suitable control for survey of the type (iii) mentioned above. The work involved depends on many factors: the density of the existing survey control (if any), the nature of the topography, and the urgency of the work, which of course is usually governed by the speed at which the country is being developed. In this last connection the opinion has been stated* that it is better for the survey not to be more than one year ahead of settlement. Otherwise beacons may have been destroyed or moved, new roads and villages may have sprung up, all of which will probably necessitate an expensive new survey before any use has been made of the original. A good example of this type of work was given in a paper entitled *Some Aspects of Demarcation for Agricultural Development in Flat Country* to the Conference of Commonwealth Survey Officers by R. C. Wakefield and J. W. Wright in 1951. It describes the cadastral survey control established in the Gezira of the Sudan, the extremely flat area of about 30,000 sq. km. lying between the Blue and White Niles, prior to its development by irrigation (rendered possible by the building of the Sennar Dam).

SURVEY OF CULTIVATED LAND BEING BROUGHT ON TO THE REGISTER. Conditions of work under the second heading, i.e. the survey of

* E.S.R., Vol. IV, No. 28, p. 335, 'Cadastral Air Survey', in which the writer (in 1935) was arguing against the suitability of air survey for this type of work.

land already parcelled, may be just as diverse as those referred to above and methods of work will depend chiefly on the general topography and the density of the existing survey control. As already mentioned, the work may be conveniently divided into cadastral blocks, bounded by natural features, and, when the survey is being drawn, rigid sheet lines which cut through properties must be avoided. The size of these blocks should be such that the plan of them can be drawn as a whole on one single piece of paper preferably of normal map-sheet size. The scale of the plan must be decided by the degree of accuracy with which the survey is being made and, in a lesser degree, by the amount of detail it is desired to show. The degree of accuracy necessary for the survey depends entirely on local conditions such as the value of the land and the use to which it is being put, e.g. building, agriculture, etc., the latter being qualified by the value of the crop being grown.* The scales used in Switzerland are given above.

Boundaries and beacons. It is convenient if the cadastral block boundaries can follow natural features, but, quite apart from this, there must be frequent reference marks which will form the framework from which individual properties within the block may be demarcated. The emplacement and location and indeed the design and construction of these reference marks or beacons is an important function of most cadastral survey departments. The actual design of the beacon used will vary from country to country, but solidity and permanence should be their chief feature. However, the possibility of their being moved by unscrupulous persons must always be borne in mind, and any beacon whose position cannot readily be checked from adjacent beacons should be provided with witness marks preferably hidden. The siting of these cadastral reference beacons requires forethought, for they must be placed (a) where they may be easily preserved, (b) in positions which form a suitable control for the survey of the block as a whole and (c) in such position that parcel boundaries anywhere within the block may be readily and surely fixed from the fewest number of them. As a general principle, it is undesirable that the beacons marking parcel boundaries should be used as reference marks, because these parcel boundaries are constantly changing. (Dowson and

* See E.S.R., Vol. IV, p. 245, for tolerances allowed, and Vol. V, p. 368, for accuracy obtained in the Gold Coast cadastral survey.

Sheppard state that about 5% of land parcels modify their boundaries each year with consequent disturbance of their boundary marks). On the other hand, it is useful to have many subsidiary reference beacons situated in positions which by their nature are unlikely to change, such as the junction of irrigation channels; these will greatly assist the surveys necessitated by future sale, purchase and subdivision of land.

In Zanzibar, where the main crop is clove trees, the interesting innovation of using the trees themselves as beacons was proposed. The proposal was adopted in modified form by using them as witness marks to the parcel boundaries.*

PROPERTY SURVEYS. Under the general heading (iii) is the individual property survey made within the cadastral block and fixed from the reference marks there established. Again the method used will depend more on the nature of the topography and the size of the property being surveyed than anything else; but as far as generalization is possible it will usually be traverse, the measurements being made by theodolite and steel tape either in catenary or on the ground.† Tacheometric methods are seldom used. Occasionally large parcels of land being cultivated for the first time may be demarcated in the first place by compass traverse. Boundary beacons will usually be co-ordinated on the government survey grid.

It will be seen that much of the work of the cadastral surveyor is akin to that of the topographical surveyor, and much is very similar to the engineering surveying problem of setting out work.

Setting out boundaries. The surveyor will usually be responsible for placing and cementing the beacons, and it is desirable that this should be done before the survey, though obviously this cannot be so when exact acreages have to be demarcated. When the boundaries of exact areas are to be established, the work will start in the office and the boundaries will be laid down on the plan of the cadastral block within which it is contained. The co-ordinates of the corners of the property and any places where beacons or boundary marks are to be placed will be calculated, and with this

* E.S.R., Vol IX, p. 9. and Vol. X, p. 3.

† For methods adopted for cadastral traversing in Gold Coast, see E.S.R., Vol. XIII, p. 138.

information the field work for setting out the beacons on the ground and connecting them with the nearest cadastral reference marks can be planned.

When large areas are involved and the ground is suitable for triangulation, it is convenient to make a plane-table survey first and place temporary marks in the approximate positions of the final marks. Adjustments can then be made on the ground before the triangulation proper takes place, should the purchaser's legitimate rights in respect to water supply, etc., not be satisfied. During the plane-table survey fixed unalterable boundaries, particularly if they are curvilinear, should be plotted, in order that the adjusted second approximation positions of the boundary marks may be as precise as possible. The triangulation is then made and areas are calculated and, if necessary, boundary marks are adjusted (though such adjustments should usually be very slight).

When the work is being done by traverse, it will frequently be convenient to make the preliminary survey for establishing the first-approximation positions of the boundary marks by compass traverse. Adjustments can then be made, should they be badly out of position, before the main traverse is run.

The traverse should follow the best route for accurate work and should not necessarily zig-zag to include the boundary marks. It should, however, pass close enough to them to enable their final positions to be established with ease by direct measurement from a main traverse line. To enable this final establishment to be achieved with accuracy a traverse station should be made (even though no alteration in direction is involved) in the near vicinity of the

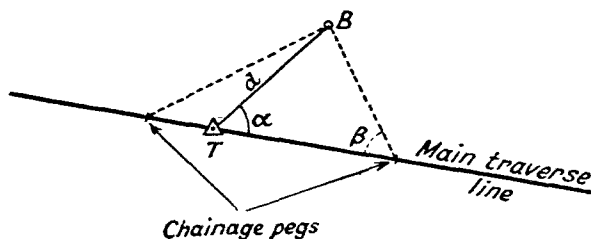


FIG. 144

boundary-mark position. The final position of the boundary mark will then be determined by bearing and distance from this station and also, as a check, by tie-lines from the traverse line.

Thus, in Fig. 144, T is a traverse station, on a main traverse line, established in the vicinity of the expected position of B, the boundary mark.

The correct position of B has been plotted on the cadastral plan and its co-ordinates calculated; the corrected co-ordinates of T have also been determined as part of the main traverse computation. The distance d and the angle α may be computed and B established on the ground, the final position being checked by measuring the angle β , and also the tie-line distances to the nearest chainage pegs. (Similar methods would be adopted, but in the reverse order, should it be required to establish the position of a beacon which was already in existence.)

A caution must be sounded concerning the clearing necessary for running the main traverse which may, unless very precise denials are issued, be mistaken by the proprietors of the land as their boundary.

Keeping of records. It should be unnecessary to tell the surveyor to keep his field book precisely and neatly, but it must be re-emphasized in cadastral work, where the drawn plan, the surveyor's field records and his computations must all be regarded as legal documents available for the court should any dispute arise concerning boundaries. Especially important is it that all points in the survey, whether natural or artificial, temporary or permanent, should be referred to by the same letter, number or name in *all* the documents or plans in which they occur. This possibility of field books and computations being required as legal documents also underlines the necessity of checking all work and recording the check. The notification 'checked' is not enough; the check measurements must be recorded, and check computations must also be available for inspection.

Check taping. In one respect the checking is frequently relaxed, and that is in the check taping of traverse lines. It is usually sufficient for the check taping to be of a much lower order of accuracy than the main taping and to be undertaken chiefly for the prevention or detection of gross errors, since the closure of the traverse, both angular and linear, provides the main check on the accuracy of the work.

Diagrams. A diagram of the individual parcel of land is usually required to be attached to the conveyance or lease for registration,

and this is also part of the surveyor's responsibility. What is shown on these diagrams and how it is depicted will depend largely on the local land laws, but a few things common to all may be mentioned.

Since the diagram is supplied to assist in delineating the property on the ground, the scale must be large enough to enable the property to occupy at least one square inch of paper. (In South Africa the scales to be used for diagrams depend on the area of property being surveyed and are laid down in instructions for surveyors.) A linear scale must be provided, and also both magnetic and grid north must be shown, the former being clearly labelled with the local declination of the compass needle at the time of the survey. All boundary beacons must be shown and also any beacons of the main survey triangulation which fall within the limits of the diagram. Above all, the date of the survey must be clearly indicated.

Other things that are usually shown are the topography, including the contours (or form lines); also the names of the adjoining properties and their proprietors. Areas are nearly always shown and frequently survey data, in tabular form, giving the length (to $\cdot 01$ of a foot) and direction (to nearest $10''$) of each boundary line, and the co-ordinates of the boundary marks.

It should be noted here that, in the past, legal disputes concerning the validity of beacons on the ground versus co-ordinates of the recorded boundary marks have been decided in different countries with different conclusions, and no hard and fast rule can be given on this point.*

RELATED METHODS

CADASTRAL MAPPING FROM AIR PHOTOGRAPHS. It has generally been accepted that cadastral surveying from air photographs is not satisfactory, largely because so much work must be done on the ground in any case that the work provided from the air is scarcely worth the expense involved. However under certain circumstances, such as existed in Zanzibar, valuable cadastral work can be and has been done from the air.

The main crop of the islands of Zanzibar and Pemba is cloves (pp. 368 and 370) and coconuts. The clove trees grow about 40–50 feet high, with foliage extending about 15 feet in diameter; they

* See various articles in E.S.R., notably Vol. I, No. 3, 'Beacon versus Deed-Plan', and Vol. I, No. 5, 'Relations of Beacon and Deed-Plan in South Africa'.

are planted in rows 21-25 feet apart, thus leaving a fairway 6-10 feet wide between the rows. The coconut palms are planted irregularly, but both can be distinguished as individual trees on air photographs taken at a suitable scale (1:5000). Wild undergrowth flourishes under the trees and makes ground-work difficult.

When the problem of making a cadastral survey became imperative, it was reckoned that there were about 50,000 clove holdings in the islands, about 6,000 of them being of less than $\frac{1}{4}$ acre. Under Islamic law the crop belongs to the planter irrespective of the ownership of the land; in consequence the accurate delineation of land boundaries becomes relatively unimportant provided the crop lies on the right side of the boundary. For this reason ± 3 feet was considered adequate accuracy for the cadastral maps (p. 368), and, since individual trees are distinguishable on the air photographs, it has been found possible to plot the parcel boundaries on the photographs themselves, which then become the cadastral record.

It will be seen that the result is a *record* and not a cadastral *map* as understood elsewhere in this chapter but, although 'it is unnecessary to emphasize the fundamental importance of a well designed and precise general framework . . . it may not be so obvious that the primary purpose of cadastral survey may be effectively and economically served by purely localized controls and plans. For this primary purpose is . . . the record (of the parcellation of land) in a manner which will enable the limits of any and every individual parcel to be located readily, authoritatively and unambiguously at need in its full-scale natural dimensions and setting on the ground. . . . Inability to establish a general framework of proper quality in useful time should not prevent the execution of urgently needed cadastral work in any territory'.

The words quoted are by Sir Ernest Dowson in the first of two articles entitled 'Direct Use of Air Photographs for Cadastral Purposes in Zanzibar'*; the second article was by Mr. L. G. Chambers, and the two discuss at length all the problems involved. These problems will not be dealt with in detail because, up to date (1952), the method does not appear to have been adopted elsewhere. The chief defect seems to have been the height distortion at sheet edges, especially acute when it is considered that the tree-tops are 40 feet or so above the ground on which the boundary is to be delineated. This was reduced considerably by substituting a 20"

* E.S.R., Vol. IX, pp. 1-14, and Vol. X, pp. 1-13.

lens and a correspondingly greater flying height for the 8" lens and contact scale of 1/5000 with which the work was started. Another major difficulty has been the gaps or duplication at the edges when joining adjacent photographs to form the mosaic of the cadastral blocks, although these have purposely been kept smaller than normal. This difficulty has been especially acute in Pemba, which has very uneven topography.

COMPUTATION OF AREAS. The cadastral surveyor is frequently confronted with problems concerning areas, either the computation of the area of a property or the laying down on the ground of a given area (which has been referred to on pp. 370 and 371). In addition he is often concerned with the subdivision of areas.

Areas are computed by the usual formulae, but, since a large proportion of properties will be bounded by straight boundaries with the corners co-ordinated, it is usually convenient to compute their areas by these co-ordinates by the so-called *double longitudes* method.

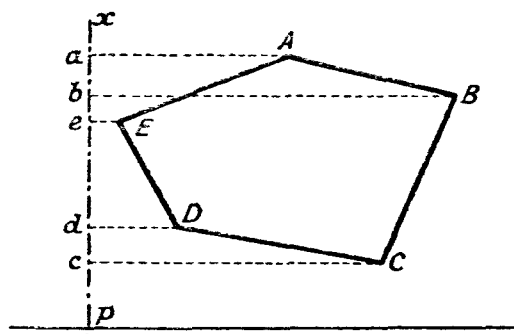


FIG. 145

In Fig. 145 ABCDE is a straight-sided area of which the rectangular co-ordinates of each corner are known. Draw a false meridian of origin, px , through the even hundred of easterly co-ordinates which is just west of the most westerly corner (in this case the corner E). From each corner draw lines in a (co-ordinate) westerly direction to cut this meridian at right angles in a , b , c , d and e , which points will obviously have the same northerly co-ordinates as A, B, C, D and E.

The method is to compute the areas of the trapeziums $ABba$ and $BCcb$, and subtract from their sum the areas of the trapeziums $CDdc$, $DEed$ and $EAae$.

The areas are those enclosed between each boundary line and its projection on to the meridian of false origin and are:

$$\frac{aA + bB}{2} \times ab$$

$$\frac{bB + cC}{2} \times bc$$

etc.

i.e. for each line the area is half the sum of the (false) eastings of its end points multiplied by their difference of latitude.

For ease in computation it is simpler to omit the fraction $\frac{1}{2}$ and to use the sum of the (false) eastings, i.e. double their mean longitude. This means that double areas will be obtained in each case, and so at the end of the computation the algebraic sum of all the trapezium areas must be divided by two.

The d. lats. are labelled N and S as usual and the resulting areas are also considered N and S. Consequently the area required is $\frac{1}{2}(\Sigma N \text{ areas} \sim \Sigma S \text{ areas})$.

Example:

E	N	False eastings	Double long.	d. lat.	Double areas	
A 92 864.4	82 001.4	564.4			N	S
B 93 392.3	81 836.6	1 092.3	1 656.7	164.8 S		273 024.2
C 93 128.5	81 572.6	828.5	1 920.8	264.0 S		507 091.2
D 92 507.5	81 664.0	207.5	1 036.0	91.4 N	94 690.4	
E 92 385.6	81 772.4	85.6	293.1	108.4 N	31 772.0	
A 92 864.4	82 001.4	564.4	650.0	229.0 N	14 885.0	
					141 347.4	780 115.4
					2 638 768.0	
					Area	319 384 sq. units

A minor point which arises in countries situated at a high elevation is that boundary lengths and areas in land transactions must

correspond to horizontal measurements on the ground, i.e. the true lengths and areas, and not those which have been reduced to datum of mean sea level.

Subdivision of areas. Several cases arise of which, perhaps, the most common are (a) subdivision of an area into given parts by a straight line drawn from a given point on the boundary and (b) subdivision of an area into given parts by a line drawn on a given bearing.

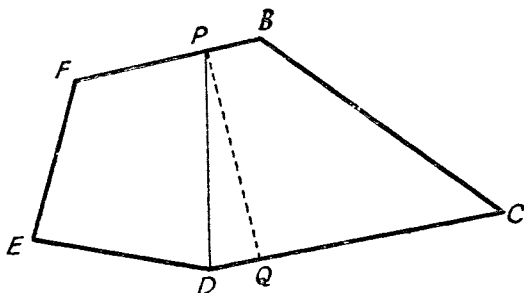


FIG. 146

(a) In Fig. 146 let BCDEFB be a plot of land, surrounded by straight boundaries, the co-ordinates of the corners being known. It is required to subdivide the plot into two equal parts by a straight line from a point, P, on the boundary.

The area of BCDEFB should be calculated (call this $2A$), and the figure should be plotted on a fairly large scale. By inspection it can be seen that a line joining P to D will nearly achieve the object sought but that the true dividing line PQ will be somewhere to the east of this.

If not already known, compute the co-ordinates of P and the direction of the line DC, then

(i) compute the length and direction of DP and hence the angle PDQ,

(ii) calculate the area of PDEF and call this A' .

Now, when Q is located correctly the area of PQDEF will = A .
 \therefore the area of the triangle PQD = $A - A'$

It also = $\frac{1}{2} DP \cdot DQ \sin PDQ$

In the above all the quantities are known except DQ. Therefore

(iii) calculate $DQ = \frac{2(A - A')}{DP} \operatorname{cosec} PDQ$, from which the

co-ordinates of Q may be found.

(b) In Fig. 147 let BCDEFGHB be a plot surrounded by straight boundaries, the co-ordinates of the corners being known, which is to be divided into two equal parts by a straight line drawn parallel to BH. As before, calculate the area of the whole figure, $2A$,

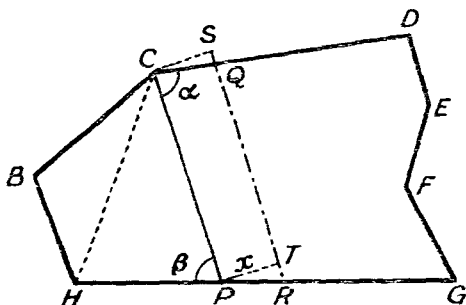


FIG. 147

and plot on a fairly large scale. Draw in the line CP [see note 1 below] which will divide the figure, though obviously, in this case, with the western part smaller than the eastern. Let QR be the true dividing line required. The bearing of CP has been given, and if not already known the bearings of CD and HG should be calculated; and hence the angles $DCP = \alpha$ and $CPH = \beta$.

(i) From their co-ordinates calculate the bearing and distance CH and hence the angle HCP.

(ii) Solve the triangle CHP to find the lengths CP and HP and hence the co-ordinates of P.

(iii) By the co-ordinates of H, B, C and P calculate the area of $HBCP = A'$.

Then the area of the figure $CQRP = A - A'$

From C and P drop perpendiculars CS and PT on to QR, produced, and let

$$CS = PT = x$$

then

$$QS = x \cot \alpha \text{ and } RT = x \cot \beta$$

and area $CQRP = \text{area rectangle CSTP} + \text{area RTP} - \text{area QSC}$

$$= x \cdot CP + \frac{x}{2}(x \cot \beta) - \frac{x}{2}(x \cot \alpha)$$

So

(iv) [See note 2 below] solve the equation

$$x \cdot CP + \frac{x^2}{2}(\cot \beta - \cot \alpha) = A - A'$$

to obtain x and hence

$$CQ = x \operatorname{cosec} \alpha \text{ and } PR = x \operatorname{cosec} \beta$$

Notes. (1) If BD was a straight boundary a point C' would be selected on it close to its true position Q' and its co-ordinates calculated.

(2) If the figure has been drawn on a fairly large scale the first-approximation line can be drawn close to QR, thus making the areas of the triangles QSC and RTP so small that their areas may be estimated with fair accuracy from the drawing; suppose their estimated areas = Z and Y then for step (iv) can be substituted the 'trial and error' equation

$$x \cdot CP = (A - A') + Y - Z$$

by which x can be found and so CQ and PR.

(A third-approximation line may be needed if the second approximation has been misjudged.)

CHAPTER 14

THE REPRODUCTION OF MAPS AND PLANS

THE PROCESSES described in this chapter concern only the making of the finished map after all the field work has been done. In the case of plans a little more detail is given about field work, as the method is new and in many phases is closely allied to reproduction processes.

CLASSIFICATION OF MAPS BY SCALES

Maps are often classified by scales under the following headings:

- (a) Small scales.
- (b) Medium scales.
- (c) Large scales.

SMALL SCALES. These vary between about 1 inch to one mile ($1/63,360$) and smaller, and include the Atlas types of map such as one in one million or about 16 miles to one inch, one in four million, and similar very small scales.

MEDIUM SCALES. Those scales lying between the 1 inch to one mile and the $1/25,000$. The British 6 inches to one mile ($1/10,560$) is very well known and the $1/25,000$ or about $2\frac{1}{2}$ inches to one mile, although a comparatively new map in Great Britain, is becoming increasingly popular.

LARGE SCALES AND PLANS.

- (a) Anything larger than the above.
- (b) *Plans*. Very large scale maps are usually called 'plans'. By far the best known in England at present (1954) is the $1/2,500$ or the 25-inch plan, but in the future it may well become the $1/1,250$ or 50-inch plan.

DRAWING AND REPRODUCTION OF PLANS IN GREAT BRITAIN

THE $1/1,250$ PLAN. The re-survey of the larger towns in Great Britain, which is now (1954) being made on the $1/1,250$ scale, is being carried out on thin ($\frac{1}{16}$ ") aluminium plates 20 cm. \times 20 cm.

surfaced with white enamel which can be butt-jointed one to the other. These plates then have plotted on them in the office the following information:

(a) all known fixed points such as traverse and reference points;

(b) all chain survey detail.

Such information is obtained from work already done on the ground when fixing the traverse points and carrying out the chain survey. Four of these plates are then fitted together in a special carrier and used in the field in the same way as the paper was on the old plotting or plane-tableing boards. These small plates are very accurately made and are carefully tested to see that the edges fit exactly. Work is then completed on them in the field, using known survey methods as described elsewhere in this book.

A standard border for the whole series of 1/1,250 plans is designed and is printed down in black on a metal plate which has been sprayed with white enamel. The four field plates are now photographed at the same scale, and from the negative a *blue* key (see p. 384) is produced to fit exactly inside the standard border mentioned above. The detail is then *fair* drawn in black using the blue key as a guide. Each finished plate covers a ground area of 500 metres square, and is reproduced in one colour only, black, although an impression of grey for houses and buildings is conveyed by breaking up black into a series of tiny dots called a *stipple*.

The whole aim of the new Ordnance Survey technique described above is to get away from using a distorting medium both for the first document and the subsequent fair drawing.

THE 1/2,500 PLAN. The new 1/2,500 National Plans of Great Britain are being produced in the three following ways:

(i) by direct reduction of the 1/1,250 plans of the larger towns;

(ii) by resurvey at 1/2,500 scale;

(iii) by overhauling the old 1/2,500 plans of the County series and converting them to National Grid sheet lines.

Direct reduction of the 1/1,250 plans. As soon as the four components of the 1/1,250 have been completed (16 plates in all), the production of the 1/2,500 plan can be put in hand. First a black impression of the standard 1/2,500 border and footnotes at the 1/1,250 scale

is printed down in black on a large white enamel plate. The grid is not printed down at this stage, but a small cross is provided at the centre of the plan as a registration cross.

This enamel is then re-coated with a light-sensitive solution, and an image in black of each of the component $1/1,250$ plans is printed down in exact register from the final glass negatives of the $1/1,250$ plans.

The enamel is then passed to the areas section. In this section the first stage is the division of the plan into a series of convenient *parcels*, bounded as far as possible by natural features. Different types of land are measured separately, and no parcel is allowed to fall in two parishes.

When the plan has been subdivided into parcels, the acreage of each parcel is measured twice independently. The acreages are then adjusted to give a total area for the plan allowing for the scale factor. Final figures are rounded off to two decimal places of an acre. In the drawing section all house numbers and small names which would be unreadable as a result of the reduction in scale by one half are erased.

Some new names and all the area parcel numbers and acreages are added by being stuck on to the enamel. This is done by printing letterpress type matter on to a thin clear film. The film is then made up into a sandwich, having a pressure-sensitive adhesive on the reverse side which is in turn protected by a backing paper. For use each name is cut out with a very sharp knife and the film with its adhesive is lifted away from the backing paper, placed into position on the original drawing and pressed down. As the film is transparent, it does not obscure any map detail underneath should it happen to fall on it slightly.

The enamel is now passed to the studio and photographed in a special fixed focus reduction camera and a final negative at $1/2,500$ scale is produced. Later the house stipple is made as for the $1/1,250$.

Re-survey at the $1/2,500$ scale. Certain $1/2,500$ plans are also re-surveyed and butt-joint plates are used in an almost identical manner to that described for the $1/1,250$.

Overhauling existing $1/2,500$ County series plans. There is also the problem of the overhauling of certain existing $1/2,500$ plans. Two separate operations are involved in this work:

(a) The conversion of the old County sheets to National Grid sheet lines.

(b) The actual revision of the sheets in the field.

The operation in (a) above is carried out by taking prints in black on thin *Astrafol* (see p. 384) from the existing negatives of the County series plans. These are then very carefully gridded in blue from adjusted grid values of the four corners, an adjustment being made to allow for bow in the edges of the old plans and to ensure that the new trig points are correctly plotted in relation to both grid and detail. The *Astrafol* prints are then cut up and carefully stuck down on a master grid in black printed on plate glass. The fitting is done by registration of the drawn blue grid on the *Astrafols* over the black printed grid on the glass.

From these compilations, red prints on this *Astrafol* are made by 'contact' and sent to the field for field revision. It should be noted that the grid on these documents is derived directly from the standard master grid and is *not* drawn. After the return of the corrected red *Astrafols* from the field the subsequent drawing and reproduction are similar to those already described for the 1/1,250.

DRAWING AND REPRODUCTION OF MAPS

Almost all modern maps are now reproduced by a process known as photo-lithography. The word 'lithography' is derived directly from the Greek words *lithos*, a stone, and *graphein*, to write. Slabs of a special stone 2 to 3 inches thick and of all sizes were originally—and even now are occasionally—used in the process. Thin metal plates, usually of zinc or aluminium, have now taken the place of stones. An even newer process using bi-metal plates is being used in the trade where very long runs are required.

The basic difference between lithographic and other methods of printing is that, whereas in letterpress (i.e. newspapers, books, etc.) and line blocks and half-tone blocks (i.e. pictures in the daily Press), the matter to be printed is in relief, and in the various forms of engraving the work is in intaglio (i.e. hollows below the surface), lithography, on the other hand, relies on the mutually repellent properties of water and grease. The printing plate has a flat surface on which the design is in greasy ink, and, so long as the plate is kept damp, the inking rollers transmit ink only to the design which is then transferred to the paper. In the new bi-metal process the plate is a polished, stainless steel plate, which is coppered on the

ink-carrying surface. Copper is ink-attracting while stainless steel is ink-repellent in a remarkable way.

In the case of coloured maps, a litho plate must be made for each colour. To produce a good clear plate, two things are essential. All lines and other drawn work of the map must be sharp with clean edges against a clear background, and secondly every plate must fit or register most exactly with all the other plates so that the different colours fall into their correct places.

The sequence of operations to achieve these two objects is as follows: (1) Fair drawing, (2) photography, (3) photo-writing, (4) printing down, (5) proving, (6) machine printing.

FAIR DRAWING. To ensure that the final printing plates are of the correct size it is absolutely essential that the fair drawings should be most accurately made on a material which will not distort through atmospheric or other conditions. Paper is no longer used for map-drawings, and maps are now mostly made on white-enamelled zinc or aluminium plates, or, more recently, on plastic sheets. The Americans have, for some time, been using plastic sheets to draw on, and now, after much research, a British firm produces a transparent plastic sheet which is almost non-distorting. It is called *Astrafoil*. Before the War a German firm produced a similar material called Astralon. This is again available in England through the agents for the German firm. The advantages of a light and plastic material over white-enamelled metal plates are obvious both in working and storage. The technique of drawing on these hard and almost non-absorbent surfaces is quite different from drawing on paper, but suitable inks and tools are now available and draughtsmen, both young and old, soon get used to the new materials.

The key to which the draughtsman works is supplied by sensitizing the plastic *Astrafoil*, or the white-enamelled zinc plate, with a ferro-prussiate emulsion and printing down from a glass negative of the field plate in the case of plans or of the compilation drawing in the case of the map. These keys show on the white plate in light blue (a non-photographic colour), one for each colour of the map. These keys are prepared on the scale selected for drawing, which is usually larger than the finished map, sometimes as much as twice as large. This is in order to secure greater fineness of line and lettering after the subsequent reduction. The work to appear in any

one colour is drawn by hand in black ink (Indian or special plastic) on one of the blue keys.

The names, which most draughtsmen find difficult to draw, may be produced in a number of ways such as by hand-drawing, typing with a hand-typing machine, letterpress printing by machine on sheets of thin plastic film with a sticky backing such as 'Duraseal', 'Claritex', etc., from which they are cut out and stuck on the enamel in the appropriate places, or by a phototype-setting machine method as used in the War Office and other cartographical departments.

The actual specification of the map, together with the sizes of every conceivable symbol, should be decided before mass drawing of a new map-series commences. Such a specification may take time to draw up, but it does ensure uniformity of production and prevents constant queries in the drawing office. On completion of all the drawings, they are carefully examined, as corrections at this stage are far easier to do than they are later on. Even with such a specification much depends on the individual draughtsman's skill and experience of cartography.

PHOTOGRAPHY. The set of fair drawings is now sent to the camera studio, where the procedure is similar to that of any ordinary camera. The camera itself is, however, very different. It is a large high-precision instrument with an expensive lens corrected for distortion within the very small tolerance allowed. Its movements allow for reduction or enlargement of scale up to two or three times. In the most modern cameras the settings for scale and focus are done automatically by means of electric motors once the camera has been calibrated. A camera of this type known as a *monotype* is now in use at the War Office, and will take all sizes of negative up to 40" × 40".

The drawings or other material to be photographed are then set up in turn on the copy-board of the camera. They are illuminated by arc lights at the side and in front of the copy-board. The image is focussed on to a ground glass screen in the camera and either enlarged, reduced or photographed with no change in scale as desired. The ground glass screen is then replaced by a sensitized glass plate and the necessary exposure made. This sensitized glass plate can be either *wet plate* as in the Ordnance Survey or *dry plate*. Both methods have their merits, and some departments use one

method and some the other. In the 'wet-plate' method the negative is made fresh for each exposure, but in the 'dry plate' method the plate is pre-coated by a photographic firm and sold in a light-proof box ready for use. In this case the sensitive emulsion only remains good for a certain period of time. Each drawing, except that for the detail plate, carries four corner 'registration' marks corresponding exactly with the four corners of the map border as on the detail drawing, and all the other colours are made to fit by measurements made on the ground glass focussing screen in the camera. It must be noticed that on each drawing all the items which are to appear in another colour have not been inked over by the draughtsmen, and so remain in the ferro-prussiate blue colour. As previously mentioned, the photographic emulsion is non-sensitive to this blue colour, and it does not appear on the negative. The plates are developed, fixed, dried and varnished and then sent to the negative retouching section.

PHOTO WRITING. Every negative made, no matter how carefully, will have some blemishes on it due to causes such as poor originals, dust in the dark room, etc. The negative retouchers, or *photo writers* as they are sometimes called, make the background of the negatives opaque by applying a liquid called 'Photopake' with a small painting brush. They also make the detail sharp and clear where necessary by using a sharp needle-pointed tool for actually cutting through the photographic emulsion on the glass plate. Dimensions of each negative must also be again carefully checked.

Great skill and patience are required in this section to ensure that as perfect a negative as possible goes on to the printing down department. If light comes through the glass of the negative where it is not required, or does not come through where it is required, then the resultant printing plate made from the negative will not be perfect. Small corrections and amendments can also be carried out on the negative at this stage.

Bromide prints, which are really the same as the prints of the ordinary camera, are a quick method of producing what is on the negative. They are often used for the multi-production of footnote arrangements on maps. Large bromide prints are awkward to handle as they cockle badly and change scale appreciably with weather changes.

PRINTING DOWN. The touched-up negatives are now ready for the transfer of the image from them to the printing plates. This is called the albumen process. Positives can be used as well as negatives, and either can be on film or glass. The plates are normally of zinc and the usual sizes are as follows:

<i>Trade name</i>	<i>Plate size</i>
Demy	$24\frac{1}{2}" \times 24"$
Double demy	$37\frac{1}{2}" \times 30"$
Quad crown	$45" \times 38"$

All printing plates are smooth when received, and are subsequently given a slightly matt surface in order to retain some moisture when actually being printed. The machine for doing the graining has a shallow tray in which the plates are securely clamped and then covered all over with glass marbles. These marbles are then mixed with some fine sharp sand and water. A rotary shaking movement of the tray gives a fine *grain* or matt surface to the plate after about half-an-hour. This process of graining can, if not done properly, spoil the final map. It is largely a matter of skill and experience in the machine-operator to know exactly how much water to use, what size of marbles, the actual texture of the sand and for how long the tray should be oscillated.

The plate is now evenly coated with a light-sensitive solution in a machine called a whirler. The action of the whirler is that the plate is held by clamps on a revolving turn-table. The solution (egg albumen and ammonium bichromate) is poured in the centre of the revolving plate, so that an even coating is ensured, and warm air from a heater and fan passing over the plate dries it quickly.

After coating, the dried plate, which is not so sensitive to light as an ordinary photographic film, is placed in contact with the glass negative, and both are put into a frame either face up or face down. The frame has a thick clear glass top with a rubber blanket underneath. The air is exhausted between the rubber and glass, and thus the glass negative and coated metal plate are held in very close contact.

The frame and contents are then exposed to arc lamps, the light penetrating to the coated plate through the clear portions of the negative and hardening and making insoluble that portion so exposed. The plate is removed from the frame and covered with a special greasy ink and developed under water. The water dissolves the parts of the coating which have not been acted upon by light;

that is, all except the lines of the work. Printing ink will now only adhere to the image to be printed, provided, of course, that the rest of the plate is kept damp.

A printing plate can also be made by using a positive which can be either glass, film, tracing paper or any other translucent material. The standard practices for preparing zinc plates from a positive are:

- (a) The gum reversal process.
- (b) The glue or Vandyke process.

The following stages occur in all positive reversal processes:

- (a) A print from a positive is made on a metal plate previously coated with a bichromated colloid whose characteristic is to be rendered insoluble and hard when exposed to light.
- (b) The unexposed colloid is washed away to leave, as it were, a stencil and the metal bare in the 'work' areas.
- (c) The bared metal is filled in with a greasy ink.
- (d) The colloidal stencil is removed and the plate is prepared in the usual way for printing.

Hence can be seen the difference between an albumen plate when the ink rests on top of the light-hardened coating on top of the plate and a gum reversal where the ink is rubbed into the actual clean surface of the metal plate.

PROVING. After an albumen or gum reversal plate has been made, it goes to the proving department whose main work is:

- (1) Pulling proofs in full colours of maps, etc.
- (2) Duplicating plates.
- (3) Adding stipples, layer rulings, etc.
- (4) Making offsets.
- (5) Generally preparing plates for the printing machine.

Pulling proofs. The zinc plate when received from the printing down department contains the detail in greasy ink, and the entire plate is covered with a solution of gum arabic and is non-sensitive.

The first process in the proving department is the *etching* of the plate. The term etching is misleading, as a true etch consists in removing portions of the plate surface so that the work is left standing up in relief. This is not done in lithographic printing. The so-called etching process aims at converting the metallic surface of the zinc into certain salts of zinc, which—unlike the metallic zinc

—have no affinity for grease. Etching also has the effect of assisting the surface to retain moisture for a longer period.

After the etching the plate is washed out with an asphaltum solution sometimes called *doctor* in the trade. This solution dissolves any of the ink which has been decomposed by the etch solution, but it does not remove the fatty acids which have combined with the zinc. It also leaves a solid deposit of tallow, beeswax and asphaltum on the lines where the ink has been. This deposit strengthens the work and forms a deposit for the printer's ink to adhere to. After washing out, the plate is washed with water, and whilst still damp is rolled up in printer's ink. This is done by several applications of a hand nap roller, which is charged with ink and passed backwards and forwards over the plate which is kept damp all the time. When the plate has received a full charge of ink, surplus moisture is dried off and it is then ready for pulling the first proof on paper.

All modern proving presses are now what is known as *offset*. An impression from the plate is offset on to a rubber *blanket* on a cylinder which rotates first over the plate and then over a sheet of paper.

If the map is in one colour only, this completes the proof, but more often the finished map is required in many colours. The outline plate is then washed out and removed, the offset impression on the rubber blanket is cleaned off and a new plate inserted in the machine. This is then proved in whatever colour required in exactly the same way as described above for the black or outline plate. By measurements from the corner registration marks on the plate the prover ensures a near fit of each colour. Some slight final adjustments will be necessary on the *grip* (back and front) and *lay* (sideways) stops to ensure exact register before proving can commence.

As a rough guide the number of pulls required from the original outline plate, to allow for setting sheets to get correct register, should be the number of final pulls required plus three times the number of colours; i.e. 12 proofs in six colours require 30 pulls of the outline. The finished proof is examined, marked with corrections and returned to the negative retouchers, who carry out the corrections on the negatives. New printing plates are then made and the proof plates cancelled. The proof plates could themselves be corrected by a litho-draughtsman, and often are on service maps where

speed is essential. It is, however, bad practice, as, if the work on the plate breaks down in printing, it is essential to go back to the negative and make a new plate. If this negative has not been corrected errors are inevitable, and much time is lost.

Duplicating. Sometimes when a plate is of poor quality or in a bad condition it can be rolled up and one good impression taken on to the blanket. Here large areas of bad marks, etc., can be removed and an impression of the old plate taken from the blanket on to a clean new sensitized zinc plate.

Stipples and layer rulings. It is often necessary to lay down stipples, i.e. a series of dots over an area to portray sand, etc., on the finished map.

The area where the stipple is not to fall on the printing plate is gummed out. An impression in litho ink from a master plate of stipple has already been taken on the blanket, and is now put down all over the printing plate. The dots only fall on the ungummed portion of the plate, i.e. where they are wanted, and the rest are washed away with the gum.

Layers or hypsometric tints, as they are often called on maps, are a method of showing relief by colours. It is possible to get several apparent different steps in colour by using only one colour. It is a sort of optical illusion produced by single lines, double lines, crossed lines at varying spaces all of one colour. All cartographers use this artifice in some form to give the effect of height. The areas of the different rulings are, of course, governed by the contour lines on the map, and to produce layers one method is to gum out up to a given set contour line and then from master-line plates, as described above, to transfer these lines on to the printing plates.

Some layer systems are very complicated and difficult to reproduce, but the fundamental principle is as described above, i.e. to produce an orderly build-up of colour to give the effect of height using the minimum number of printing colours. The contour lines need not be on the map (cf. the new Oxford Atlas). The difficulty with any large series of maps is to devise a layer system which applies all over the area. For example, a bump in Holland is important, but is difficult to show on any overall layer system which also covers the Alps.

Set-offs. It sometimes happens that a non-printing duplicate of a printing plate is required as a guide for work to be done by a litho-draughtsman on the actual plate itself. Such duplicates are known as *set-offs* and may be either:

(a) Permanent; i.e. they remain on the plate during printing but are non-receptive to printing ink, and are therefore not reproduced.

(b) Semi-permanent; i.e. they remain on the plate for a sufficient time to enable the printing portion of the new plate to be prepared.

PRINTING. A printing machine or press is in effect a fast-running proving machine in that it produces at great speed a large number of paper copies from a printing plate. There are many different types and sizes of printing machines, each with their advantages and disadvantages. They can be divided into the following types:

(a) Rotary offset

(b) Flatbed

The paper can be hand-fed or automatically fed into either of the above types of machine.

The only type described here is the rotary offset with automatic feed which is by far the most common in map-printing to-day. Rotary machines may be single-colour, two-colour or more. Their sizes are usually called by such names as demy, royal, double demy, quad crown, etc. In a two-colour machine, the most usual type in map-printing, there are two complete *decks* each consisting of one set of cylinders and damping and inking rollers (see Fig. 148). The first set prints one colour and the second another, and the paper then passes automatically from one to the other as shown in the figure. More time is required at the outset on these machines to obtain register, but in the end, especially on long runs, this is more than made up, and the difficulty of register due to paper distortion is easier.

In the commercial trade there are three-, four- and even five-colour machines for printing the enormous runs required for cigarette cards, food labels and the like, but they are not usual in map-printing. A good map is judged not only on its accuracy but on its printing, and this means the register of its colours. Where a road, a railway and a river run parallel in a deep valley, bad printing may even show the railway on the wrong side of the river, and

the red colour band of the road falls into the river. The blue water must also go under the bridges.

These things are very important for maps for the Armed Forces. Even in single-colour black printing of large-scale plans, it is possible to see bad printing in the form of thick or conversely grey lines and for the whole to lack the dense black extremely sharp lines one expects in Ordnance Survey plans of Great Britain.

In these modern machines the plate is bent to a curve and wrapped round a large cylinder. This cylinder is in contact with another, round which has been stretched a rubber blanket. Rotation of these cylinders causes the plate to come first in contact with the damping rollers, then with the inking rollers, and finally with the blanket on to which the image is transferred. Single sheets of paper from a stack are fed in, and are made to flow along feed-boards by means of suckers, one set to lift the sheets off the stack, another to separate the sheets and a third to move them forward.

As the paper flows along the feed-boards, and just before it reaches the first cylinder, it is positioned against grip and side lay stops similar to those already described for the proving machine. Grippers on the paper or impression cylinder catch the paper fed from the feed-boards and keep the paper in correct position until it just touches the blanket carrying the impression on the printing cylinder. After the impression has been made, grippers on the take-off chains remove the paper from the impression cylinder and carry it to the delivery stack. Fig. 148 shows the machine in use in the War Office Cartographical Department. There are many other types and sizes of printing machines which feed the paper and deliver the printed map in a variety of different ways, but the principle is the same in all.

The plate is clamped on to the plate cylinder as shown in Fig. 148. On revolving the plate is damped by the damping rollers and then inked by the last of the inking rollers. The large number of inking rollers as shown are necessary in order to make sure that the ink reaches the last roller spread evenly along its length. This even inking is ensured by some of the rollers having a lateral movement. After being inked the plate just touches the printing cylinder, leaving an image on the rubber blanket for off-setting on to the paper.

This process goes on continuously at the rate of about 2500-3000 copies an hour until all the copies of one colour are printed.

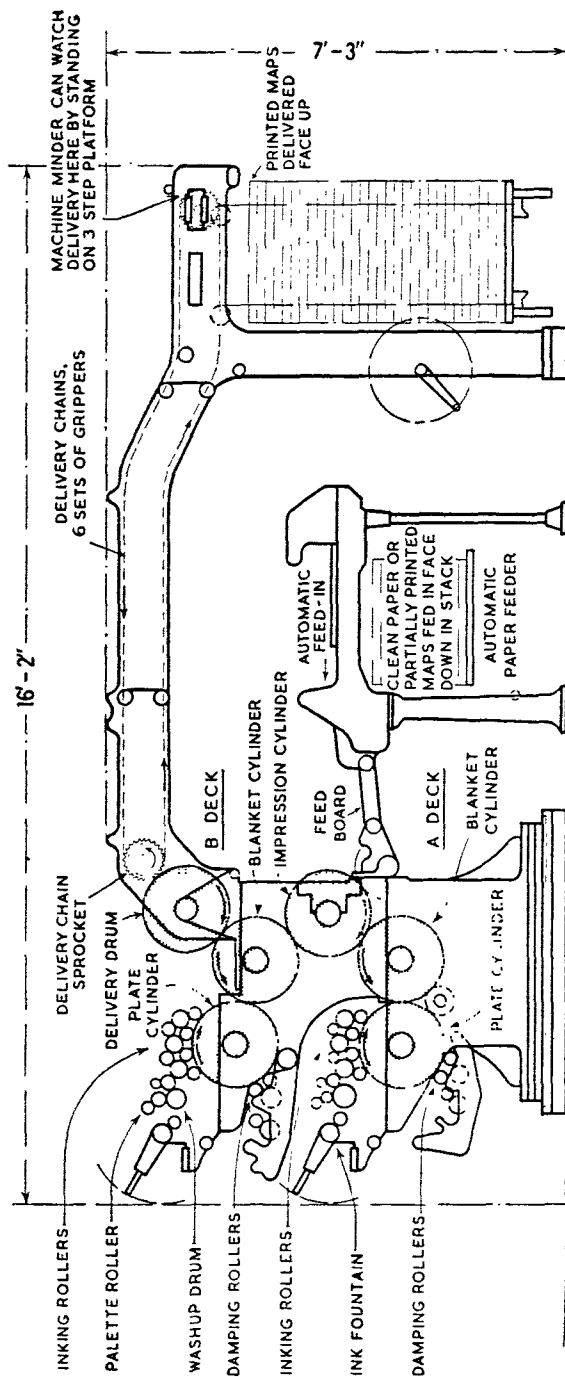


FIG. 143. DIAGRAM OF DOUBLE DEMY TWO-COLOUR OFFSET MACHINE WITH AUTOMATIC FEEDER

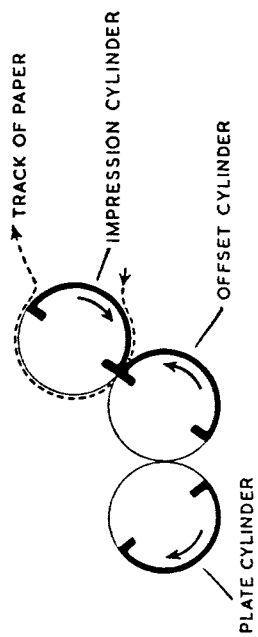


Fig. 149. DIAGRAM OF SINGLE-COLOUR
OFFSET MACHINE

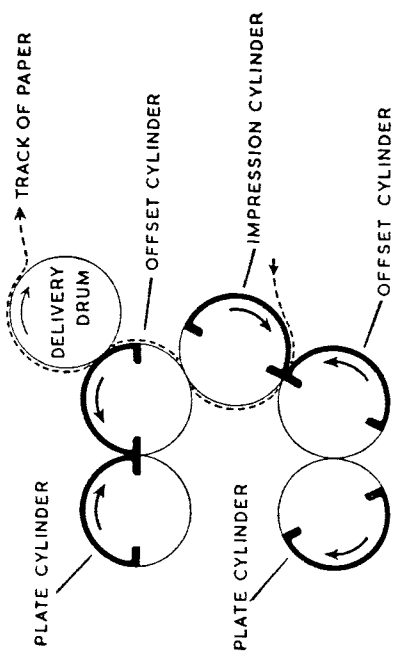


Fig. 150. DIAGRAM OF TWO-COLOUR
OFFSET MACHINE

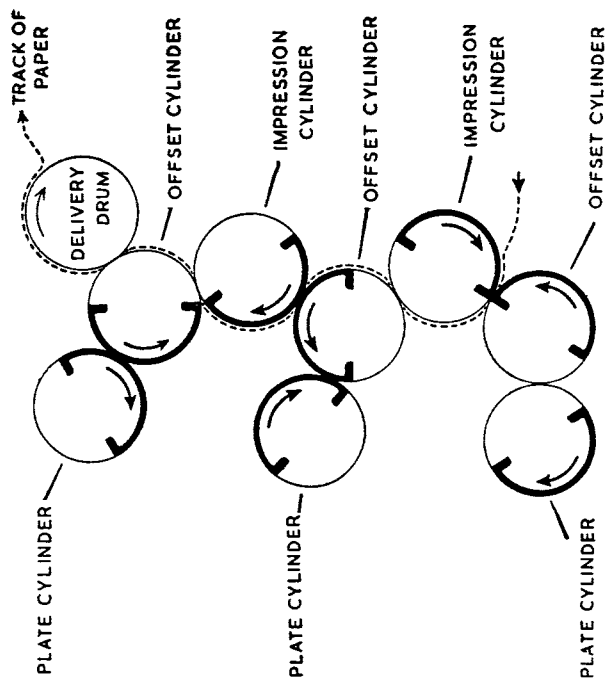


Fig. 151. DIAGRAM OF THREE-COLOUR
OFFSET MACHINE

The colour on the ink rollers is then washed off with turpentine and charged with the next colour. The second plate is fitted and adjusted to fit the first by measurements on the plate clamp and on the cylinder by the machine-minder. This double process on a single-colour machine takes about $1\frac{1}{2}$ –2 hours, and on a two-colour machine about 2–3 hours, and so on until all colours are printed. The precision of the machines is such that successive sheets of paper are fed into exactly the same position on the cylinder within a few thousandths of an inch, but the work does need constant watching by the printer as a number of things can, and often do, go wrong. As copies come off the machine at about 50 a minute, a great many can be spoilt if the printer is not fully alert all the time.

The chief worry in printing is due to variation in paper-size, caused by weather changes, in the course of a run. Air-conditioning as to both temperature and humidity is a cure but a very expensive one, and as yet few printing installations have such a luxury. There are other expedients to overcome this, and without some trouble being taken to condition the paper to the actual conditions in the printing shop, serious troubles over colour registration will occur.

(Line drawings showing the working of a single-, double-, and three-colour offset printing machine are given in Figs. 149, 150 and 151 on page 394 opposite.)

CHAPTER 15

FIGURE OF THE EARTH

THAT THE earth is more or less spherical in shape has been known for many centuries. The radius being large, about 3956 miles, no account need be taken of the curvature of the surface, in surveys of moderate areas, up to say 75 square miles; it has no measurable effect on the angles or sides of a survey polygon, which may accordingly be computed by the formulae of plane trigonometry.

But when the survey covers larger areas, the curvature must be taken into account; a triangle on the surface may be treated as a spherical triangle on a sphere of radius equal to the earth's mean radius, as an approximation. For a still closer approximation, the radius of the sphere for computation is taken as that appropriate to the area concerned, as deduced from accepted values of the earth's form and dimensions.

The actual physical land surface of the earth is of course highly irregular; the elevations and depressions are however negligibly small compared with the earth's radius, and in any case they can be allowed for, and a determination of the general curvature made by astronomical methods. The curvature in any direction is the rate of deflection of the horizon plane per unit distance along that direction. In principle, this is not difficult to determine, star altitudes above the horizons of two places of known distance apart would give it; but in practice it involves elaborate work of a high order of precision to give the average curvature between the two points with the necessary accuracy.

ELLIPSOID OF REFERENCE

Measurements of meridian arcs, with a view to the determination of the meridian radius of curvature, have been made in many parts of the world. These, together with accurate triangulation of large areas, show that the figure of the earth approximates closely to an ellipsoid of revolution, and that the axis of rotation, the polar axis, is the minor axis of the elliptical meridian section; the figure is accordingly an oblate spheroid, or at least approximates very

closely to it. The meridian curvature is greatest at the equator, and diminishes towards the poles, where it is least.

It is therefore rational to select the spheroid which appears to conform most closely to the figure as determined by triangulation over large areas, and to refer the positions of points to that spheroid by normals to it. This is called the 'ellipsoid of reference'. The determinations of its axes which have been made in different countries at various times do not agree exactly, e.g. the ellipsoid which appears best to fit say N. America may not agree with determinations made in Europe or elsewhere. Consequently, several ellipsoids of reference are in use or have been used. They do not differ greatly.

Roughly, the semi-axis major is 3963 miles, and the semi-axis minor 3950 miles. The departure from sphericity is small, it would not be detected by the eye in a scale model. In the usual notation for an ellipse, where a is the semi-axis major and b the semi-axis minor, $\frac{a-b}{a}$ is called the ellipticity or compression c ; for the earth

it is about $\frac{1}{297}$. The eccentricity e of an ellipse is $\frac{\sqrt{a^2 - b^2}}{a}$ which

for the earth is about 0.082.

It can be shown mathematically that an ellipsoid of revolution is a figure of equilibrium for a rotating fluid mass subject to no forces but its own gravitational attraction and the centrifugal force due to its rotation; without rotation its form would be a sphere, the attraction of which on all external matter is directed towards its centre, the external spherical surface, and also all concentric spherical surfaces would be equipotential surfaces and the lines of force as indicated by the plumb line would be radial.

But when there is rotation these conditions are altered by the centrifugal force, and the form would become ellipsoidal with the axis of rotation as the minor axis. The concentric equipotential surfaces are ellipsoids, and the resultant lines of force, being normal to the equipotential surfaces, are curved. The effect is quite negligible, except in the most refined geodetic operations. It is therefore not surprising to find by measurement that the shape of the earth, which has passed through fluid stages in its evolution, approximates to an ellipsoid of revolution.

From the many geodetic surveys which have been made on the land surface of the earth over a long period of years, a number of slightly differing 'Figures of the Earth' have been deduced for the dimensions of the ellipsoid of reference. These can be found tabulated in works dealing with geodesy; in most cases the dimensions are quoted in metres.

In the *Text Book of Topographical and Geographical Surveying* by Close and Winterbotham, 1925, the dimensions of Clarke's Figure, 1858, are given, and they are followed by a number of geodetic tables for facilitating computation on that figure. The dimensions are

$$a : 20,926,348 \text{ feet}$$

$$b : 20,855,233 \text{ ,,}$$

$$c : \frac{a-b}{a} = \frac{1}{294.26}$$

Corresponding tables for other ellipsoids of reference are also available in other publications, e.g. in *Notes on the Minor Trigonometrical Work of the Ordnance Survey*, published by the O.S. in 1933, the dimensions of Airy's Figure, of 1831, are given, also followed by a number of useful tables. The dimensions of Hayford's Figure, 1910, adopted by the International Union of Geodesy and called the Madrid 1924 figure, may also be quoted here, they are:

$$a : 6,378,388 \text{ metres}$$

$$b : 6,356,909 \text{ ,,}$$

$$c : \frac{1}{297}$$

The departure of the Figure of the Earth from sphericity has to be taken into account in geodetic work and to some extent also in work of a lower order of precision. An obvious effect is on the length of an arc of given angular measure, say θ'' . On a sphere of radius R this length is $R\theta'' \sin 1''$; for a short arc on a spheroid it is $\theta'' \sin 1'' \times$ the radius in the latitude and azimuth of the line, $M\theta'' \sin 1''$ for an arc of the meridian and $N\theta'' \sin 1''$ for a line perpendicular to the meridian. Geodetic tables include one which gives the length of $1''$ of arc along the meridian from the equator to 60° , or in some tables as far as the pole.

THE GEOID. The actual physical ocean surface, when cleared of disturbances and supposed still, is of necessity an equipotential surface, i.e. a level surface, with the plumb line everywhere normal

to it. If this quiescent ocean surface be imagined to be continued by narrow canals intersecting the land surface throughout the continents, the complete surface so defined would be level and everywhere perpendicular to the plumb line. This is the *geoid*. Because of irregularities in the disposition and density of the matter forming the earth, the geoid does not exactly coincide with the ellipsoid of reference but the latter is a surface amenable to mathematical treatment, and accordingly suitable as a reference surface.

For many purposes in surveying, as also in navigation, where great accuracy is not called for, the sphere is a near enough approximation. In fact, even in work of a higher order of accuracy, a sphere is quite good enough, but in such cases the radius of the sphere is taken as that which best fits the ellipsoid at the centre of gravity of the area concerned. The geometrical properties of the sphere and spheroid must accordingly be understood by the surveyor; the more important of these will be considered.

At any point, the tangent plane and the normal to the ellipsoid are taken as being the true horizon and the true vertical at the point, any small departures therefrom, shown by astronomical observations for latitude, longitude and azimuth, being regarded as due to local deflection of the plumb line, called 'station error', which is the angle between the plumb line and the normal to the ellipsoid of reference.

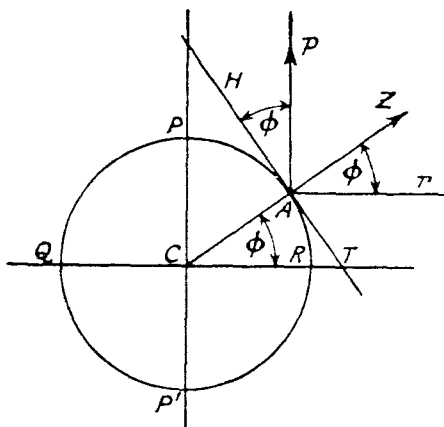


FIG. 152

LATITUDE AND LONGITUDE. The position of a point on a spheroid is defined by its latitude and longitude, just as on a sphere. The

longitude is the angle between its meridian plane and the plane of the prime meridian; but in the case of latitude, owing to the fact that the normal to the figure is not directed towards the centre of the meridian section in the ellipsoid as it is in the sphere, it becomes necessary to distinguish between two kinds of latitude, according to whether it is defined by the direction of the normal or of the radius.

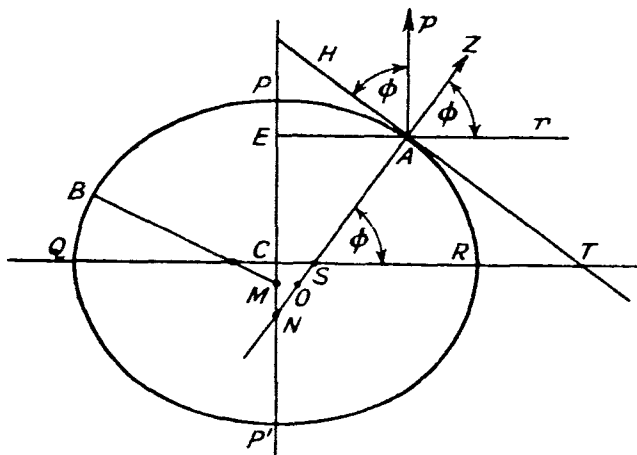


FIG. 153

Figs. 152 and 153 show (a) a meridian section of a sphere
(b) a meridian section of a spheroid.

In each case, the tangent plane at a point A is represented by HAT, PP' is the axis and QR the equator. AZ is the normal, Ar a plane parallel to QR, and Aφ parallel to PP'.

On the sphere, the latitude ϕ is any one of the three equal angles,

I $H\hat{A}\phi$, the altitude of the celestial pole;

II $Z\hat{A}r$, the inclination of the vertical to the plane of the equator;

III ACR , the angle subtended at the centre C by the arc of meridian from A to the equator.

On the spheroid, ZA produced meets QR in S and PP' in N. The angles (I) $H\hat{A}\phi$, (II) $Z\hat{A}r$ and (III) ASR are equal; the latitude ϕ measured by these is called the *ellipsoidal latitude* or the *geodetic latitude*.

ACR, the angle subtended at C, is called the *geocentric latitude*, it is less than ASR by the angle CAS, which is the *reduction of the latitude*. It can be shown that the reduction r is given approximately by

$$r \text{ in radians} = c \sin 2\phi$$

where $c = \frac{a-b}{a}$ (called the compression or ellipticity)

a = equatorial semi-diameter

b = polar semi-diameter

When $\phi = 45^\circ$, r has its maximum value of about $11'5$.

The nearer A is to the equator, the nearer is N to C, the centre of the ellipse; e.g. the normal at B in a lower latitude than A meets the polar axis in M.

On the ideal ellipsoidal rotating earth, the tangent plane HAT and the normal ZAN are respectively the horizon and the vertical, Z being the zenith of A. The actual direction of the vertical, the plumb line, may deviate by some seconds of arc from the normal to the ellipsoid of reference—this is the station error already mentioned.

The latitude and longitude of any station as determined by astronomical observations, and any azimuths measured there are of course based on the actual direction of the plumb line; the geographical co-ordinates so determined are the *astronomical latitude and longitude*, and they may both differ somewhat from the geodetic or ellipsoidal values, as the deviation of the plumb line may be in any direction. Azimuths are also slightly affected by any local deviation of the plumb line, as they are referred to a horizon which is tilted to the ellipsoidal horizon. When latitude and longitude are referred to without any qualification the geodetic co-ordinates are meant. They are those shown on maps. It is often convenient to regard the latitude as the *declination of the zenith*.

CURVATURE OF EARTH'S SURFACE. The curvature of the earth's surface and its reciprocal, the radius of curvature, enter into many survey computations. On a sphere, where the curvature of the normal section is the same at all points and in all directions, there is no difficulty in computing the lengths of arcs, the spherical excess of a triangle, etc.; the data must of course include the radius of the sphere. But on a spheroid, account has to be taken of the

fact that the curvature varies from place to place, and also with the direction or azimuth of the line concerned.

All plane sections, whether normal or oblique, are either ellipses or circles perpendicular to the axis of the spheroid. There are two theorems, applicable to curved surfaces in general, which are of use in treating of the curvature on the ellipsoidal earth. They are as follows:

Euler's theorem. There are at every point of a curved surface two normal sections, in one of which the curvature is a maximum, in the other a minimum; and these sections are at right angles to each other.

Further, calling the radius of maximum curvature M and that of minimum curvature N , the radius of curvature R in a direction inclined at α in azimuth to the direction of maximum curvature is given by

$$\frac{1}{R} = \frac{\cos^2 \alpha}{M} + \frac{\sin^2 \alpha}{N}$$

The curvature referred to is in each case that of a normal section at the point.

Meunier's theorem. The radius of curvature of an oblique section of a surface is equal to that of the normal section through the same tangent line multiplied by the cosine of the angle between the two sections.

At a point on a spheroid the curvature of the sections made by planes containing the normal at the point varies according to the direction or azimuth of the section. It is evident that the maximum curvature is that of the meridian section. It follows from Euler's theorem that the minimum curvature is that of the prime vertical section, which is the section made by a plane containing the normal and perpendicular to the plane of the meridian.

The symbols in general use are:

a : the semi-axis major, i.e. the equatorial radius CQ

b : the semi-axis minor, i.e. the polar radius CP

c : the compression or ellipticity, which is $\frac{a-b}{a}$

e : the eccentricity, which is $\frac{\sqrt{a^2 - b^2}}{a}$

ϕ : the geodetic latitude

M, R_M or ρ : the meridian radius of curvature

N or ν : the prime vertical radius of curvature

The radius M of the meridian curvature at a point is found from the equation to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the general formula

$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

For the ellipse this can be reduced to

$$M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}} = \frac{b^2}{a(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}}$$

which expresses M as a function of the latitude ϕ .

From the general equation to the ellipse it can also be derived that

$$x = \frac{a \cos \phi}{\sqrt{1 - e^2 \sin^2 \phi}}, y = \frac{a \sin \phi (1 - e^2)}{\sqrt{1 - e^2 \sin^2 \phi}}$$

The centre of curvature of the meridian lies on the normal AN (Fig. 153); it will be found to be at a point O between S and N . The radius N of the prime vertical curvature is easily derived by Meunier's theorem, as follows: the parallel of latitude through A may be regarded as an oblique section obtained by rotating the PV section at A about a tangent at A perpendicular to the plane of the meridian, through an angle ϕ . The radius of curvature of the parallel of latitude, which is AE , is accordingly equal to the radius of curvature of the PV section multiplied by $\cos \phi$. The radius of curvature N of the PV section is therefore $AE \sec \phi$, which is AN , the normal, and N on the minor axis is the centre of curvature.

Further

$$N = \frac{AE}{\cos \phi} = \frac{x}{\cos \phi} = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

The radii of curvature are therefore

$$M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}} = \frac{b^2}{a(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}}$$

and

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

At the equator, where $\phi = 0$, the formulae give

$$M = \frac{a(1 - e^2)}{1} = \frac{b^2}{a}$$

and $N = a$, the major axis,

which is self-evident, as a is the radius of the equatorial section.

At the poles, where $\phi = 90^\circ$,

$$M = N = \frac{a}{\sqrt{1 - e^2}} = \frac{a}{b \div a} = \frac{a^2}{b}$$

These values $\frac{a^2}{b}$ and $\frac{b^2}{a}$ are respectively the greatest and least radii of curvature of normal sections.

As already mentioned, under Euler's theorem, the radius of curvature R in an azimuth α is given by

$$\frac{1}{R} = \frac{\cos^2 \alpha}{M} + \frac{\sin^2 \alpha}{N}$$

It can also be shown that the mean radius of curvature at a point is \sqrt{MN} ; this is taken as the radius of the sphere which most nearly approximates to the ellipsoid at the point.

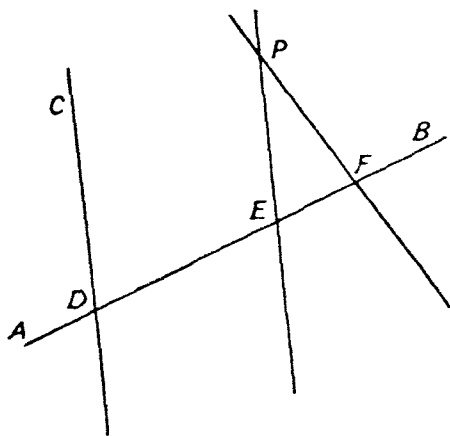


FIG. 154

CONVERGENCE OF MERIDIANS. On a plane surface if a straight line AB (Fig. 154) meets two parallel straight lines CD and PE , the angles CDB and PEB are equal, but if it meets two lines PE and PF

converging to P, the angles PEB and PFB are not equal; the difference $PFB - PEB$ gives the convergence of PE and PF, viz. the angle EPF.

The line on a sphere which corresponds to the straight line on a plane is a great circle, being the shortest or direct line joining two

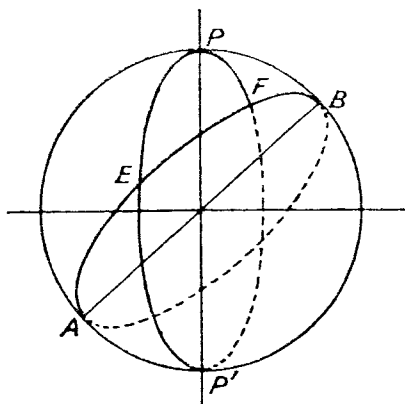


FIG. 155

points. PEP' and PFP' (Fig. 155) represent two meridians intersected at E and F respectively by an oblique great circle AB. The angles PEB and PFB are in general unequal; but the difference between them, $PEB \sim PFB$, is not equal to the dihedral angle EPF, which is the difference in longitude between the two meridians PE and PF. The difference $PEB \sim PFB$ is however called the *convergence* between E and F; it is a function of the latitudes of E and F and of the difference in longitude EPF, and can be readily computed in any given case.

The great circles of the equator and the meridians keep a constant azimuth, viz 90° and 00° respectively, but the azimuth or true bearing of any oblique great circle such as AEFB varies continuously from point to point, being due E and W at the two points of its nearest approach to the north and south poles respectively.

If the latitudes and longitudes of two points E and F are known, the convergence c between them, being the change of azimuth of the great circle AEFB between E and F, is

$$PFB - PEB = (180^\circ - PFE - PEB)$$

$$\text{i.e.} \quad c = 180^\circ - (PFE + PEB)$$

The value of $(PFE + PEF)$ can be computed by Napier's formula

$$\tan \frac{1}{2} (PFE + PEF) = \frac{\cos \frac{1}{2} (PE - PF)}{\cos \frac{1}{2} (PE + PF)} \cdot \cot \frac{1}{2} EPF$$

The resulting value of $PFE + PEF$ is then subtracted from 180° to get the convergence c , as $c = 180 - (PFE + PEF)$

$$\begin{aligned} \text{Alternatively, thus: } \frac{1}{2}c &= 90 - \frac{1}{2} (PFE + PEF) \\ \cot \frac{1}{2}c &= \tan \frac{1}{2} (PFE + PEF) \\ &= \frac{\cos \frac{1}{2} (PE - PF)}{\cos \frac{1}{2} (PE + PF)} \cdot \cot \frac{1}{2} EPF \end{aligned}$$

which gives $\frac{1}{2}c$ directly.

Now $\frac{1}{2} (PE - PF) = \frac{1}{2}$ (difference of polar distances)
and $\frac{1}{2} (PE + PF) =$ mean polar distance

Inverting the equation just above gives

$$\begin{aligned} \tan \frac{1}{2}c &= \frac{\cos (\text{mean polar distance})}{\cos \frac{1}{2} (\text{diff. of latitude})} \cdot \tan \frac{1}{2} (\text{diff. of longitude}) \\ &= \frac{\sin (\text{mean latitude})}{\cos \frac{1}{2} (\text{diff. of latitude})} \cdot \tan \frac{1}{2} (\text{diff. of longitude}) \end{aligned}$$

This will give the convergence between two points of known latitude and longitude however far apart, on a spherical earth.

In surveying other than geodetic the points will not usually be many miles apart and an approximation may be used, viz. that

$$\begin{aligned} \cos \frac{1}{2} (\text{diff. of latitude}) &= 1 \\ \text{and } \tan \frac{1}{2}c &= \frac{1}{2}c \text{ in radians} \\ \text{and } \tan \frac{1}{2} (\text{diff. of longitude}) &= \frac{1}{2} (\text{diff. of longitude}) \text{ in radians} \end{aligned}$$

The formula then gives

$$c = \text{change of longitude} \times \sin (\text{mean latitude})$$

Further, as $c = 180^\circ - (PFE + PEF)$

$$\text{and } \angle PFE + \angle PEF + \angle EPF = 180^\circ + \epsilon$$

where ϵ is the spherical excess of the triangle EPF

$$PFE + PEF = 180 + \epsilon - EPF$$

$$\begin{aligned} \text{and } c &= EPF - \epsilon \\ &= \Delta L - \epsilon \end{aligned}$$

where ΔL is the difference in longitude; and the convergence, which would be ΔL on a flat earth as in Fig. 154 with meridians

radiating from a pole, falls short of ΔL by the spherical excess of the triangle EPF, i.e. by $\frac{A}{R^2 \sin 1''}$, where A is the area of the tri-

angle EPF and $c = \Delta L - \frac{A}{R^2 \sin 1''}$

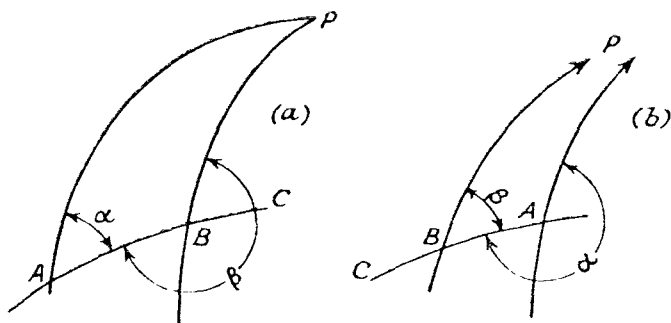


FIG. 156

If a survey be regarded as starting from a point A, Fig. 156(a), from which a sight is taken to B, the azimuth of B from A may be called the forward azimuth; it is the angle marked α in Fig. 156(a), reckoned clockwise from AP the direction of true north at A. The azimuth of A from B is the back or reverse azimuth, it is reckoned clockwise from BP, the direction of true north at B, and it is marked β in Fig. 156(a). The convergence c is $\beta - 180^\circ - \alpha$.

In Fig. 156(b) the forward and reverse azimuths are again marked α and β respectively and the convergence is

$$c = \beta + 180^\circ - \alpha$$

In general, it may be expressed as $\beta \pm 180^\circ - \alpha$ in either the north or south hemisphere and for any direction of the line AB. If there were no convergence, the forward and back azimuths would differ by 180° .

Any survey of an area, or any traverse, must depart in an east-west direction (i.e. in longitude) to some extent from the meridian on which it starts. The effect of convergence may be appreciable when the departure is as little as a mile or so. The convergence is in fact, as will presently be shown, directly proportional to the departure multiplied by the tangent of the latitude; in latitude $\pm 30^\circ$ it is about $30''$ for one mile of departure.

If for instance starting from A (Fig. 157) the azimuth of AB is found astronomically to be 60° , and at B an angle of 275° is measured clockwise from BA to BC, the *bearing* of the line BC is $60^\circ + 275^\circ - 180^\circ$, i.e. 155° . This 'bearing' is however not the *true bearing* (which is identical with azimuth), because it is referred to the meridian of A and not to the meridian of B, and those two meridians are not parallel. The bearing of BC (155°) will differ from its azimuth or true bearing by the amount of the convergence c between A and B, where $c = \text{difference of longitude} \times \sin(\text{mean latitude})$.

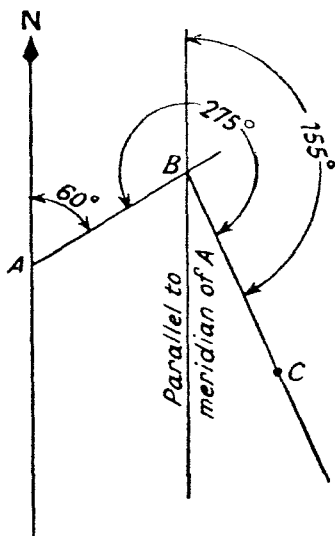


FIG. 157

The bearings of any other lines deduced from the angles turned off will also be affected by the convergence and must be corrected if their azimuths are wanted. There are several other formulae by which the convergence may be found, according to which quantities are known.

The convergence may be found from the 'departure', i.e. the distance east or west of the initial meridian.

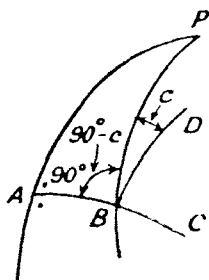


FIG. 158

Let PA (Fig. 158) be the initial meridian of a survey starting at A, and let AB be a great circle set out at A at right angles to PA. The distance $AB = s$ is the departure, and AB is the prime vertical section at A. A line BD set out from B at right angles to AB would be part of a great circle coinciding at B with a small circle parallel to AP, and if the figure were plotted by rectangular co-ordinates BD would be parallel to AP. The meridian at B is BP and the convergence c at B is the angle $PBD = PBC - PAB$.

In the right-angled triangle PAB,

$PAB = 90^\circ$, $PBA = 90^\circ - c$ and AB is the departure
Four consecutive parts are PA, 90° , AB, $(90^\circ - c)$.

The four-part formula gives

$$\cos AB \cos 90^\circ = \sin AB \cdot \cot PA - \sin 90^\circ \cdot \cot (90^\circ - c)$$

$$\therefore 0 = \sin AB \tan \phi_A - \tan c$$

$$\therefore \tan c = \sin AB \tan \phi_A$$

As AB is likely to be in units of length, say feet, and as the quantities c and AB will be small this equation becomes

$$c \text{ (in radians)} = AB \text{ (in radians)} \times \tan \phi_A$$

$$\text{and } c \text{ (in radians)} = \frac{s}{\text{radius of earth}} \times \tan \phi_A$$

where s is the departure in feet

$$\text{and } c'' \text{ in secs of arc} = \frac{s}{R \sin 1''} \times \tan \phi_A$$

where R is the earth's radius, in feet.

More properly, as the radius of the prime vertical section is N

$$c'' = \frac{s}{N \sin 1''} \times \tan \phi_A$$

The same result may also be obtained as follows. In Fig. 159 the parallel of latitude through A is a small circle AB of radius $AC = R \cos \phi$, where ϕ is the latitude and R the earth's radius. A length $AB = s$ along the parallel subtends at C an angle $\frac{s}{AC}$ radians, and this is the change of longitude ΔL between A and B.

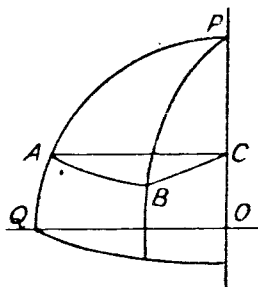


FIG. 159

$$\therefore \Delta L'' = \frac{s}{R \sin 1'' \cos \phi}$$

The convergence for a change of longitude ΔL was shown to be

$$c = \sin (\text{mean latitude}) \times \text{change of longitude}$$

The convergence c between A and B is accordingly

$$\frac{s \sin \phi}{R \sin 1'' \cos \phi}$$

$$\text{i.e. } c'' = \frac{s \tan \phi}{R \sin 1''} \text{ as before}$$

The great circle distance AB is of course slightly less than the distance along the parallel; this is negligible for short distances.

Also A and B will usually differ in latitude; in this case the formula becomes

$$c'' = \frac{s \tan (\text{mean latitude})}{R \sin 1''}$$

It is to be understood that the formulae given are not exact, but only approximations near enough except for geodetic work, for which more precise formulae are in use.

Example: From A in latitude $54^{\circ} 50'$ a line AB is set out at right angles to the meridian of A, and is extended to B at a distance of 25 miles from A (Fig. 158). It is part of a great circle ABC and if produced far enough it would meet the equator at Q the pole of the meridian of A.

Find the azimuth at B of the line BC, and also the change of latitude and longitude from A to B.

The formula for convergence is

$$c'' = \frac{s \tan \phi}{R \sin 1''}$$

where R is the earth's radius, which may be taken approximately as 3958 miles.

If geodetic tables are available a more accurate result will be given by

$$c = \frac{s \tan \phi}{N \sin 1''}$$

as AB is a prime vertical section.

$$s = 25 \times 5280 \text{ feet} = 132,000$$

$$\phi = 54^{\circ} 50'$$

$$\log s$$

$$5.1205739$$

For the change of longitude ΔL

$$\log \tan \phi$$

$$0.1520873$$

$$\log \frac{1}{N \sin 1''}$$

$$3.9927448$$

$$\Delta L = \frac{s \sec \phi}{N \sin 1''}$$

$$\log c''$$

$$3.2654060$$

$$c = 1842''.5$$

$$= 30' 42''.5$$

	logs
s	5.1205739
$\sec \phi$	0.2396101

$$\text{The arc AB is } \frac{s}{N \sin 1''}$$

$\frac{1}{N \sin 1''}$	3.9927448
------------------------	-----------

of which the logarithm is 3.1133187

$$\text{and arc AB} = 1298''.1$$

$$= 21' 38''.1$$

$\log \Delta L$	3.3529288
-----------------	-----------

ΔL	= 2253''.9
------------	------------

$$= 37' 33''.9$$

As the convergence c is change of longitude $\times \sin$ (mid lat.) the figures may be checked thus

$$\log \Delta L \quad 3.3529288$$

$$\log \sin \phi \quad 1.9124772$$

$$\log c \quad 3.2654060$$

$$c = 1842''.5$$

$$= 30' 42''.5$$

It should be noted that the latitude is not constant from A to B, but decreases from A, slowly at first; the change in 25 miles may be neglected for the present purpose, being only about 6".

The change of latitude from A to B may be found as follows:

A point moving along AB from A to B is at first moving at right angles to the meridian of A and its motion has no component along the meridian. As it moves along AB its direction at any point is inclined to the meridian at 90° less the convergence up to that point, until at B its direction is inclined at $(90^\circ - c)$, c being the already computed convergence at B, viz. $30' 42''$. Hence the mean value of the inclination may be taken as $(90^\circ - 15' 21'')$ and the change of latitude ($\Delta\phi$) is, in feet,

$$\begin{aligned} 25 \times 5280 \times \sin 15' 21'' \\ \log (25 \times 5280) &= \log 132,000 = 5.12057 \\ \log \sin 15' 21'' &= \bar{3}.64984 \\ \log \Delta\phi \text{ in feet} &= \frac{2.77041}{2.77041} \\ \Delta\phi, \text{ feet} &= 589.4 \end{aligned}$$

To reduce this to angle, if geodetic tables are available

$$\Delta\phi'' = \frac{\Delta\phi \text{ in feet}}{M \sin 1''}$$

where M is the meridian radius of curvature in the latitude $54^\circ 50'$

$$\begin{aligned} \log \Delta\phi &= 2.77041 \\ \log M \sin 1'' &= \bar{2}.00627 \\ \log \Delta\phi'' &= \frac{0.76414}{0.76414} \\ \Delta\phi &= 5''.81 \end{aligned}$$

B is accordingly $5''.81$ nearer to the equator than A.

The change of longitude ΔL , A to B, was already found to be $37' 33''.9$. If there are no tables available, the mean radius $R = 3958$ miles, substituted for M, will give an approximate result.

The complete solution then is:

Azimuth of line AB at B is $90^\circ 30' 42''.5$

Change of latitude A to B is $- 5''.81$

Change of longitude A to B is $37' 33''.9$ eastwards

Azimuth of line BA, i.e. the reverse azimuth, is

$$90^\circ 30' 42''.5 \div 180^\circ = 270^\circ 30' 42''.5$$

The example just given is a particular case of a common problem, viz. a line AB of given length is set out at a given azimuth α from a point A of known latitude and longitude, find the latitude and longitude of the point B, and the azimuth α' of the line ABC at B, also the reverse azimuth β , viz. that of the line BA.

On a sphere, this problem and others of a similar nature can be solved by the ordinary rules of spherical trigonometry, but there may be a want of accuracy due to the smallness of one or more of the sides or angles involved. The formulae can however be modified

or adapted in such a way as to obviate this, and at the same time to take into account the spheroidal form of the earth, i.e. of the ellipsoid of reference.

For the longest lines used in geodetic triangulation, where the utmost precision is wanted, every refinement of accuracy is used in all measurements, in the adjustment of observational errors and in computation. The formulae used are rather elaborate. These are beyond the scope of the present volume.

Mid-latitude formulae. For lines up to say 30 miles long, simpler formulae have been devised, which will give an accuracy of the order of about $0''.1$ or even closer in low latitudes, which is quite good enough for most purposes.

One set of these formulae, known as the 'mid-latitude' formulae, will be given here.

The problem may be stated thus:

The quantities given are:

ϕ_A . . . the latitude of A

L_A . . . the longitude of A

l . . . the length AB in linear units

α . . . the azimuth of AB at A

The quantities to be found are:

$\delta\phi$. . . the change of latitude A to B (i.e. $\phi_B - \phi_A$)

δL . . . the change of longitude A to B

$\delta\alpha$. . . the change of azimuth A to B (i.e. $\alpha' - \alpha$, the convergence)

For the solution, it is necessary first to find approximate values for $\delta\phi$ and δL , as those quantities are used in the more accurate complete formulae. It is thus a case of successive approximation. Approximate $\delta\phi$ in linear units, say feet, can clearly be found by projecting AB on to AP, the meridian of A; this gives (Fig. 160)

Approx. $\delta\phi = l \cos \alpha$ feet

and if M be the meridian radius of curvature at A the length of $1''$ of meridian arc at A is $M \sin 1''$

$$\therefore \text{Approx. } \delta\phi'' = \frac{l \cos \alpha}{M \sin 1''}$$

The length $M \sin 1''$ of meridian arc (or its logarithm) can be taken from the geodetic tables.

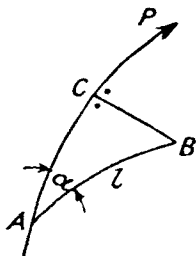


FIG. 160

For δL : the approximate departure is $l \sin \alpha$ feet; the appropriate radius of curvature is that of the prime vertical, i.e. N , and the length of $1''$ of arc is $N \sin 1''$; the departure is $\frac{l \sin \alpha}{N \sin 1''}$ seconds of arc, and the resulting approximate change of longitude is

$$\delta L = \frac{l \sin \alpha}{N \sin 1'' \cos (\phi + \frac{1}{2} \delta \phi)}$$

In this expression the value of $\delta \phi$ just found is used.

The complete formulae known as the mid-latitude formulae are:

$$(a) \quad \tan \frac{1}{2} \delta \alpha = \tan \frac{1}{2} \delta L \sin (\phi_A + \frac{1}{2} \delta \phi) \sec \frac{1}{2} \delta \phi$$

$$(b) \quad \delta \phi = \frac{l}{M \sin 1''} \times \cos (\alpha + \frac{1}{2} \delta \alpha)$$

$$(c) \quad \delta L = \frac{l}{N \sin 1''} \times \frac{\sin (\alpha + \frac{1}{2} \delta \alpha)}{\cos (\phi_A + \frac{1}{2} \delta \phi)}$$

The formula (a) for $\tan \frac{1}{2} \delta \alpha$ is identical with that given above for $\tan \frac{1}{2} c$, c being identical with $\delta \alpha$, the convergence.

RECTANGULAR CO-ORDINATES.* The system of recording and plotting the positions of points on the earth's surface by rectangular co-ordinates is very extensively used and will now be described.

The system of geographical co-ordinates, viz. latitude and longitude, has already been fully explained. In it the reference system consists of a great circle, the equator, and a series of secondaries to it, the meridians, which are great circles perpendicular to the equator. The latitude of a point is its angular distance from the equator, and its longitude is the angle between its meridian and a prime meridian, usually that of Greenwich.

In the system of rectangular co-ordinates, the primary great circle or central meridian is the meridian through a selected convenient point, and the secondaries are great circles perpendicular to this central meridian.

In Fig. 161 POP' represents the central meridian, on which a point O is selected as the origin of co-ordinates. The position of any point, such as A, is defined by its perpendicular distance AM from the central meridian and by the distance OM along the meridian from the origin O to M the foot of the perpendicular. These two co-ordinates OM and AM are the rectangular spherical co-ordinates of A. The perpendicular AM forms part of the great circle

* See also pp. 65-73.

QAR; Q and R are the poles of the central meridian POP' and lie on the equator 90° from POP'. In the figure, A lies to the north and east of O. OM is generally called the x co-ordinate or northing and AM the y co-ordinate or easting, and north and east are taken

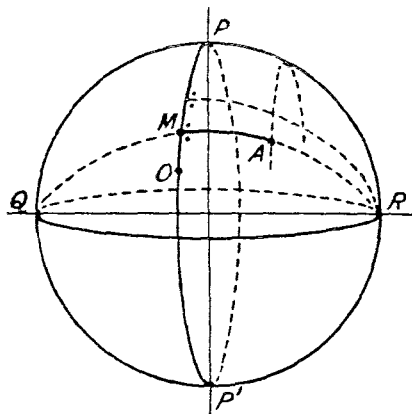


FIG. 161

as the positive directions of these. They correspond to the latitude and departure of plane surveying and are always expressed in linear units, feet or metres. South and west of O are of course negative directions of x and y .

A line set out with a theodolite at M at right angles to the meridian of M, i.e. due east and west at that point, would if produced far enough both ways, meet the equator at Q and R, 90° in longitude from M. All lines set out at right angles to POP' converge to Q and R, just as all meridians converge to the poles.

All points having the same x co-ordinate as A, viz. OM, lie on the great circle QMA, and all points having the same y co-ordinate as A, viz. AM, lie on the small circle through A parallel to POP'. The latitude of any point such as A is clearly less, numerically, than that of M, as M is nearer to the pole P than is A.

Rectangular co-ordinates provide a perfectly definite system of recording the position of points on the earth. If the survey is not a very extended one, the rectangular co-ordinates may be computed and plotted from the measured angles and sides by plane trigonometry but any bearings deduced from these angles will not be

true bearings or azimuths, from which they will differ by the amount of the 'convergence of the meridians' as already explained.

CASSINI PROJECTION. Many of the maps of the Ordnance Survey are drawn on what is called the Cassini or Cassini-Soldner projection. This is a rectangular co-ordinate system, with a central meridian and a series of great circles perpendicular to it, plotted as parallel straight lines; distances along all these great circles are plotted true to scale. On the central meridian a suitable point more or less central to the area to be shown is chosen as the origin of co-ordinates, from which distances along the meridian are plotted to scale. The geodetic tables include one which tabulates lengths of meridian arcs from the origin to each degree of latitude from 49° to 61° , advancing by 10' intervals.

In any representation of a spherical surface on a plane, as in a map, distortion of some kind is unavoidable. The nature of the distortion in the Cassini projection will be explained.

Let OP in Fig. 162 represent the central meridian with O the origin of co-ordinates, A and B two points on the survey equidistant from OP , i.e. the y co-ordinates AM and BN are equal. The x co-ordinates are respectively OM and ON . If the survey be plotted to scale by rectangular co-ordinates, MA and NB will be two parallel straight lines at right angles to OP , each of length to represent y to the chosen scale, and MN will represent $x_2 - x_1$, to the same scale. But MA and NB on the ground are not parallel but are

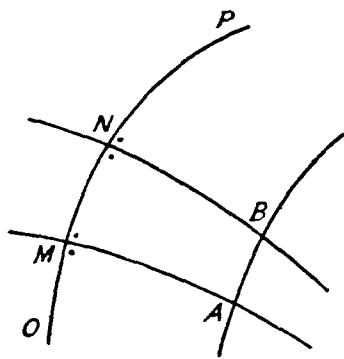


FIG. 162

great circles converging towards a point on the equator, 90° in longitude from OP , where they meet, the rate of convergence being slow at first. On the ground the actual distance AB is less than the distance MN , whereas on the plot the two are equal and parallel. The scale in the north-south direction therefore increases as the survey gets further from the central meridian, but it is correct in the direction of AM and BN and also along the central meridian. There is a 'scale error' which is greatest in lines parallel to the

central meridian, and which also increases with increased distance from the central meridian.

On a sphere, the distance along a parallel of latitude between two meridians is the distance between the same two meridians measured along the equator, multiplied by the cosine of the latitude. Similarly, the distance $AB = MN \times \cos AM$ in angular measure, or

$$AB = MN \cos y \text{ in angular measure}$$

$$= MN \cos \frac{y}{R}$$

where R is the radius of the sphere and

$\frac{y}{R}$ is the angle (in radians) subtended at the centre by AM or BN .

In seconds of arc it is $\frac{y}{R \sin 1''}$.

If geodetic tables are available it is better to take the radius of curvature of the prime vertical section in the latitude of the survey, ν or N , as this is the relevant radius.

It is not necessary to evaluate the angle subtended at the centre:

$$MN = AB \secant \frac{y}{\nu}$$

The first two terms of the expansion of $\sec \frac{y}{\nu}$ are $1 + \frac{y^2}{2\nu^2}$; further terms may be neglected.

Accordingly
$$MN = AB \left(1 + \frac{y^2}{2\nu^2} \right)$$

and this equation gives the exaggeration of scale in the direction AB at the distance y from the central meridian.

In latitude 52° it amounts to rather less than one foot per mile when $y = 75$ miles. The increase of scale in any direction depends on the bearing of the line; it is a maximum for lines such as AB parallel to the central meridian.

The use of the prime vertical radius ν in the formula given in the preceding paragraph implies that account is being taken of the spheroidal form of the earth. For surveys of small extent this is of course quite unnecessary, the area may be treated in all respects as being a plane surface, and lengths and relative bearings and areas computed by plane trigonometry.

From what has been said, it follows that at any point on the map the scale is not the same in all directions, bearings of lines are affected and areas are not true to shape, in other words the projection is not what is called orthomorphic or conformal. Near the central meridian the distortion is negligible. The old 6-inch-to-1-mile maps and a few others of the Ordnance Survey are on the Cassini projection. The area shown on each of the 6-inch maps is limited so that no point on it is more than 75 miles east or west of the central meridian, to keep the scale error within agreed limits. Each area has its own central meridian.

TRANSVERSE MERCATOR PROJECTION.* All or most of the recent maps of the O.S. are on the Transverse Mercator projection, which is derived from the Cassini by applying to it the principle first used by Mercator in producing the projection which is known by his name and which is so useful to navigators. On this, the Mercator projection, as the meridians are shown as parallel straight lines, the parallels of latitude are all equal on the map, whereas on the actual surface they are progressively shortening as their distance from the equator increases. There is, therefore, a progressive increase of scale. Mercator devised the principle of progressively increasing the scale along the meridians, so that at any point the scale along the meridian should be the same as that along the parallel at that point. Small areas are therefore true to shape, i.e. orthomorphic; the map distances between parallels increase with increased latitude and areas on the map in high latitudes are very greatly exaggerated.

The Transverse Mercator projection is one of rectangular coordinates with a central meridian more or less central to the area to be shown, drawn as a straight line. In the Ordnance Survey maps of G.B. it is the meridian 2° W of Greenwich. There is a system of great circles at right angles to the central meridian: these are drawn as straight lines, as in the Cassini projection, but the scale of distance along them is progressively increased, as in the Mercator, so as to agree at any point with the scale parallel to the central meridian at that point (which scale has necessarily increased because the converging east-west lines have been plotted as parallel straight lines). The map is then orthomorphic.

Even in the case of Great Britain, which has a much greater

* See also p. 72.

extent in latitude than in longitude, the scale error at the east and west edges of the map of G.B. would be inconveniently large. To reduce this scale error, the device of introducing a small negative scale error at the central meridian has been adopted, whereby the positive scale error at the points (east and west) most remote from the central meridian is reduced to an unimportant amount — about $\frac{1}{2000}$.

The National Grid. In the National Grid reference system of the O.S. maps of G.B. the central meridian is the meridian 2° W of Greenwich, as in the Transverse Mercator projection to which the grid is applied. The origin of co-ordinates is the point in latitude 49° N on that meridian. The grid itself consists of a series of lines parallel to the central meridian, spaced at equal distances apart, with a series of lines at right angles to the central meridian, spaced at the same equal distances apart, the zero of these passing through the origin. The National Grid is now a metric one, with 10-kilometre squares, subdivided according to the scale of the map. Grid north, being parallel to the central meridian, differs from true north by the amount of the convergence, at any point. It runs east of true north if the point is east of the central meridian, and west of true north if west of the central meridian. On most of the recent maps the amount of this convergence is stated in the margin, either for the two edges, east and west, or for the centre of the map.

To avoid the inconvenience of having negative y co-ordinates, i.e. negative eastings, which would be the case for all that part of Great Britain lying to the west of the central meridian, an amount of + 400 kilometres is added to all easting co-ordinates, thereby making them all positive. Further, in order that no northing or x co-ordinate may exceed 1000 km. all northings are reduced by 100 km. The origin of the grid is by these operations virtually transferred to the point 100 km. north and 400 km. west of the true origin. This point is referred to as the 'False Origin'. The whole of Great Britain lies to the north and east of the 'False Origin', and all co-ordinates are positive. In grid references the easting always precedes the northing.

APPENDIX

SYNOPSIS OF SPHERICAL GEOMETRY AND TRIGONOMETRY

It is assumed that the reader is familiar with the elements of plane geometry and trigonometry. These suffice for the treatment of the survey of a small area, which may be considered plane; i.e. the positions of the various survey points are referred to a surface, supposed plane, by lines perpendicular thereto, without appreciable error.

But when a larger area is under survey certain effects of the curvature of the surface have to be taken into account. The first approximation, which is usually sufficient, treats the surface as spherical; a knowledge of the geometry and the trigonometry of lines and figures on the surface of a sphere is therefore necessary for the surveyor. The essentials will be given in the following paragraphs.

GEOMETRY OF THE SPHERE

Every plane section of a sphere is a circle. If the section passes through the centre of the sphere, the section is a circle of the same diameter as the sphere; it is called a *great circle*. If the cutting plane does not pass through the centre, the section is a circle of less diameter than the sphere, and it is called a *small circle*.

In Fig. A are shown two great circles ABCD and EDFB, from which it is evident that any two great circles must intersect each other in two diametrically opposite points, B and D in the figure, the straight line BOD being the intersection of their planes.

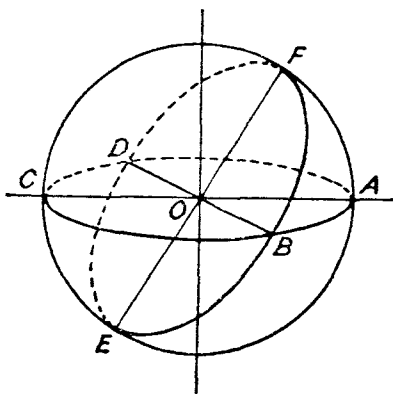


FIG. A

In general, there can be only one great circle joining or passing through two given points on a sphere; it is the section of the sphere by the plane containing the two given points and the centre. In the particular case, where the two points are diametrically opposite to one another, the centre lies on the diameter joining them, and any plane containing that diameter gives a great circle joining the two points; for instance, any plane containing the straight line AOC will cut the surface of the sphere in a great circle joining A and C.

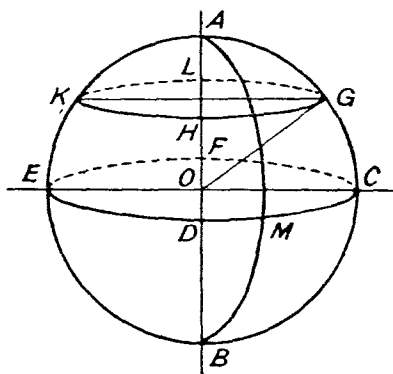


FIG. B

AB being a diameter of the sphere (Fig. B), any plane perpendicular to AB which cuts the sphere does so in a circle of which A and B are called the *poles*. A and B are the poles of the great circle CDEF and also of the parallel small circle GHKL, both of which are sections by planes perpendicular to AB, which is called the *axis*. It may be taken as obvious that all points on a great or small circle are equidistant from the poles of that circle, i.e. that

$$AG = AH = AK = AL$$

and

$$AC = AD = AE = AF = AM$$

Planes containing the axis or diameter AB cut the surface of the sphere in great circles such as AMB which are called *secondaries* to the great circle CDEF, and to any small circles parallel to it, such as GHKL. Their intersections are at right angles. The measure of a great circle arc is the angle it subtends at the centre O; that of the arc AG for instance is $\angle AOG$, and that of AC is 90° or a quadrant.

All secondaries to any great circle, e.g. to CDEF, converge towards the poles A and B of that great circle. Secondaries to AMB, of which CDEF is one, converge towards and meet at the poles of AMB, which are necessarily on the great circle CDEF and 90° from M.

The angle between two great circles is the angle between their planes, or, which is the same thing, the angle between their axes,

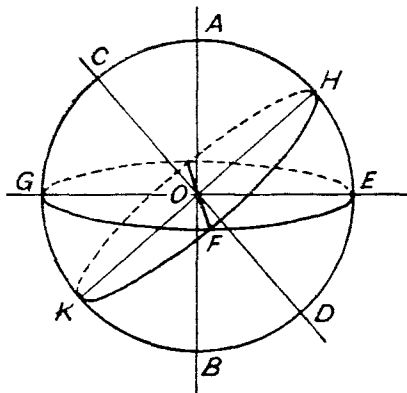


FIG. C

or the angular distance between their poles. In Fig. C, AOB and COD are the axes of two great circles EFG and HFK intersecting at F. AB and CD being in the plane of the diagram, the intersection OF is perpendicular to the plane of the diagram, and F is the pole of the great circle CAEB. The angle between the planes is $\angle HOE = \angle AOC = \text{arc HE} = \angle HFE$. It is the angle at F, i.e. $\angle HFE$, on the surface which is measured in surveying.

SPHERICAL TRIGONOMETRY

A *spherical triangle* is one formed by joining three points on a spherical surface by great circle arcs. The points could of course be joined by other curves, which would be either small circles of the sphere or non-planar or tortuous curves, but the term 'spherical triangle' is restricted as stated. The shortest distance between two points on a sphere is the great circle arc joining them; it is evident that this must be the case, because no other curve joining them, and lying in the surface, can have so large a radius, viz. that of the sphere – be in fact so nearly a straight line.

The three sides and the three angles are all in angular measure, usually degrees, minutes and seconds of angle. The length l of a side in linear measure, if wanted, is the product of the radius R of the sphere and the angular measure θ of the side, in radians. The sum of the three angles is not 180° as in a plane triangle, but exceeds 180° by an amount, called the *spherical excess*, which is proportional to the area of the triangle, for a sphere of given radius. The proof of this will not be given here.

If e be the spherical excess of a triangle it can be shown that

$$e \text{ in radians} = \frac{\text{area of triangle}}{R^2}$$

and
$$e^\circ = 360^\circ \times \frac{\text{area of triangle}}{\text{area of hemisphere}}$$

and
$$e'' = \frac{\text{area of triangle}}{R^2 \sin 1''}$$

It is easily seen that this holds for a triangle such as ABC , Fig. D, where each of the three angles is a right angle, in which case each vertex must be the pole of the opposite side and each side is also 90° .

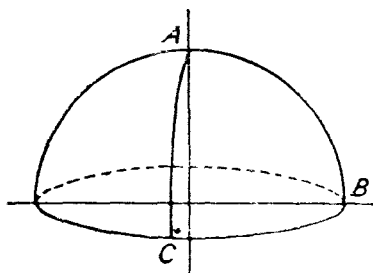


FIG. D

Such a triangle is one quarter of the hemisphere, and the above formula rightly gives the spherical excess as 90° . On the earth, the radius is large and the spherical excess is only $1''$ for an area of about 75 square miles.

The solution of spherical triangles is necessary in many problems of directions and distances on the earth in surveying and in

navigation, and applied to the celestial sphere in the operations of field astronomy. A sufficient knowledge of spherical trigonometry is easily acquired. The formulae are analogous to those of plane trigonometry; they are derived in the first instance by plane trigonometry applied to the appropriate lines and angles on a sphere; no new principles are involved.

SOLUTION OF SPHERICAL TRIANGLES. A few of the general properties of spherical triangles may be stated before giving formulae for their solution:

(a) The sides or angles may have any value up to 360° , but it is always possible and usually convenient, if any part exceeds 180° , to substitute an associated triangle of which the appropriate part is the supplement of the part of the original triangle.

(b) The sum of any two sides exceeds the third side.

(c) The greater side subtends the greater angle.

(d) The sum of the three angles exceeds 180° (by the spherical excess).

Assuming then that no part exceeds 180° , care must be taken that the proper sign is given to the trigonometrical functions of the parts. When any part is determined from its sine, an ambiguity will arise, as the sine is positive from 0° to 180° . Other considerations will then indicate which of the two supplementary angles is the correct one.

Let ABC, Fig. E, represent a spherical triangle. The same notation is used as in plane trigonometry; the sides a, b and c are opposite to A, B and C respectively, and $2s = a + b + c$, the sum of the sides. All the parts are in angular measure.

There are a number of formulae for the solution of spherical triangles. The selection of a suitable one depends both on the data and on the requirements. A few of the more important are given here.

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \quad . \quad (1)$$

with of course corresponding formulae for $\cos b$ and $\cos c$. Formula (1) provides a solution when two sides and the included angle are given. It is not very well adapted to logarithmic computation; the two terms of the right-hand side have to be evaluated separately and added. The sum is the natural cosine of the side a . If there is not a table of natural cosines available, the logarithm of this natural cosine is taken out and the table of log cosines entered with that value. An alternative solution is given by:

$$\cos a = \frac{\cos (c - \phi) \cos b}{\cos \phi} \quad . \quad . \quad . \quad (2)$$

where ϕ is an auxiliary angle found from $\tan \phi = \tan b \cdot \cos A$.

The data included the two sides b and c , and the angle A; the third side a having now been found, other formulae are available to determine the remaining angles B and C if wanted.

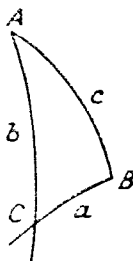


FIG. E

A useful formula, directly applicable where two angles and the side opposite to one of them are given, e.g. A, B and a , is:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad . \quad . \quad . \quad (3)$$

As the required part will be determined from its sine, the ambiguity referred to above may arise as to whether the required part is θ or $180^\circ - \theta$. Usually there will be no difficulty in deciding, from other considerations.

$$\sin \frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \cdot \sin c}} \quad . \quad . \quad (4a)$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \cdot \sin c}} \quad . \quad . \quad (4b)$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin s \sin(s-a)}} \quad . \quad . \quad (4c)$$

These formulae are useful when the three sides are known.

$$\left. \begin{aligned} \tan \frac{1}{2}(A+B) &= \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cdot \cot \frac{1}{2}C \\ \tan \frac{1}{2}(A-B) &= \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cdot \cot \frac{1}{2}C \end{aligned} \right\} \quad . \quad . \quad (5)$$

These formulae are known as Napier's Analogies. They are used when, two sides and the included angle being known, the other two angles are required. If only the third side were wanted, formula (1) or (2) would be better.

A formula called the 'Four Part' or the cot formula is often useful. Four consecutive parts of a spherical triangle are taken in order either way round the triangle, for example:

$$\begin{array}{llllll} b, A, c, B & . & . & . & . & \text{I} \\ \text{or} & & & & & \\ C, a, B, c & . & . & . & . & \text{II} \end{array}$$

The formula referred to is, in words,

$$\cos(\text{inner side}) \cos(\text{inner angle}) = \sin(\text{inner side}) \cot(\text{other side}) - \sin(\text{inner angle}) \cot(\text{other angle}) \quad . \quad (6)$$

Applied to I and to II above, this gives, respectively,

$$\begin{aligned} \text{from I} \quad \cos c \cdot \cos A &= \sin c \cdot \cot b - \sin A \cdot \cot B \\ \text{and from II} \quad \cos a \cdot \cos B &= \sin a \cdot \cot c - \sin B \cdot \cot C \end{aligned}$$

When one part of a spherical triangle is 90° , the formulae are simplified. When one angle is 90° , the simplified formulae can all

be comprehended in two rules, known as 'Napier's Rules for Circular Parts'. Omitting the right angle, the remaining five parts of the triangle are taken in order, either way round the triangle; but for the hypotenuse and for the two oblique angles the complements of these quantities are substituted. The actual values of the two sides which include the right angle are however taken. E.g. if C, in Fig. F, be the right angle, the two sides including it are a and b ; the five quantities taken in order are then a , b , $90^\circ - A$, $90^\circ - c$, $90^\circ - B$. As stated above, C is omitted.

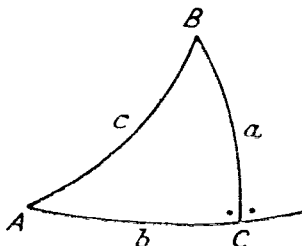


FIG. F

Any one part being called the *middle part*, the two *adjacent parts* are the one immediately before and the one immediately after it. E.g. if $90^\circ - B$ is taken as the middle part, the adjacent parts are $90^\circ - c$ and a (as the series may be supposed continued in the same order). The other two parts are called the *opposite parts*, they are b and $90^\circ - A$. The rules are:

- I. \sin (middle part) = product of tangents of adjacent parts.
- II. \sin (middle part) = product of cosines of opposite parts.

The agreement between the vowels on the right-hand side of each of I and II will be noticed, as an aid to memory.

By taking each part in turn as the middle part all the necessary formulae for the solution of a right-angled triangle are obtained. It is helpful to draw five radiating lines, as in Fig. G, and to write in the five spaces, or on each of the lines, one of the five remaining parts (the right angle being excluded) in order, one in each space, taking care to put the complements of the hypotenuse and the two base angles; calling

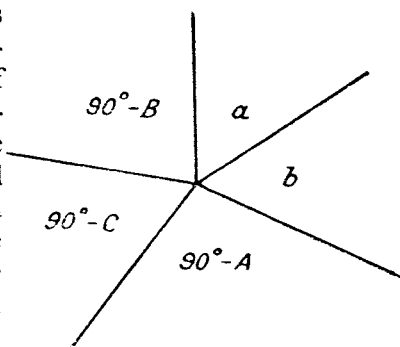


FIG. G

the right angle C. The diagram will be as in Fig. G, and the rules given above can be applied, taking any part as middle part.

For instance, given a and b ; to find A . Three consecutive parts are a , b and $(90^\circ - A)$, in the diagram, and taking b as the middle part

$$\begin{aligned}\sin b &= \tan a . \tan (90^\circ - A) \\ &= \tan a . \cot A\end{aligned}$$

or

$$\cot A = \sin b . \cot a$$

Again, if the three parts (two given and one to find) are B , b , and A , $(90^\circ - B)$ is seen from the diagram to be opposite to b and $(90^\circ - A)$

$$\begin{aligned}\sin (90^\circ - B) &= \cos b . \cos (90^\circ - A) \\ \cos B &= \cos b . \sin A\end{aligned}$$

and similarly for any three parts.

The resulting formulae can of course be obtained from the fundamental general formulae by putting $C = 90^\circ$, but Napier's Rules are simple and convenient. The foregoing formulae are in constant use for the solution of spherical triangles, both on the celestial sphere and on the earth, which, although not perfectly spherical, approximates to a sphere. Where necessary, allowance is made for the departure from that form.

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